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MATHEMATICAL MODEL FOR FROST FORMATION ON PLANE SURFACE

أنموذج رياضي لنمو الصقيع على سطح مستو

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يغطي الانموذج المقترح أهمية حساب تأثير نمو الصقيع على جدار المبخر على كفاءة النقل الحراري بين وسط التبريد والهواء الرطب الموجود في مخازن التبريد . وقد صيغت معادلات انتقال الحرارة التي تصف الحالة المدروسة وحولت الى الصورة اللابعدية وشملت مجموعة من الفروض المناسبة لتبسيط المعادلات مما أعطى الفرصة لحلها حلا تحليليا صحيحا . وقد وضع الحل في صورة معادلات لحساب توزيع درجات الحرارة في كل من جدار المبخر وطبقة الصقيع المتكونة ومعامل انتقال الحرارة الكلي وتغير سمك طبقة الصقيع مع الزمن وقد أثبتت النتائج دقة الحل عند مقارنتها ببعض النتائج العملية الموجودة في الابحاث السابقة .

ABSTRACT

Suggested model covers the necessity of calculating the effect of frost formation, on the evaporator wall in cooling storage rooms, on the heat transfer efficiency between refrigerant and wet air. In it obtained analytical relationships for temperature distribution in both wall material and frost layer, overall heat transfer coefficient and the time of frost layer formation as a function of frost layer thickness.

INTRODUCTION

Frost formation on plane wall surface of refrigerating devices, reduces the effectiveness of their work, tends to disturb technological regime of a sectional cooler, over expenditure of electrical energy and deterioration of the performance of cooling plant. In the known methods of heat calculation of air cooler considered the effect of frost on the thermal resistance of frost layer (Δ/k_g). In this case the frost surface temperature was taken equal to the wall surface temperature, which is not corresponding to the true value of the frost surface temperature and leads to an appreciable error in calculations.

Many experimental researches in frost formation on a cooling surface in cryogenic technology and normal cooling were made. Some of these investigations [1-5] have been conducted in cold chambers, where real operating conditions were simulated. The particular relationships given in these works for the change of frost layer thickness and thermal conductivity can be used only for very limited cases of experimental study.

Another parts of these investigations [6-8] was carried out of a stand of the aerodynamic channel type, where the air parameters could be varied within broader limits. Testing by these methods however, has the serious disadvantage that the testing conditions differ, to a greater or less degree, from those of real exploitation.

The chief aim of the work reported here is to give a universal equation for various experimental works on air-cooler and it's design with a predicted change in refrigerating capacity, to do this we should study their dynamical characteristics which can be obtained by arranging the mathematical model of the system (apparatus).

MATHEMATICAL MODEL

Before constructing the mathematical model of frost formation on a plane surface we take into consideration the following assumptions:

1- heating and physical characteristics of frost layer are constant , 2- heat and mass transfer coefficients are constant, 3- no longitudinal heat transfer, and 4- heat flux due to thermal conductivity and diffusion in both longitudinal and lateral directions are negligibly small.

Let us consider a one-dimensional heat transfer process between wet air and a refrigerant through a plane wall and the formed frost layer on it's surface, see Fig. (1), where energy equations and the associated boundary conditions are as follows:

a) for the wall

$$\frac{\partial T_2}{\partial t} = \alpha_2 \frac{\partial^2 T_2}{\partial x^2} , \qquad 0 < x < \delta \qquad (1)$$

$$k_2 \frac{\partial T_2}{\partial x} = h_1 (T_2 - T_1) , \qquad x = 0 \qquad (2)$$

$$T_2 = T_3 , \qquad x = \delta \qquad (3)$$

$$k_2 \frac{\partial T_2}{\partial x} = k_3 \frac{\partial T_3}{\partial x} \qquad x = \delta \qquad (4)$$

b) for frost layer

$$\frac{\partial T_3}{\partial t} = \alpha_3 \frac{\partial^2 T_3}{\partial x^2} , \qquad \delta < x < \delta + \Delta \qquad (5)$$

