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TWO-DIMENSIONAL TEMPERATURE DISTRIBUTION IN NUCLEAR FUEL RODS

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ABSTRACT

The aim of this work is to propose a two-dimensional analytical model for prediction of the temperature distribution in a nuclear fuel rod in steady operation and in transient conditions. In this model, the cooling rate around the cladding is considered to be nonuniform. As a calculation example, temperature distribution in a nuclear fuel rod of a 900 MWe pressurized water reactor is predicted for quasi-steady operation. It was found that the cladding temperature reaches values which are nearly 223 C higher than that obtained for uniform cooling.

INTRODUCTION

The surface heat flux in an operating water reactor is of the order of 1 MW/m2 [1]. Such heat fluxes can be removed quite satisfactory using flowing water (or steam water mixture) at velocities of a few meters per second. In this case, the temperature difference between the cladding of the fuel rod and the coolant will be quite low-a few tens of degrees. If, however, the surface heat flux becomes too high or alternatively the coolant too low (or the steam content too high in a boiling system), then overheating of the fuel material and cladding can occur. This overheating occurs at a particular set of thermal and hydraulic transients [1,2,3].

To date, the thermal and hydraulic response of a nuclear core in normal and transient operation has been primarly demonstrated by computer anylysis. The validation of the anlytical models used is the main purpose of many experimental facilities [4,5].

One of the most important fields of study is the temperature behaviour in fuel and cladding materials during normal and transient operating conditions. In many computer codes, the temperature behaviour in the fuel rod has been treated as one dimensional problem in which the cooling rate around the cladding is considered uniform [2, 4, 5, 6]. In many one dimensional codes, solution of the discretization equations has been obtained using the fully explicit scheme which is conditionally stable [2]. In other codes the solution is performed using the fully implicit scheme [6, 7].

In the present paper, a two-dimensional analytical model for prediction of the temperature behaviour in nuclear fuel rods under steady and transient conditions is developed. In this code the heat transfer rate around the cladding is considered non-uniform.

MATHEMATICAL MODEL

The differential equation which governs the heat conduction with internal heat generation in unsteady-state conditions is:

$$\rho c(\partial T/\partial t) = (1/r)\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) + (1/r)\frac{\partial}{\partial \theta}\left(\frac{K}{r}\frac{\partial T}{\partial \theta}\right) + S \qquad (1)$$

To solve the above equation numerically, fuel and cladding materials are divided into differential volumes. Grid points and control volume faces are shown in Fig. 1 in polar coordinates. Thickness of the control volume in the third direction z, is considered unity. Multiplying Eq.1 by rdt dr dθ and integrating with respect to r, θ, t over the control volume $r\Delta r\Delta\theta$ and over the time intevral Δt, one gets the discretization for the considered volume. In the following each term in Eq.1 will be integrated separatly, and the order of integration is chosen according to the nature of the term.

I- Change of energy stored in the volume element:

To represent the term $\partial T/\partial t$, the grid point value of T is assumed to prevail through the volume element $r\Delta r\Delta\theta$, then one has

$$\int_{\theta}^{\theta+\Delta\theta}\int_{r}^{r+\Delta r}\int_{t}^{t+\Delta t} \rho c r(\partial T_{i,j}/\partial t)\, dt\, dr\, d\theta$$

$$= \rho c(T_{i,j} - T^{0}_{i,j})\, r\,\Delta r\Delta\theta \qquad (2a)$$

where $T_{i,j}$ is the new value and $T^{0}_{i,j}$ is the original value of the grid point temperature.

