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ESTIMATION OF MEAN VELOCITY IN MOUNTAINOUS STREAMS AND THE EFFECT OF SLOPE AND FROUDE NUMBER

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تقدير السرعة المتوسطة في المجاري الجبلية وتأثير الميل ورقم فراود

الخلاصة:

يتناول هذا البحث تقدير السرعة المتوسطة والتصرف في المجارى الجبلية ودراسة تأثير الميل ورقم فراود يهدف البحث لاستخدام معادلات مقاومة السريان وقد تم استخدام البيانات المتاحة لبعض الأنهار من الطبيعة وكذلك البيانات المعملية المتاحة. تم عمل مقارنة بين نتائج المعادلات وبذلك يمكن استخدامها في تحديد السرعة المتوسطة وبالتالى التصرف المطلوب.

ABSTRACT:

It is necessary to estimate the mean velocity and thus the corresponding discharge in the ungauged mountainous streams, but the flow resistance equations available for this purpose require further testing and development for mountainous streams. Such streams are characterized by coarse bed materials, steep slopes and small water depths. For these conditions, boulders, cobbles and gravels protrude well into or completely through the flow, and the bed roughness is very large. In the present attempt, equations of Darcy – Weisbach friction factor were developed already to address this problem from which the mean velocity can be predicted. Data of different mountainous streams and laboratory flumes were used to test the equations.

INTRODUCTION

It is often necessary to estimate the mean velocity and discharge in ungauged mountainous streams. This is the case because of increasing pressure on mountain streams for water resources development, fisheries, recreation and forestry. To develop mountain regions while minimizing adverse impacts on the fluvial system requires engineering, geomorphic and ecological studies of the watersheds and streams.

Central to such knowledge is the analysis and explanation of flow resistance in mountainous streams. Flow resistance described the processes by

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which the physical shape and bed roughness of a channel control the depth, width and mean velocity of flow in the channel. These processes are accounted for by a flow resistance coefficient. The three coefficients which are in common use are; Manning 'n' Chezy 'c' and Darcy – Weisbach 'f' . Of these , only the Darcy- Weisbach friction factor 'f' is dimensionless. This gives it a distinct advantage scientifically and so only the friction factor, f, is used in this paper.

However, these three resistance coefficients are easily related to each other as below:

$$(8/f)^{0.5}=C/(g)^{0.5}=R^{1/6}/n(g)^{0.5} \quad \ldots\ldots (1)$$

Where , g = acceleration due to gravity, andR=hydraulic radius

Consequently, any of the equations presented here can be written in terms of Manning or Chezy coefficients.

The Darcy – Weisbach friction factor, f, is related to the mean velocity by

$$\text{i. e, } f = 8g\, R\, S / V^2 \quad \ldots\ldots (2)$$

in which, S = the energy slope (equal to the water surface slope in uniform flow); and V= the mean flow velocity.

To use this equation to calculate the mean velocity requires an equation for the friction factor that is based on the physical shape of the channel and the roughness of the boundary materials. Reliable equations of this type have been developed primarily for lowland streams, but on theoretical grounds, they should not be applicable to mountainous streams because the hydraulic processes of flow resistance in mountainous streams are different to those in lowland streams. There are three main reasons for this:

A- The bed material in mountainous streams is very coarse, usually consisting of gravels, cobbles and boulders. In contrast, lowland steams have sand and gravel beds;

B- The bed slopes in mountainous streams are of the orders of 1-5% much steeper than those in lowland streams; and

C- The relative roughness, i.e. the ratio of bed material size to flow depth, is very much higher mountainous streams, boulder, cobbles or the mixture of boulder, cobbles and gravels protrude well into or even completely through the flow. Under these circumstances the relative roughness is close to unity. In lowland streams, bed material size is only a few percent of the flow depth, and relative roughness is of the order of 0.01, Bathurst (1). For large relative roughness, most of the flow resistance is caused by the form of the gravels, cobbles, boulders, free surface distortion, hydraulic jumps, and the roughness is said to be " large scale" In lowland streams, with small relative roughness, resistance is mostly due to skin friction and roughness is " small scale".

Recently, three flow resistance equations specially for large roughness have been developed (3,10). However, these equations have not been independently applied by researchers other than responsible for one or more of the equations, to evaluate their usefulness. Also all of these equations use empirical coefficients derived from limited data sets. As the data mostly came from laboratory flumes and artificial channels which may not represent real mountain streams.

In this paper, Keulegan (7) tested the actual mountainous streams problem and by regression analysis of different mountainous data (2, 3, 4, 5 , 6 , 8 , 11) available ,

a set of equations were obtained for different sizes of bed roughness. The present form of equations illustrated that in their present form the equations are prone to maximum errors of around + 20 to + 35% . The possible sources or error in the equations are examined and recommendations for improving the accuracy of mean velocity estimates are made.