$$k_3 \frac{\partial T_3}{\partial x} = h_4 (T_4 - T_3) + \rho_3 r \frac{\partial \Delta}{\partial t} , \qquad x = \delta + \Delta \qquad (6)$$

$$k_3 \frac{\partial T_3}{\partial x} = h_4 \beta (T_4 - T_3) \qquad x = \delta + \Delta \qquad (7)$$

where:

$$\beta = 1 + \frac{(d_4 - d_3)(r - I_f)}{c_3 (T_4 - T_3)} , \qquad I_f = c_3 T_3 \qquad (8)$$

It is convenient to write the above equations and the corresponding boundary conditions in the dimensionless form using the following dimensionless groups:

$$\Theta_2 = (T_2 - T_1)/(T_4 - T_1), \quad \Theta_3 = (T_3 - T_1)/(T_4 - T_1), \quad X = x/b, \quad S = \Delta/b,$$
$$Z = \delta / b, \quad \alpha_{32} = \alpha_3 / \alpha_2 , \quad Bi_2 = h_1 b/k_2 , \quad Bi_3 = h_4 b/k_3 ,$$
$$k23 = k_2/k_3, \quad \tau = t/\tau o , \quad \varepsilon = c_3 (T_4 - T_1)/r = b^2 / \alpha_3 \tau o . \qquad (9)$$

The characteristic time (τo) is defined as the ratio between the energy stored in the frost layer during it's formation to the heat flux through the maximum frost layer thickness and the wall. It can be determined from the following relation:

$$\tau o = \frac{\rho_3 b^2 r}{k_3 (T_4 - T_1)} \qquad (10)$$

Equations (1-8) may be rewritten in the following forms:

a) for the wall

$$\varepsilon \alpha 32 \frac{\partial \Theta_2}{\partial \tau} = \frac{\partial^2 \Theta_2}{\partial X^2} , \qquad 0 < X < Z \qquad (11)$$

$$\frac{\partial \Theta_2}{\partial X} = Bi_2 \Theta_2 , \qquad X = 0 \qquad (12)$$

$$\Theta_2 = \Theta_3 , \qquad X = Z \qquad (13)$$

$$k23 \frac{\partial \Theta_2}{\partial X} = \frac{\partial \Theta_3}{\partial X} , \qquad X = Z \qquad (14)$$

b) for the frost layer

$$\varepsilon \frac{\partial \Theta_3}{\partial \tau} = \frac{\partial^2 \Theta_3}{\partial X^2} \qquad Z < X < Z+S \qquad (15)$$

$$\frac{\partial \Theta_3}{\partial X} = - Bi_3 (\Theta_3 - 1) + \frac{\partial S}{\partial \tau} , \qquad X = Z + S \qquad (16)$$

$$\frac{\partial \Theta_a}{\partial X} = - Bi_a (m \Theta_a - n) \quad , \qquad X \geq Z + S \tag{17}$$

where:

$$m = 1 + \frac{2.1 (d_s - d_a)}{c_a} \tag{18}$$

$$n = 1 + \frac{2.1 \,(d_s - d_a)}{c_a} + \frac{(d_s - d_a)\,(r - 2.1\, T_s)}{c_a \,(T_s - T_f)} \tag{19}$$

As τ is very high value i.e $\varepsilon \ll 1$ and tends to zero and equations (11,15) can be written in the following form neglecting the left hand sides:

$$\frac{\partial^2 \Theta_z}{\partial X^2} = 0 \, , \qquad \frac{\partial^2 \Theta_a}{\partial X^2} = 0 \tag{20}$$