II- Net heat flow in the radial direction

To get an expression for the net heat flow in both radial and tangentional directions, the following reasonable relation is applied

$$\int_{t}^{t+\Delta t} T\, dt = T\,\Delta t \qquad (2b)$$

According to Eq.2b, it is assumed that the new value of T prevails throughout the entire time step t (fully implicit scheme). Applying Eq.2b to the first term in the right hand side of Eq.1, one obtains

$$\int_{t}^{t+t}\int_{\theta}^{\theta+\Delta\theta}\int_{r}^{r+\Delta r}\frac{\partial}{\partial r}\left(rk\frac{\partial T_{i,j}}{\partial r}\right) dr\; d\theta\; dt$$

$$= \int_{t}^{t+\Delta t}\left[\left(kr\frac{\partial T}{\partial r}\right)_{r_{i+1}} - \left(kr\frac{\partial T}{\partial r}\right)_{r_{i}}\right]\Delta\theta\, dt \qquad (2c)$$

The term $(kr\frac{\partial T}{\partial r})_{r_i}$ in Eq. 2c can be expressed in terms of the neighbor temperatures as follows:

$$-(kr\frac{\partial T_{i,j}}{\partial r})_{r_i}\Delta\theta = CR_{i-1,i}(T_{i-1,j} - T_{i,j}) \qquad (2d)$$

where $CR_{i-1,i}$ is the heat conductance at the interface i-1,i and is given by

$$CR_{i-1,i} = \frac{r_i\Delta\theta}{\left[\frac{\Delta r}{2k_{i-1}} + \frac{1}{h_{i-1,i}} + \frac{\Delta r}{2k_i}\right]} \qquad (2e)$$

Similarly, an expression for the term $(kr\frac{\partial T}{\partial r})_{r_{i+1}}$ can be obtained. It is given by

$$-(kr\frac{\partial T}{\partial r})_{r_{i+1}}\Delta\theta = CR_{i,i+1}(T_{i,j} - T_{i+1,j})$$

Substituting in Eq.2c one gets the following expression for the net heat flow in r-direction

$$=[CR_{i,i+1}T_{i+1,j} + CR_{i-1,i}T_{i-1,j} - CR_{ii}T_{i,j}]\Delta t \qquad (2f)$$

where $CR_{ii} = CR_{i,i+1} + CR_{i,i-1}$

III- Net heat flow in the tangentional direction

Following the same procedure in item II, the net heat flow in the θ-direction is given by

$$=[CT_{j,j+1}T_{i,j+1} + CT_{j-1,j}T_{i,j-1} - CT_{jj}T_{i,j}]\Delta t \qquad (2g)$$

where the heat conductance $CT_{j-1,j}$ at the interface j-1,j is given by

$$CT_{j-1,j} = \frac{\Delta r}{\left[\frac{r_i\Delta\theta}{2k_{j-1}} + \frac{1}{h_{j-1,j}} + \frac{r_i\Delta\theta}{2k_j}\right]}$$

and

$$CT_{jj} = CT_{j,j+1} + CT_{j-1,j}$$

IV- Heat generation

Assuming that the thermal source strength S is uniform throughout the volume element and over the time step Δt, one obtains

$$\int_{t}^{t+\Delta t}\int_{\theta}^{\theta+\Delta\theta}\int_{r}^{r+\Delta r} S\, r\, dr\, d\theta\, dt = S\, r\Delta\theta\, \Delta r\, \Delta t = S\,\Delta v\, \Delta t \tag{2h}$$

Substituting from Eqs. 2a, 2f, 2g, 2h in Eq.1 and rearranging one obtains the discretization equation for the grid point i,j in the form

$$A_{ij}\, T_{i,j} = CR_{i,i+1}\, T_{i+1,j} + CR_{i-1,i}\, T_{i-1,j} + CT_{j,j+1}\, T_{i,j+1} + CT_{j-1,j}\, T_{i,j-1} + d_{ij} \tag{3}$$

where

$$A_{ij} = CR_{i,i+1} + CR_{i-1,i} + CT_{j,j+1} + CT_{j-1,j} + (\rho c \Delta v / \Delta t)$$

and

$$d_{ij} = S\Delta V + (\rho c \Delta v / \Delta t)\, T^{0}_{i,j}$$

Solution of Eq.3 is performed for the following boundary conditions (Fig.1):