FORMULATION OF FLOW RESISTANCE EQUATION:

Using the prandtl – von Karmen's universal velocity distribution equations , i . e.

$V/V_* = 5.75 \log(y/Ks) + 8.5$ .... (3)

In which v= point velocity at a height of y from the bed; Ks = roughness height . Keulegan (7) has derived equations for mean velocity of turbulent flow in open channels as:

$Q = v.dA = V.A$ ....(4)

Substituting the pertinent expression for v in the foregoing equation and by subsequent simplification , the equation of mean velocity can be obtained as:

$V/V_* = 5.75 \log(R/Ks) + 6.25$ ....(5)

The dimensionless Darcy- Weishach friction factor 'f' is widely used as a measure of resistance to flow in open channels. Thus , from the Chezy's formula i, e

$V = C(RS)^{0.5}$ and from the definition of friction velocity,

$V/V_* = V/(gRS)^{0.5} = C^* = C/(g)^{0.5} = (8/f)^{0.5}$ ....(6)

and thus from Eqns . (5) and (6)

$V/(g R S)^{0.5} = (8/f)^{0.5} = 5.75 \log(R/Ks) + 6.25$ ....(7)

or , $V/(g R S)^{0.5} = (8/f)^{0.5} = A \log(R/Ks) + B$ ....(8)

in which A and B are numeric constants and Ks is the equivalent roughness of the boundary which can be recommended for different size of very rough bed materials as given by different investigators (3,4,6,11) i, e.:

For D50 size of bed materials;

$Ks = 4.50\, D_{50}$

For D65 size of bed materials;

$Ks = 4.00\, D_{65}$

For D84 size of bed materials;

$Ks = 3.50\, D_{84}$ and

For D90 size of bed materials;

$Ks = 3.26\, D_{90}$

Thus, the friction factor defined by the equation

$f = 8g\, RS/V^2$ can be also evaluated by a semi- logarithmic type of equation such as,

$V/(gRS)^{0.5} = (8/f)^{0.5} = A \log(R/Dxx) + B$ ....(9)

Where Dxx is the size of the bed particles for which "xx" the percent are finer (i, e D50, D65 , D84 and D90 in the present study)

Eq. (8) can also be written as a power type of equation i . e.

$V/(g R S)^{0.5} = (8/f)^{0.5} = a\,(R/Dxx)^b$ ....(10)

Here again a and b are some other numeric constants.

RESULTS AND DISCUSSION

In the present study, the field data of different mountainous streams and of different size of bed materials by different investigators such as Bathurst (2,3); Bray (4); Griffiths (5); Hey (6); Maihotra (8); paul , et al. (9) and Thompson (11) have been used. By the regression analysis of these data with the help of Eqs. (9) and (10) two sets of equations for different size of bed materials have been obtained as discussed below:

(A)-Logarithmic Type Formula

$V/(g RS)^{0.5} = (8/f)^{0.5} = 5.76 \log(R/D_{50}) + 3.25$ 11 (a)

$V/(g RS)^{0.5} = (8/f)^{0.5} = 5.99 \log(R/D_{65}) + 3.36$ 11 (b)

$V/(g RS)^{0.5} = (8/f)^{0.5} = 6.10 \log(R/D_{50}) + 3.55$ and 11 (c)

$V/(g\,RS)^{0.5} = (8/f)^{0.5} =$
$6.16\log(R/D_{50}) + 3.58$ 11 (d)

(B)- power type formula:

In the same way by the regression analysis of data as discussed ealier and with the help of Eq. (10) following power type of resistance equations have been obtained for different size of bed materials:

$V/(g\,RS)^{0.5} = (8/f)^{0.5} =$
$3.76\,(R/D_{50})^{0.287}$ ....12 (a)

$V/(g\,RS)^{0.5} = (8/f)^{0.5} =$
$4.15\,(R/D_{65})^{0.276}$ ....12 (b)

$V/(g\,RS)^{0.5} = (8/f)^{0.5} =$
$4.15\,(R/D_{84})^{0.268}$ and ....12 (c)

$V/(g\,RS)^{0.5} = (8/f)^{0.5} =$
$4.15\,(R/D_{90})^{0.258}$ ....12 (d)

