



Solve the Following Questions

(Question Number-1):(30 Marks)

(A) Find the general solution of the differential equations:

$$i) \frac{dy}{dx} = \frac{2x \sinh \frac{y}{x} + 3y \cosh \frac{y}{x}}{3x \cosh \frac{y}{x}}$$

$$ii) \frac{dy}{dx} = \frac{y}{2y \ln y + y - x}$$

$$iii) \frac{dy}{dx} = \frac{2x + y - 1}{4x + 2y + 5}$$

$$iv) \frac{dy}{dx} + y \sec x = \cos^2 x$$

(B) Find the solution of the initial value problem

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} = 1, \quad x = 0, \quad \frac{dx}{dt} = 0 \quad \text{when } t = 0$$

(C) Find the orthogonal trajectories of the family of the curves $y^2 = cx^3$.

(D) Find the solution of the following differential equations:

$$x^2 \left(\frac{d^2y}{dx^2} \right) - 4x \left(\frac{dy}{dx} \right) + 6y = x$$

(E) Solve the following system of ordinary differential equations:

$$\frac{dx}{dt} = 3x - 2y \quad \text{and} \quad \frac{dy}{dt} = 2x - y$$

(Question Number-2):(20 Marks)

(A) Find the moment of inertia about x -axis of the area bounded by $y = 2\sqrt{x}$, $x + y = 3$ and $y = 0$.

(B) Find the volume of the pyramid bounded by $x = 0$, $y = 0$, $z = 0$, and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

(Question Number-3):(25 Marks)

(A) Find Laplace transform of the following functions:

$$i) f(t) = t^2 e^{2t} \int_0^t \frac{1 - \cos t}{t} dt$$

$$ii) \text{The periodic function } f(t) = 3t, \quad 0 < t < 1$$

(B) Find inverse Laplace transform of the following functions:

$$i) F(s) = \frac{2 - 6e^{-3s}}{s^2 - 9}$$

$$ii) F(s) = \tan^{-1}(s + 1)$$

$$iii) F(s) = \frac{1}{s^2 (s + 1)^2}$$

(C) Solve the differential equation $y'' - 3y' + 2y = 6e^{-t}$ using Laplace transform method with the initial conditions $y(0) = 0$, $y'(0) = 1$.

(Question Number-4):(25 Marks)

(A) Test the convergence of the following series:

$$i) \sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$$

$$ii) \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

$$iii) \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(\ln n)^2}$$

$$iv) \sum_{n=1}^{\infty} ne^{n^2}$$

(B) Find the interval of convergence of the series: $S_n = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$

(C) Find the Fourier series of the function: $f(x) = \begin{cases} x & 0 < x < \pi \\ 2 & -\pi < x < 0 \end{cases}$, Then, show that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$$

(D) Find the Fourier coefficients of the function $f(x)$ if it is even then, write its Fourier expansion.

With our best wishes

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This exam contributes " " by measuring in achieving Programme Academic Standards according to NARS								
Question Number	Q1(A) - Q2(A) - Q3(A)	Q3(C)	Q4(A,B)	Q4(D)	Q4(C)	Q1(C,D) - Q2(B)	Q1(E) - Q3(B)	Q1(B) - Q2(B)
Skills	a-1-2	a-1-4	a-5-1	a-5-2	a-5-3	b-3-1	b-3-3	c-1-1
	Knowledge & Understanding Skills				Intellectual Skills		Professional Skills	