- The temperature gradient at the center line of the rod $(\partial T/\partial r)_{r=0}$ is approximatly zero, which is a reasonable assumption especially for uniform cooling. To satisfy this condition, the coefficients $CR_{0,1}$ and $CR_{1,2}$ must have the following values:

$$CR_{0,1} = 0.0$$

$$CR_{1,2} = \frac{r_2\,\Delta\theta}{\left[\dfrac{\Delta r}{2k_1} + \dfrac{1}{h_{1,2}} + \dfrac{\Delta r}{2k_2}\right]}$$

- At the outer surface of the cladding

$$-k\,(\partial T/\partial r)_{r_0} = \alpha\,(T_N - T_F)$$

where α is the convective heat transfer coefficient between cladding surface and coolant of temperature T_F. To satisfy this condition, the coefficient $C_{N,N+1}$ must have the value given by

$$CR_{N,N+1} = r_{N+1}\,\alpha\,\Delta\theta = r_0\,\alpha\,\Delta\theta$$

- At the diameter AC

$$(\partial T/\partial\theta)_{\theta=0} = (\partial T/\partial\theta)_{\theta=} = 0.0$$

To satisfy the above condition, the coefficients CT have the following values:

$$CT_{0,1} = CT_{M,M+1} = 0.0$$

Solution of the discretization equations is performed by using the line-by-line method which converges fast [7]. Solution of the temperature distribution in each line is obtained using the TDMA method explained in [6,7]. To account for the temperature dependent properties of the fuel and cladding materials, a few internal iterations are required.

As a calculation example, the above developed analysis is applied to predict the temperature behaviour in a fuel rod of a 900 MWe pressurized water reactor.

RESULTS AND DISCUSSION

The following data are used [8]:

volumetric thermal source strength $s = 7.2x10^{8}$ w/m^{3}

cladding outside radius $R_{O} = 4.68x10^{-3}$ m

cladding inside radius $R_{i} = 4.18x10^{-3}$ m

coolant pressure and temperature $P = 15$ MPa

$T_{f} = 300$ C

Fuel radius $R_{f} = 4.0985x10^{-3}$ m

material properties:

	Density, Kg/m3	specific heat J/KgK	thermal conductivity, W/mK
fuel material (UO_2)	$1.02x10^{4}$	296	2.59
cladding material (Zircalloy)	$6.3x10^{3}$	319	15.13

For simplicity, properties of the fuel and cladding materials are considered to be temperature independent. Two values for the heat transfer coefficient are chosen to apply as shown in Fig. 3 and given by

$$\alpha_{1} = 4x10^{-3} \; w/m^{2} K$$

$$\alpha_{2} = 4x10^{4} \; w/m^{2} K$$

Figs. 1,2 illusterate the temperature behaviour in the fuel material for different number of grid design. It is clear that for number of radial layers NF equals or more than 20, the solution is convergent. The gain in the accurcy obtained for higher values of NF is on the expense of the calculation time. The figure also indicates that the fuel in the region of inefficient cooling is exposed to higher level of temperature which is physically expected.

Figure 4 shows the temperature behaviour of the cladding surface. The figure indicates that the cladding material is subjected to higher values of temperature where the heat transfer rate is low. In this case, the difference between the maximum and minimum cladding temperatures is about 223 C. This means that the cladding material is subjected to severe thermal stresses.

In comparison with other results, the temperature of the cladding in the region where the high value of heat transfer coefficient applies is 325 C which is nearly the same value obtained using the one dimensional code [6] for the same reactor.

CONCLUSION

The proposed analytical two dimensional model is a highly efficient tool to investigate the temperature behaviour in a nuclear fuel rod. With some development in the proposed code, deformation of the cladding and fuel material can be taken into consideration.