The observed and corresponding computed values of mean velocity from Eqs. 11 (a,b,c & d) have been plotted as shown Figs . 1 (a,b,c & d) respectively. All are prone to a maximum percentage error of the order of nearly + 20 to + 35% . In the same way, the observed and computed values of mean velocities from Eqs. 12 (a,b,c & d) have been plotted as shown in Figs . 2 (a, b, c& d) and the computed values from Eqs. 12 are also prone to a maximum percentage error of the order of + 20 to + 35% . But it can be observed from the Figs. 1 and 2 that in each case the maximum concentration of data is around the average line. These equations have been derived for all size of bed such as gravels, cobbles, and boulders. So these equations can be used for mountainous streams comprising of bed materials of mixture of gravels, cobbles and boulders . Quantification of the resistance effect of bed material size distribution is not possible with the few available data. However , the foregoing suggests that much of the effect is accounted for by the inclusion of D_{84} (or similarly large percentile of the size distribution) in the resistance relationship and that any remaining size distribution influence is small. Apart from the inclusion of D_{84} or D_{90} in the relative submergence size distribution , therefore neglected in the approach developed here and also it was observed that with the increase in percentage size distribution percentage error in computed values was reduced.

EFFECT OF SLOPE AND FROUDE NUMBER

The recommendation of task force on friction factors in open channels, it is recommended to apply Darcy – Weisbach friction equation i, e . $f = 8g\,RS/V^2$ which can also be written as $f = 8S/(V^2/gR) = 8S/Fr^2$, where Fr = Froud number of the flow. Thus the friction factor, f, is influenced by both Fr and S But slope is not directly related to resistance , but tends to have an indirect influence, e. g., via the agency of Froude number.

It is also well known that in steep channels, the flow becomes unstable when the Froude number, Fr, exceeds some critical value. Instead of obtaining uniform flow, at a short distance from the channel inlet, waves of various lengths, amplitudes, and phase appear. These waves traveling downstream and occasionally overtaking each other , are called roll waves. This phenomenon results in increase in friction factor and thus decrease in discharge. The friction factor 'F' is influenced by a Froude number 'Fr' for flows in which the Froude number exceeds certain critical value, Fs, i.e. unstable flow conditions ($Fr > Fs > 1$). Rouse (10) suggested a formula to compute this increase of (f) as a function of F and Fs. Accordingly , Eqs. 11 and 12 for stable flow must be multiplied by a factor $(Fr/Fs)^{2/3}$. Thus , we can obtain resistance equation for unstable flow conditions i ,e, for Fr> Fs> 1, where Fr = Froude number given by

$V/\sqrt{g\ D \cos \Theta/\partial}$

where,

D= Hydraulic mean depth;

Θ= energy or friction slope;

∂= K.E. correction factor; and for a rough boundary in stability number, Fs , as per Rouse (10) is given as:

Fs= {(0.5)+ 0.87f0.5- 0.78f)2 – 0.78f(1+0.78f)}-0.5 ....(13)

In this way, depending upon the different hydraulic parameters and streams characteristics the more accurate Darcy- Weisbach friction factor 'f' can be calculated and from the equation, $f= 8g\ RS/V^2$, the value of mean velocity can be computed.

Certainly , for large roughness, Froude number does seem to affect flow resistance via the development of surface wave drag around protruding bed elements, but this effect dies away at the larger relative submergence, and the only effects which can related to froude number at surface instabilities and waves . However, these would be expected to increase resistance{i.e,reduce (8/f)0.5 }

While it has been observed by different investigators that the higher the Froude number, the higher is the value of $(8/f)^{0.5}$ and lower is the resistance. It is probable , therefore , that the high the Froude number results from a reduced resistance, rather than vice versa as it is also clear from foregoing discussions and equation proposed by Rouse (10) ti take into account the effect of Froude number.

APPLICATIONs

The equations presented here have two main applications in river mechanics. First, they may be used to estimate; flow resistance, mean velocity and thus discharge in natural streams with steep slope and coarse bed materials. Secondly, they may also be applied to natural or artificial channel partly or wholly lined with rip- rap. All of the equations derived above are vulnerable to maximum errors from + 20% to + 35%, and thus must be applied with caution. Whenever possible, the predicted velocities should be checked against field data from the actual site in question.

SCOPE FOR FURTHER STUDY

Application of the resistance equation is limited to the conditions of flow for which the equations have been derived. It seems likely that the discrepancies were caused by the factors or processes not adequately accounted for in the flow resistance equations. The above equations were developed by noting field data and it was possible that some of the parameters which may affect the flow resistance were not fully incorporated, like natural obstruction, bed material size distribution, density of bed material, effect of flood hydrograph, sediment transport and instability of flow. Consequently , further research on the effects of variation in bed material size distribution, density of bed material, flood hydrograph and instability of flow and other parameters which may affect the flow resistance is recommended. It is also recommended that further research into mountainous river hydraulics should concentrate on higher range of flows and at a site variations, particularly as they are affected by slope, vertical profile and sediment transport.