The solution of the above two equations by integration can be written in the following forms:

$$\Theta_z = A_1 X + B_1 \quad , \qquad \Theta_a = A_2 X + B_2 \tag{21}$$

where A1, A2, B1 and B2 are constants of integration. They can be determined using the boundary conditions given by equations (12-14,16-17). The final solution may be written in the following forms:

$$\Theta_z = \frac{n \,(X + 1/Bi_z)}{m \; k_{za} (S + F_1)} \tag{22}$$

$$\Theta_a = \frac{n \,(X + F_2)}{m \,(X + F_1)} \tag{23}$$

where:

$$F_1 = \frac{Z}{k_{za}} + \frac{1}{m \, Bi_a} + \frac{1}{k_{za} \, Bi_z} \, , \qquad F_2 = \frac{Z}{k_{za}} + \frac{1}{k_{za} \, Bi_z} - Z \tag{24}$$

The maximum frost layer thickness (b) exists when $dS/d\tau = 0$ at $S = 1$ (i.e $\Delta = b$), applying this condition to equation (16) gives

$$b = \frac{k_a \,(n - 1)}{h_s \,(n - m)} - \frac{k_a}{h_1} - \frac{\delta}{k_{za}} \tag{25}$$

The running time from the beginning of the cooling process to the formation of frost layer of thickness Δ can be determined by integrating equation (16), with the initial condition that $\tau = 0$ at $S = 0$, as follows:

$$\tau = \frac{m}{Bi_a\,(n - m)} \left[S + (F_1 - F_a)\, Ln \left(\frac{S + F_a}{F_a} \right) \right] \tag{26}$$

where:

$$F_a = \frac{Z}{k_{za}} + \frac{1}{k_{za}\, Bi_z} + \frac{(n - 1)}{Bi_a\,(n - m)} \tag{27}$$

The overall heat transfer h as a function of τ can be determined using the following formula:

$$\frac{h\, b}{k_a} = \frac{1}{\frac{1}{k_{za}\, Bi_z} + \frac{1}{Bi_a} + \frac{Z}{k_{za}} + S} \tag{28}$$

with $S = f(\tau)$ as seen from equation (26).

RESULTS AND DISCUSSION

The results of calculation, using IBM personal computer, acknowledge the usefulness of the given solution for the prediction of frost formation process, because they agreed with the empirical formulas given in the experimental works [2-5] as shown in figures (2,3).

In comparing with other works we use the practical values for the different parameters used in the frost formation on the surface of air coolers working in the regime of freezing product given in the experimental works [1,4] as follows:

$T_4 = -30\ ^{\circ}C$, $T_1 = -40\ ^{\circ}C$, $k_b = 0.15$ W/m K , $k_2 = 45$ W/m K ,

$\rho = 900$ kg/m^3 , $\rho_a = 52$ kg/m^3 , $r = 2.8262 \times 10^6$ J / kg ,

$h_1 = 800$ W/m^2 K , $h_4 = 40$ W/m^2 K , and $\delta = 0.005$ m .

The results of the exact solution given in this work, formulas (22-28) for the different given data lead to the following conclusions:

1- wall thickness, of a good thermal conductive material k_2, has no effect on the process of frost formation.

2- increasing the convective heat transfer coefficient in wet air side (high h_4), maximum frost layer thickness (b) decreases (nearly a proportion to $1/h_4$), and the effectiveness of heat transfer increases with the increase of h_4.

3- variation of convective heat transfer coefficient in the refrigerant side (h_1) in the range of it's practical values has no effect on the process of frost formation.

NOMENCLATURE

b maximum frost layer thickness, m

Bi Biot number, h b / k
c specific heat , J / kg K
d partial mass transfer, kg/kg
F_1, F_2 constants, defined by equation 24
F_3 constant, defined by equation 26
h film heat transfer coefficient, W/m K
I enthalpy, J/kg
k thermal conductivity, W/m^2 K
k_{23} dimensionless thermal conductivity, (k_z/k_3)
m constant, defined by equation (18)
n constant, defined by equation (19)
r latent heat of evaporation, J/kg
S dimensionless frost layer thickness, Δ/b
t time, s
T temperature, K
x coordinate, m
X dimensionless coordinate, x/b
Z dimensionless wall thickness, δ/b