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NOMENCLATURE

A, d	Coefficients
CR, CT	Radial and tangentional heat conductance
C	Specific heat
r, θ	Polar coordinates
K	Thermal conductivity
s	Volumetric thermal source strength
t	Time
T	Temperature
r_f	Fuel radius
r_i, r_o	Inside and outside radius of cladding
ρ	Density
h, α	Heat transfer coefficient

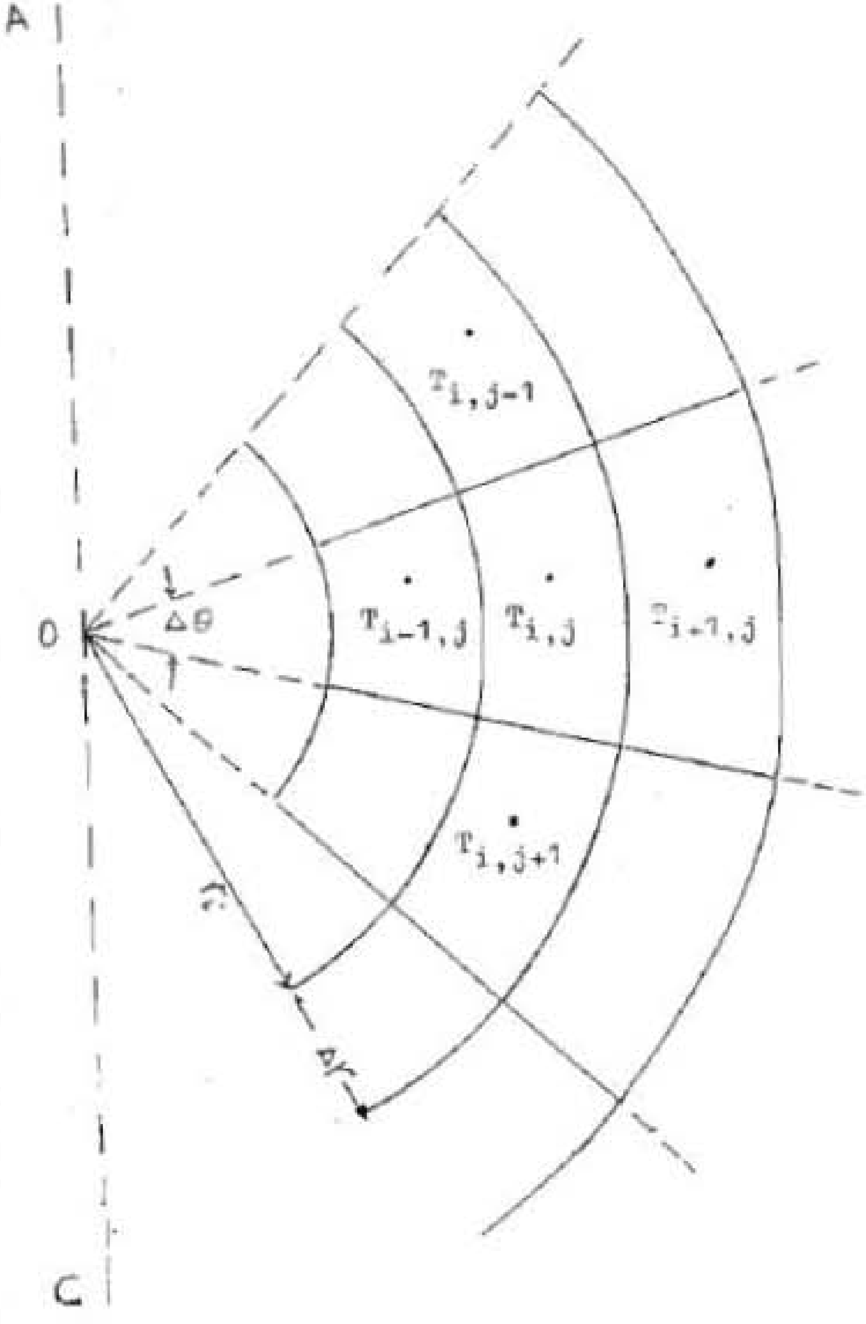

Fig.1 Grid design

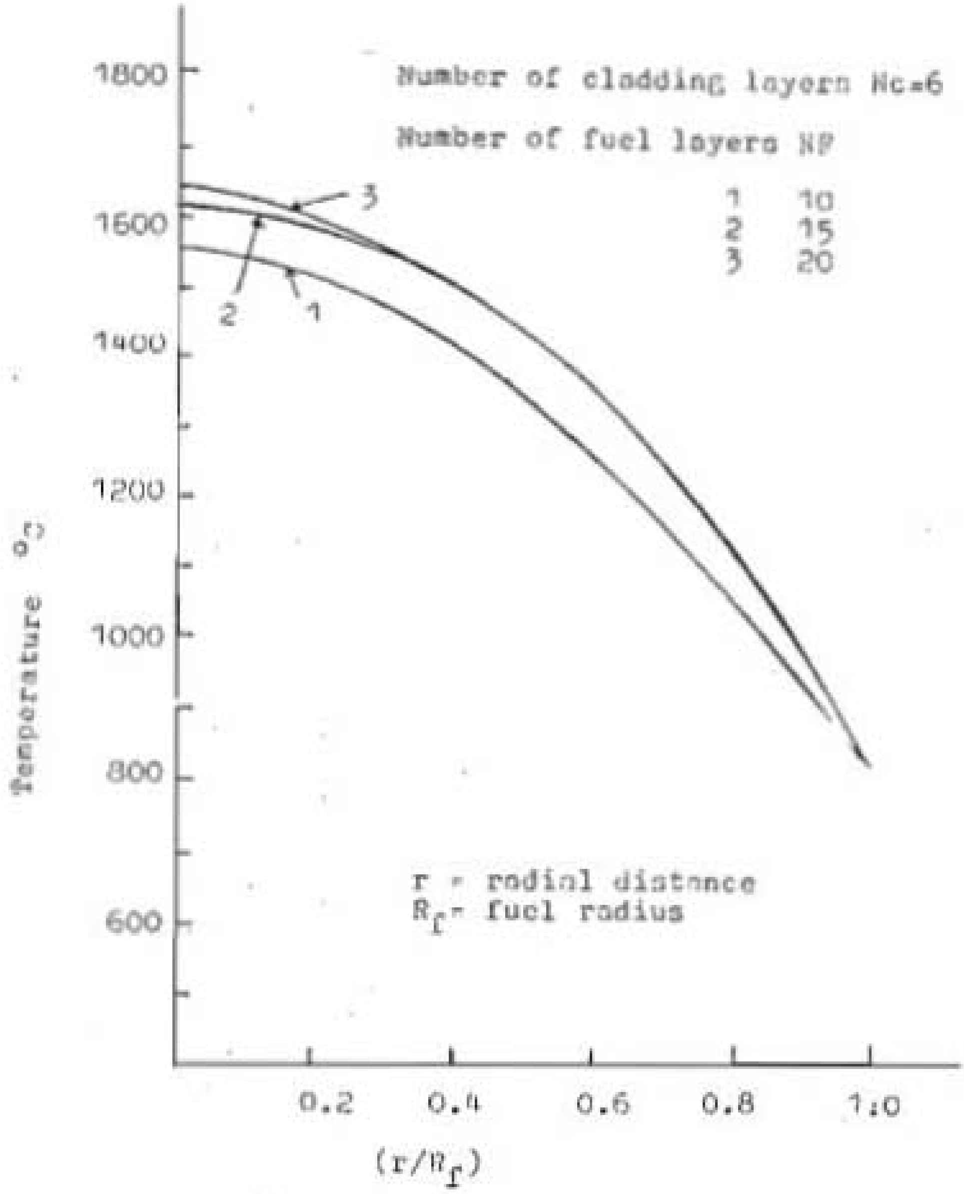

Fig.2 Temperature behaviour of the fuel material

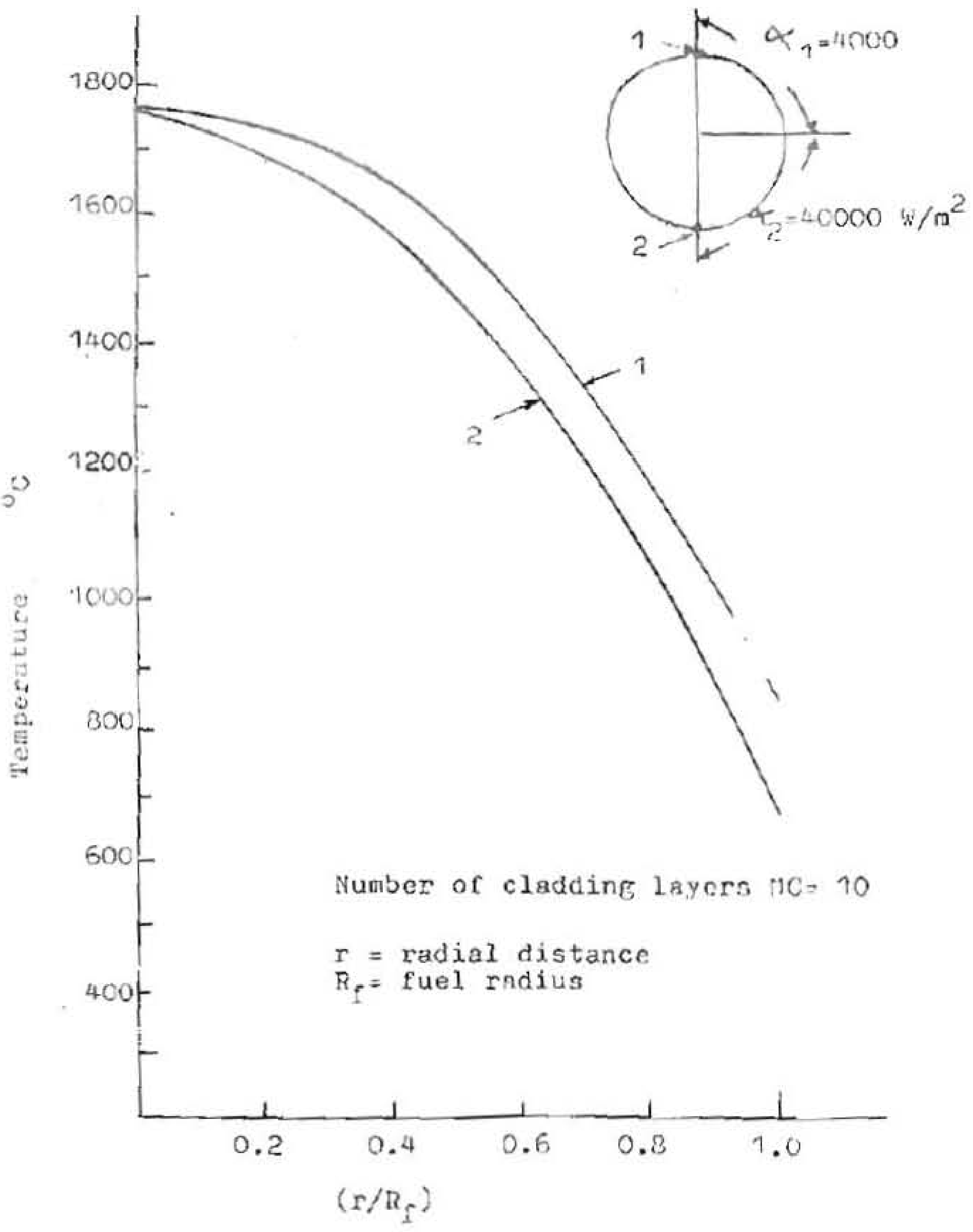