CONCLUSION

Mountainous streams differ significantly from lowland stream in several important aspects. As compared to lowland streams, the slopes of mountain streams are steeper (1-5%), the bed is formed with gravels, cobbles and boulders rather than sandy bed, and the relative roughness is larger with boulders , cobbles and boulders rather than sandy bed, and the relative roughness is larger with boulders, cobbles and / or mixture of boulder, cobbles and gravels often protruding through the free surfaces, Because of the differences, the form drag of the gravels, cobbles and boulders, free surface distortion and hydraulic jump associated with locally accelerated flow all contribute significantly to flow resistance in mountationous streams, but are not explicitly accounted for in resistance equations for lowland streams. This has led to the development of flow resistance equations specifically intended for steep streams with very rough surfaces and further research is required to produce a reliable and process based flow resistance equation for mountainous streams.

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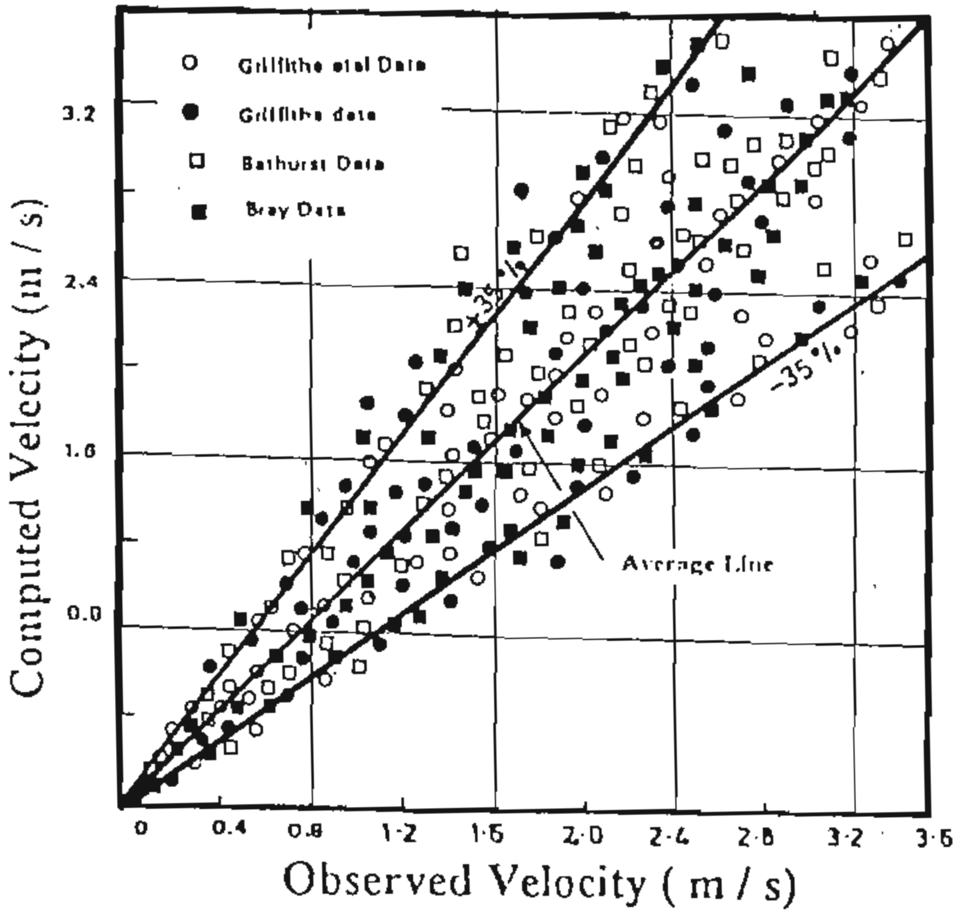

