

Note: Assume any data required, state your assumption clearly.

Question (1) (25 Marks)

Solve the following equations using second order Runge Kutta Method

$$\frac{dy}{dx} = -2y + 5ze^{-x}$$

$$\frac{dz}{dx} = -\frac{yz^2}{2}$$

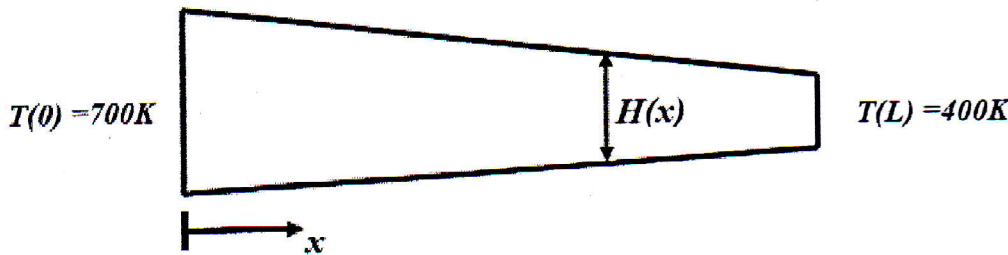
over the range $x = 0$ to 1 using a step size of 0.2 with $y(0) = 2$ and $z(0) = 4$.

Question (2) (25 Marks)

The heat transfer equation in trapezoidal fin shown in the next figure is given by

$$\frac{\partial}{\partial x} \left(kA(x) \frac{\partial T}{\partial x} \right) + hP(x)(T - T_{\infty}) = 0$$

Where, k is the thermal conductivity, $P(x)$ and $A(x)$ are the perimeter and cross sectional area of the fin at any x . given that: $k = 19$ W/m.K, $T_{\infty} = 300$ K, $h = 2$ W/m²K, the fin length is 50 cm and fin width (perpendicular to paper) is 15 cm, the fin height is $H(x) = 5 - 0.005x$ cm. Calculate the temperature distribution along the fin using five grid points

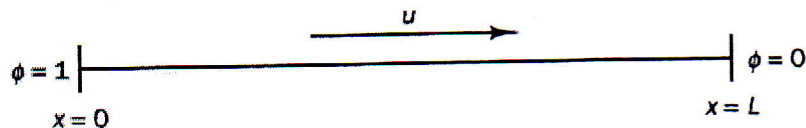


Question (3) (25 Marks)

A property ϕ is transported by means of convection and diffusion through the one-dimensional domain sketched in the figure. The governing equation is $\frac{d\rho u \phi}{dx} = \frac{d}{dx} \left(\Gamma \left(\frac{d\phi}{dx} \right) \right)$ the boundary conditions are $\phi_0 = 1$ at

$x = 0$ and $\phi_L = 0$ at $x = L$. Using five equally spaced cells and the central differencing scheme for convection and diffusion, calculate the distribution of ϕ as a function of x . The following data apply:

$u = 0.1$ m/s, length $L = 1.0$ m, $\rho = 1.0$ kg/m³, $\Gamma = 0.1$ kg/m.s.



Question (4) (25 Marks)

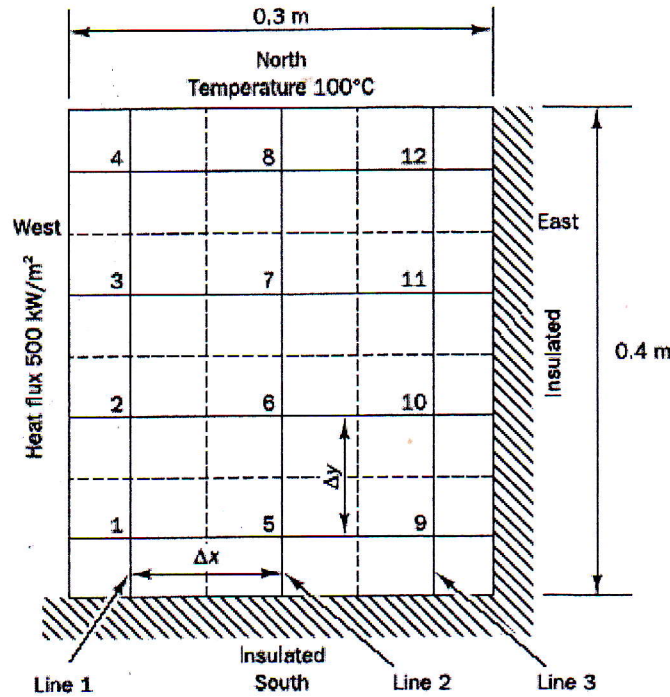
In figure 1, a two-dimensional plate of thickness 1 cm is shown. The thermal conductivity of the plate material is $k = 1000$ W/m.K. The west boundary receives a steady heat flux of 500 kW/m² and the south and east boundaries are insulated. If the north boundary is maintained at a temperature of 100°C , use a uniform grid with $\Delta x = \Delta y = 0.1$ m to calculate the steady state temperature distribution. The two-

dimensional steady state heat transfer in the plate is governed by

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) = 0.$$

Answer the following:

- Describe the solution of the aforementioned problem
- Show in details how the boundary conditions of this problem can be implemented.
- How TDMA can be used with problem?
- Write computer program for this problem.



GOOD LUCK

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