Fig.3 Temperature behaviour of the fuel material for different azimuth

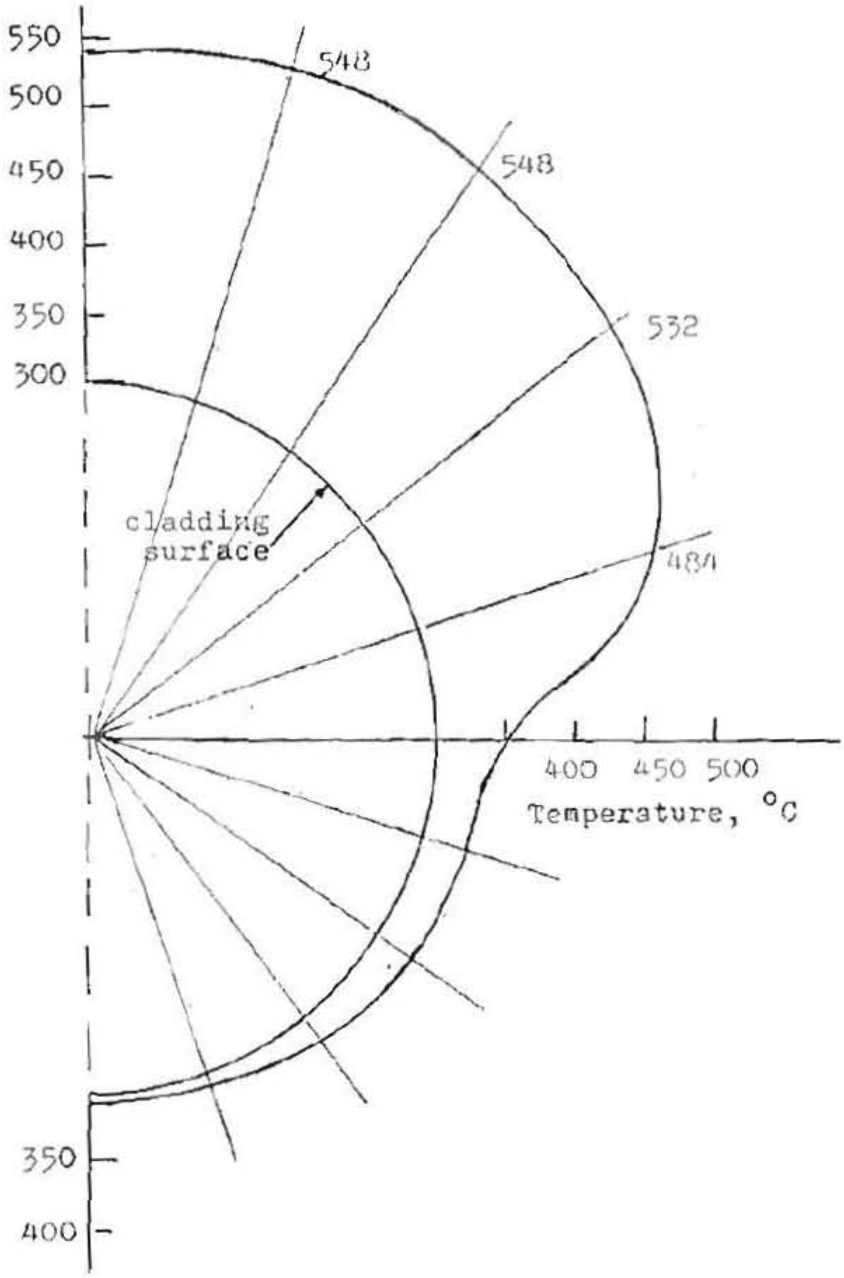

Fig.4 Temperature behaviour of the cladding surface