Fig. 1(a) Computed vs Observed mean velocity for D_{50} size

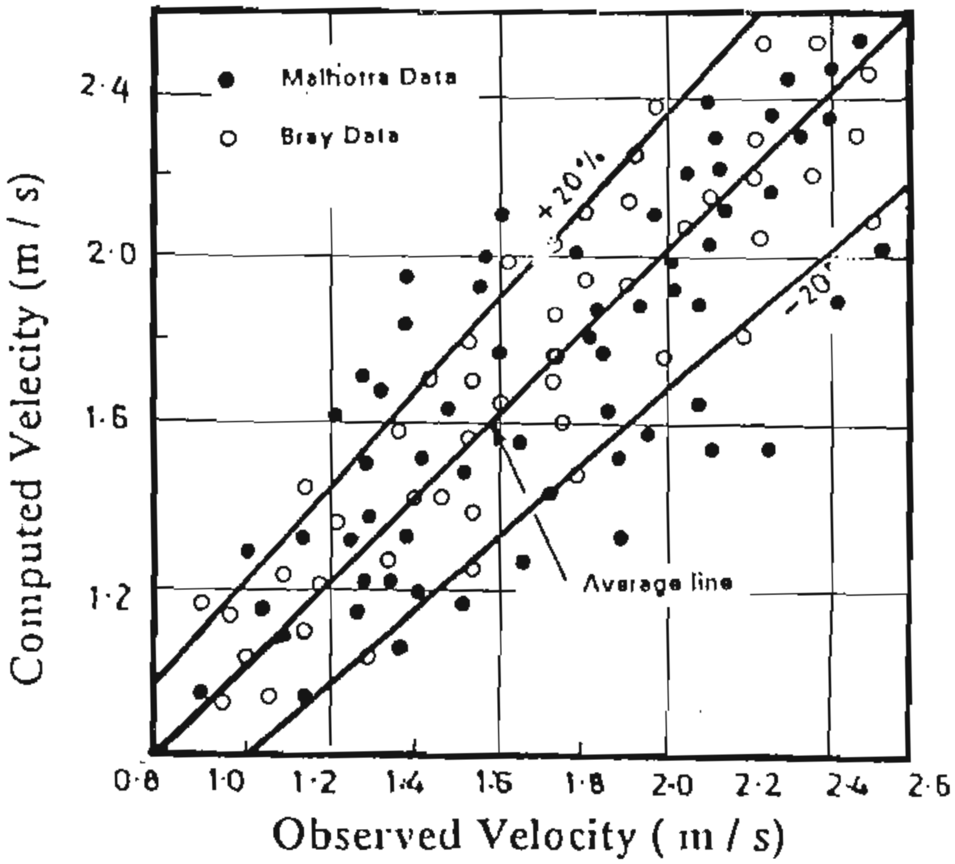

Fig. 1(b) Computed vs Observed mean velocity for D_{65} size

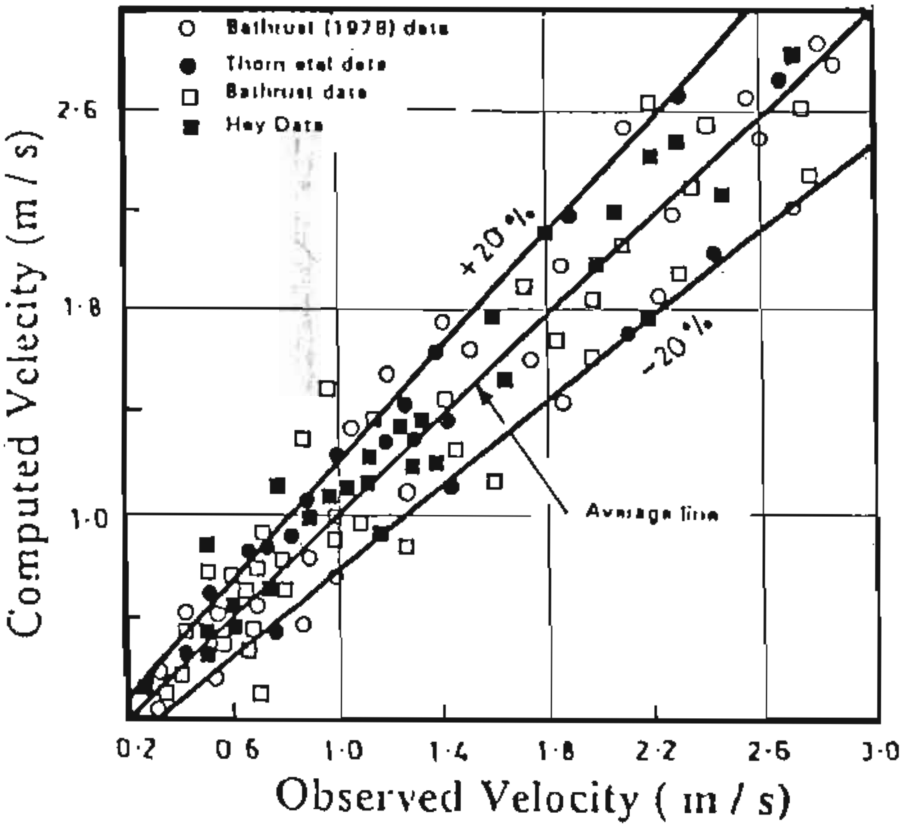

