



Note: Assume any data required, state your assumption clearly.

Question (1) (25 Marks)

Solve the following equation using Runge Kutta Method

$$\frac{d^2y}{dt^2} + 0.5\frac{dy}{dt} + 7y = 0 \text{ Where } y(0) = 4 \text{ and } \left. \frac{dy}{dt} \right|_{x=0} = 2 \text{ from } t = 0 \text{ to } 5 \text{ using step } 1.$$

Question (2) (25 Marks)

The ideal incompressible flow in a convergent-divergent nozzle can be described by

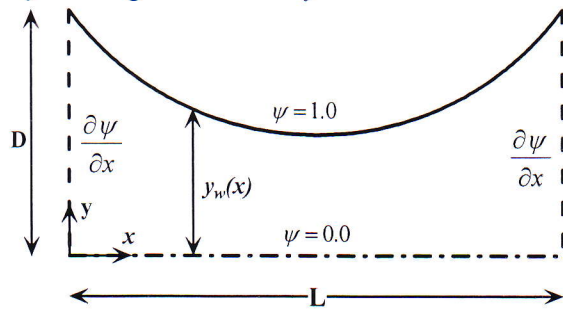
$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0, \text{ where } \psi \text{ is the stream function. Due to flow symmetry, only the upper half of the nozzle}$$

needs to be solved, see the figure. The nozzle wall can be represented by $y_w(x) = D - \frac{D}{2} \sin(\pi \frac{L-x}{L})$. The

velocity distribution through the nozzle can be calculated from $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$, where u and v are

the horizontal and vertical velocity component, respectively. Using the boundary conditions shown in the figure, answer the following:

- Describe the solution procedure of this problem using finite difference method
- Write a computer program for the solution procedure described in a.
- Show how the velocity distribution can be obtained numerically.



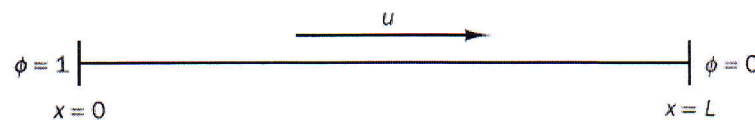
Question (3) (25 Marks)

A property ϕ is transported by means of convection and diffusion through the one-dimensional domain

sketched in the figure. The governing equation is $\frac{d\rho u \phi}{dx} = \frac{d}{dx} \left(\Gamma \left(\frac{d\phi}{dx} \right) \right)$ the boundary conditions are ϕ_0

$= 1$ at $x = 0$ and $\phi_L = 0$ at $x = L$. Using five equally spaced cells and the central differencing scheme for convection and diffusion, calculate the distribution of ϕ as a function of x . The following data apply:

$u = 0.1$ m/s, length $L = 1.0$ m, $\rho = 1.0$ kg/m³, $\Gamma = 0.1$ kg/m.s.



Question (4) (25 Marks)

Drive an expression for pressure correction equation using SIMPLE algorithm and draw a flowchart for the solution of Navier-Stokes equations using this algorithm

GOOD LUCK

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