



**Answer the following questions**

**(Question 1)**

a) Define each of the following expressions:

Algorithm- Method- Technique- Heuristic- Metaheuristic.

b) State the differences between the traditional algorithms and metaheuristic algorithms.

c) Explain the Basics of Game theory.

d) What are the necessary and sufficient conditions for the multi-variable optimization problem without constraints?

e) Determine the minimum value of the function

$$f(x) = x_1^2 + x_2^2 - 2x_1 - 4x_2$$

Subject to:

$$g_1(x) = x_1 + 4x_2 \leq 5$$

$$g_2(x) = 2x_1 + 3x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

Start from the point  $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

f) Find the minimum value for the function  $f(x) = 2x_1^2 + 2x_1x_2 + x_2^2 + x_1 - x_2$  using the Steepest Descent (Cauchy) Method, start from the point (0, 0).

g) Find the dimensions of a box of largest volume that can be inscribed in a sphere of unit radius.

h) Determine the maximum and minimum values of the function

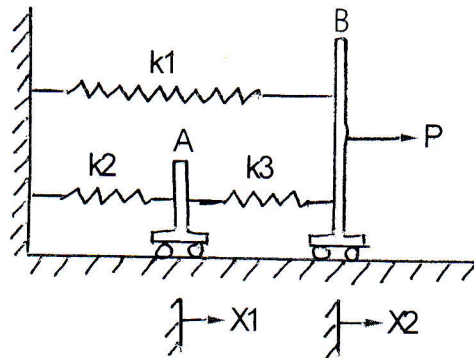
$$f(x) = 12x^5 - 45x^4 + 40x^3 + 5$$

i) How do you test the positive, negative, or indefiniteness of a square matrix [A]? Then what is the type of the following matrix?

$$A = \begin{bmatrix} 4 & -3 & 0 \\ -3 & 0 & 4 \\ 0 & 4 & 2 \end{bmatrix}$$

**(Question 2)**

- a) The following Figure shows two frictionless rigid bodies (carts) A and B connected by three linear elastic springs having spring constants  $k_1$ ,  $k_2$ , and  $k_3$ . The springs are at their natural positions when the applied force  $P$  is zero. Find the displacements  $x_1$  and  $x_2$  under the force  $P$  by using the principle of minimum potential energy.



- b) Find the function  $x(t)$  that minimizes the following cost functional  $J = \int_{-1}^1 x(t) dt$  Subject to:

$$\int_{-1}^1 [1 + \dot{x}^2(t)]^{\frac{1}{2}} dt = 1$$

- c) Solve the following multiple criteria decision problem by the weighted method, assume that the decision maker gives the weights,  $w_1 = 0.6$  and  $w_2 = 0.4$  to indicate the importance of each objective

$$\text{Max } f_1 = 0.4x_1 + 0.3x_2$$

$$\text{Max } f_2 = x_1$$

subject to:

$$x_1 + x_2 \leq 400,$$

$$2x_1 + x_2 \leq 500,$$

$$x_1 \geq 0, x_2 \geq 0.$$

- d) Solve the following problem by using  $\epsilon$ -constraint method

$$\text{Min } f_1 = x^4$$

$$\text{Min } f_2 = (x - 2)^4$$

subject to:

$$-4 \leq x \leq 4$$