Fig. 1(c) Computed vs Observed mean velocity for D84 size

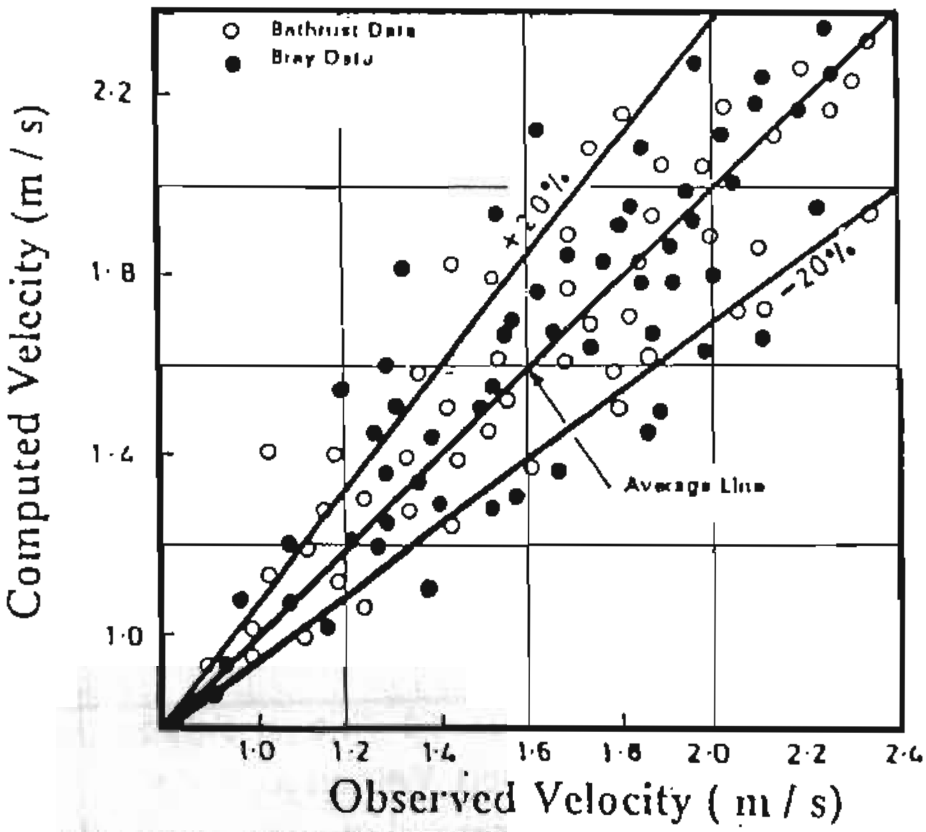

Fig. 1(d) Computed vs Observed mean velocity for D90 size

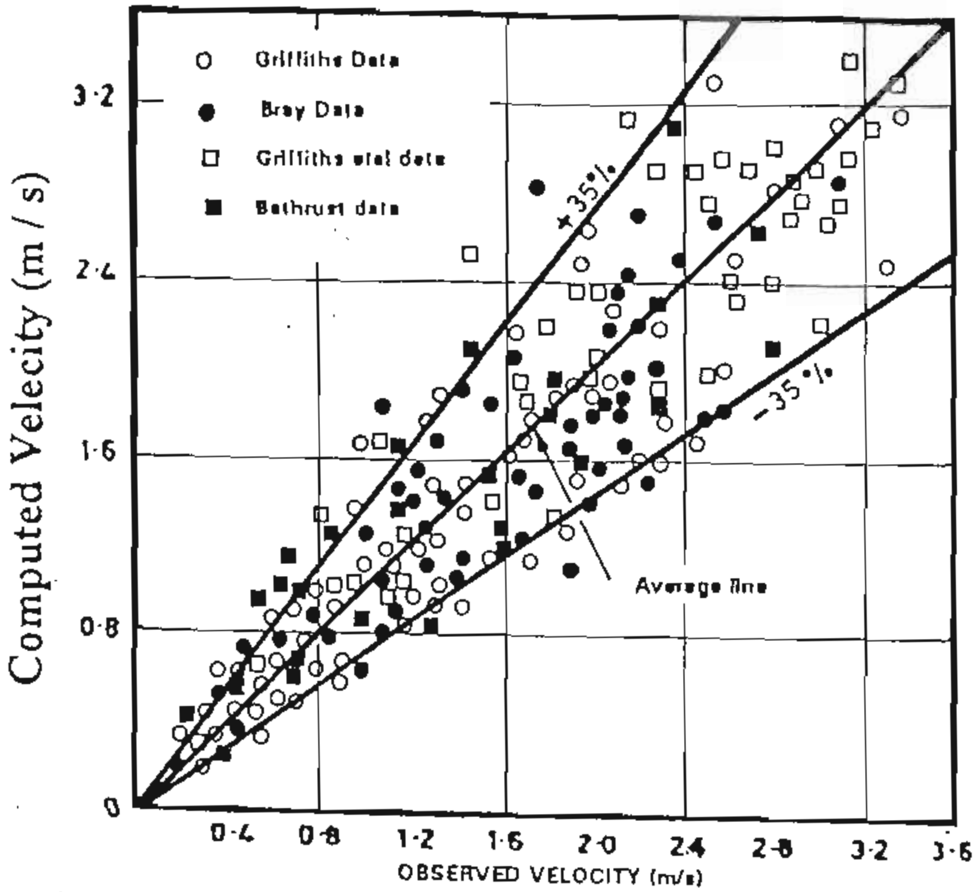