GREEK SYMBOLS

α thermal diffusivity, m^2/s
α_{32} relative thermal diffusivity, (α_3/α_2)
β moisture separation coefficient,
δ wall thickness, m
Δ frost layer thickness, m
ε dimensionless term, defined by equation (9)
θ dimensionless temperature,
ρ density, kg/m^3
τ_o characteristic time, defined by equation (10)
τ dimensionless time, t/τ_o

SUBSCRIPTS

1 refrigerant (free stream)
2 wall
3 frost layer
4 medium to be cooled (free stream)

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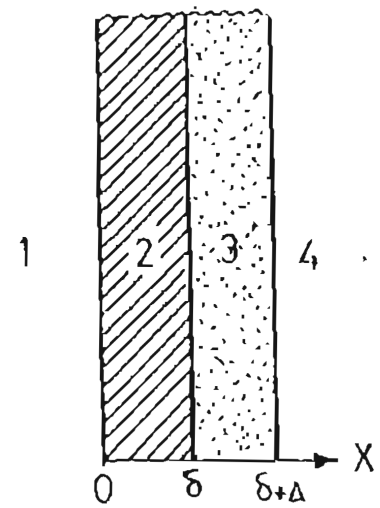

Fig. (1) Frost formation scheme on a flat surface

1- refrigerant field
2- flat wall,
3- frost layer
4- wet air cooling field

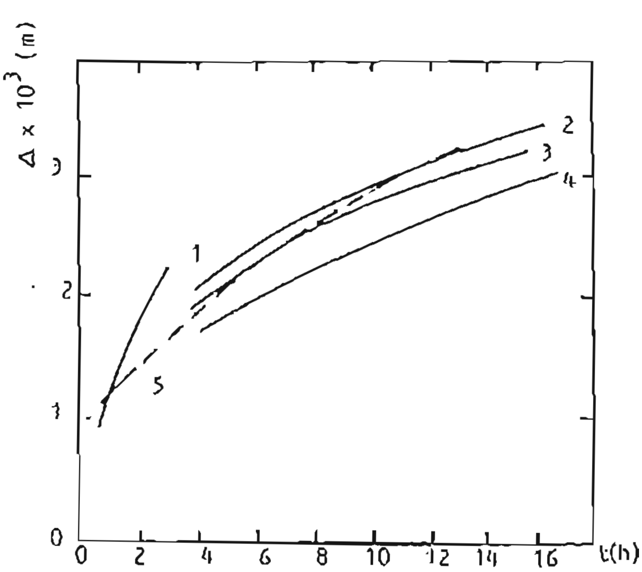

Fig. (2) Growth of frost layer as a function of time

1- Ref [4], 2- Ref [1], 3- Ref [5], 4- Ref [2] and 5- Present work

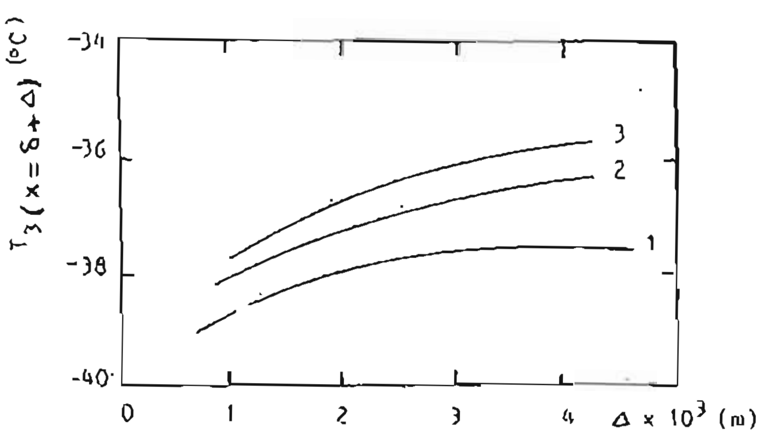

Fig. (3). Relation between temperature of frost layer surface (T_3) and frost layer thickness (Δ)

1- Ref [3], 2- Ref [4] and 3- Present work