Menoufiya University
Faculty of Engineering Shebin El-Kom
Des. & Prod. Eng. Department
First Semester Examination 2014-2015

Subject: Math. (3)
Code: BES 213
Time Allowed: 3 hours
Total Marks: 100 marks
Date of Exam: 27/1/2016

**Solve the Following Questions**

**(Question Number-1):(35 Marks)**

(A) Verify Stokes' theorem for $\bar{F}=\left(x^3+\frac{yz^2}{2}\right)\bar{i}+\left(y^2+\frac{xz^2}{2}\right)\bar{j}+(xyz)\bar{k}$ where $S$ is surface of the cube $x=0, y=0, z=0, x=3, y=3, z=3$ above the y-z plane.

(B) Find the unit normal vector and the surface area of $z=\sqrt{x^2+y^2}$ over the region D bounded by $0\le x\le 4,\ 1\le x\le 6$ .

(C) - If $\Gamma(1.6)=0.8935$ find $\Gamma(2.6)$, $\Gamma(-1.4)$, $\int_0^2 (4-x^2)^{3/2}\,dy$ and $\int_0^\infty y^{\frac{1}{2}}e^{-y^3}\,dy$ .

- Prove that $\beta(m,n)=2\int_0^{\frac{\pi}{2}}(\sin\theta)^{2m-1}(\cos\theta)^{2n-1}\,d\theta$ , and evaluate $\int_0^{\frac{\pi}{2}}\sqrt{\cot}\,d\theta$

**(Question Number-2):(25 Marks)**

(A) If $\phi=\frac{1}{\left|x\bar{i}+y\bar{j}+z\bar{k}\right|}$, prove that $grad\,\phi=-\frac{\bar{r}}{r^3}$.

(B) Show that $\bar{a}\cdot\left(\bar{b}\times\bar{c}\right)$ is in absolute value equal to the volume of a parallelepiped with sides $\bar{a}$ , $\bar{b}$ , and $\bar{c}$ .

(C) If $N(x,y)$ is defined and continuous function having continuous first partial derivatives in a closed region R bounded by C, prove that $\iint_R \frac{\partial N}{\partial x}dydx=\oint_C N(x,y)\,dy$. Why Green's theorem not applicable to the integral $\oint_C \frac{y}{x^2+y^2}dx-\frac{x}{x^2+y^2}dy$ where C is the ellipse $x^2+4y^2=4$?.

**(Question Number-3):(40 Marks)**

(A) Show that $\bar{F}(x,y,z)=\left(2xy+z^3\right)\bar{i}+x^2\bar{j}+3xz^2\bar{k}$ is conservative force field and find the scalar potential and find the work done in moving an object in the field from $(1,-2,1)$ to $(3,1,4)$.

(B) Verify divergence theorem for $\bar{F}=2x^2y\bar{i}-y^2\bar{j}+4xz^2\bar{k}$ taken over the region in the first octant bounded by $y^2+z^2=9\quad x=1$.

(C) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $x^2+y^2-z=3$ at the point $(1,1,1)$.

(D) By the simplex method, find $x_1$ and $x_2$ that maximize the sum $x_1+x_2$ subject to the constraints $x_1\ge 0,\ x_2\ge 0$, and

$$x_1+2x_2\le 4$$
$$4x_1+2x_2\le 12.$$
$$-x_1+x_2\le 1$$

With my best wishes

Dr. M.A. El- Shorbagy

| This exam contributes " by measuring in achieving Programme Academic Standards according to NARS | | | | | | | |
|---|---|---|---|---|---|---|---|
| Question Number | Q1-A | Q1-B,C | Q2, Q3-D | Q3 | Q3-D | | Q2-B, Q3-A |
| Skills | a-1-1, a-1-2, a-1-3 | a-8-1 | a-1-3 | b-3-1 | b-7-1 | | c-1-1 |
| | Knowledge & Understanding Skills | | | Intellectual Skills | | | Professional Skills |