Fig. 2 (a) Computed vs Observed mean velocity for D_{50} size

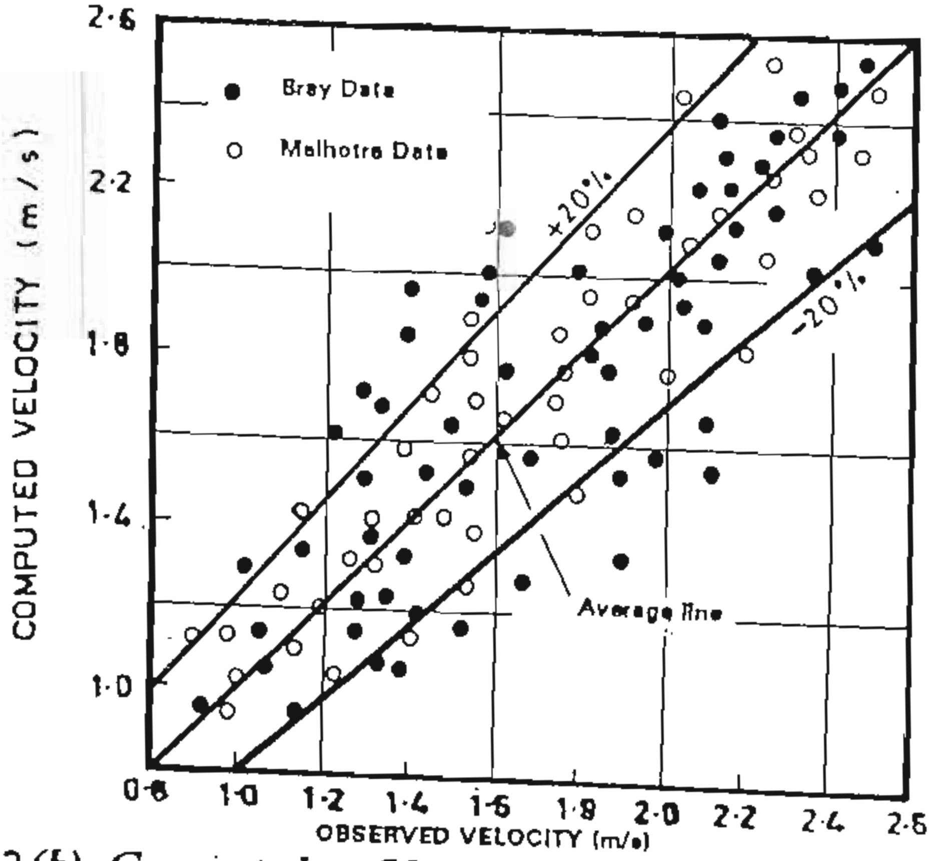

Fig. 2 (b) Computed vs Observed mean velocity for D_{65} size

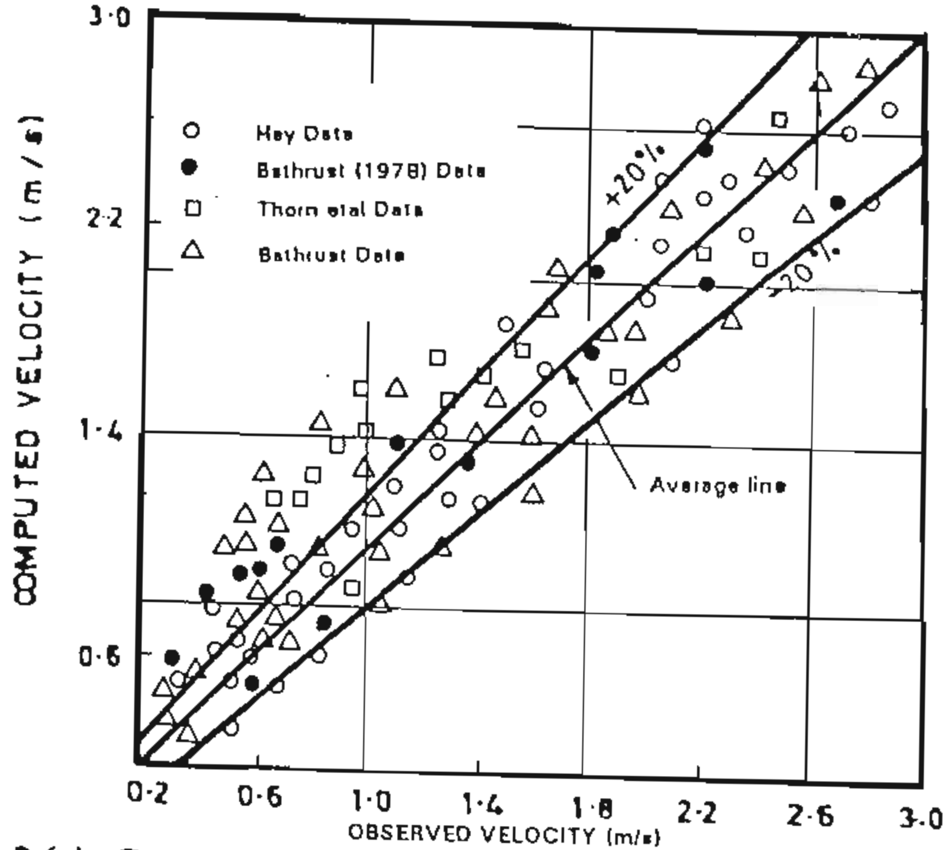

Fig. 2 (c) Computed vs Observed mean velocity for D_{84} size

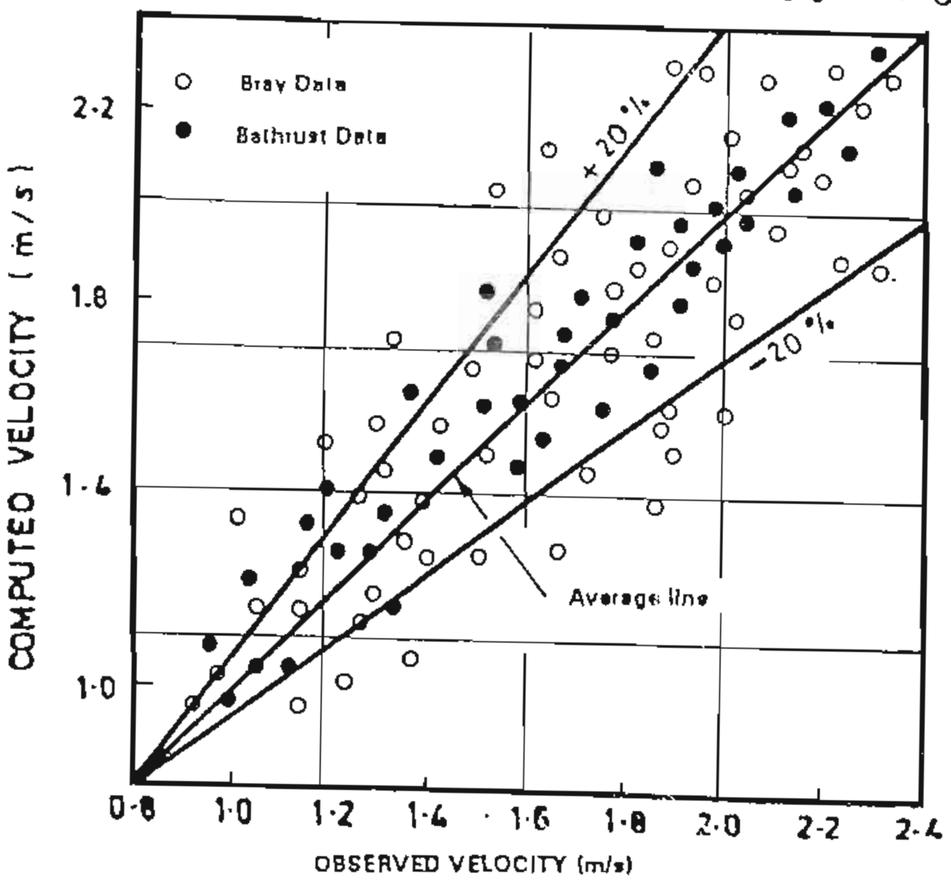

Fig. 2 (d) Computed vs Observed mean velocity for D_{90} size