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A GENERALIZATION ALGORITHM FOR SOLVING MULTI PARAMETRIC LINEAR PROGRAMMING PROBLEM

By

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ABSTRACT

In this paper a multi parametric linear programming problem is considered as that of finding the set of all optimal solutions with respect to the given domination cone and also the set of all optimal parameters .

The primal problem in which the parameters exist in the objective functions can be solved by algorithm I . The dual problem in which the parameters exist in the right hand side of the constraints can be solved by algorithm II .

1. Introduction :

The simplex-technique is an algebraic method which will solve exactly any linear multi parametric programming problem in a finite number of steps. This algorithm gives us the set of optimal solutions for any multi parametric linear programming and hence the solvability set (the set of all optimal parameters) . Here, solutions of the problem means that the cone extreme points of a given optimal set with respect to a given convex cone which specifies the domain structure of the decision-maker .

Examples are given to demonstrate the possible saving in computations of the simplex-technique algorithm parametrically .

2. Solving a Multi parametric linear programming (the parameters in the o.f.) :

Consider the following problem :

$$P_1(\lambda) = max \sum_{j=0}^{n}\{\lambda_j f_j(x)/x \in X\ ,\ \ \lambda_j > 0\ ;\ \sum_{j=0}^{n}\lambda_j = 1\}\ , \qquad (2.1)$$

Subject to : $M = \{x \in X\ /\ Ax \leq b\ ,\ \ x \geq 0\}$, where :

(i) X is a compact set on R^n , (ii) be concave for all j = 0, 1, ..., n .

To reduce 2 parameters, suppose that $\sum_{j=1}^{n} \frac{\lambda_j}{\lambda_o} = n$.

$$P_1(\lambda) = max\ \{\lambda_o f_o(x) + \sum_{j=1}^{n}\{\lambda_j f_j(x)/x \in X;\ \ \lambda_o, \lambda_j > 0\ ,\ (j = 1, ..., N)\} \qquad (2.2)$$

put $\eta_{j-1} = \frac{\lambda_j}{\lambda_o}$, j = 1, ..., n .

From the assumption we get : $\sum_{j=1}^{n} \eta_{j-1} = n$, and $\eta_1 = n - \sum_{j=2}^{n} \eta_{j-1}$

Therefore :

$$P_1'(\eta) = \lambda_o\ max\ \{f_o(x) + nf_1(x) + \sum_{j=2}^{n}\eta_{j-1}(f_i(x) - f_1(x)/x \in X\ ,$$

$$\eta_{j-1} > 0\ ,\ (j = 2, ..., n)\ \ and\ \sum_{j=2}^{n}\eta_{j-1} = n\} \qquad (2.3)$$

This procedure can be applied to our problem with at most three objective functions to get an equivalent parametric problem with only one parameter . Hence problem (2.3) takes the form :

$$P_1(\eta) = \ max\ \{\sum_{j=0}^{n}\bar{c}_j x_j / x_j \in X\ ,\ x_j > 0\} \qquad (2.4)$$

where

$\bar{c}_j = c_j + \eta\hat{c}_j$; $\eta \in (\underline{\eta}.\overline{\eta})$ be a scalar parameter subjected to

$$M = \{x_j \in X/Ax_j \leq b_j\ ,\ \ x_j \leq 0\} \qquad (2.5)$$

To solve this problem, we use the following algorithm

Algorithm I :

Step (0) :

Put $\eta = 0$, and solve the linear programming problem with compact simplex method to have an optimal basic feasible solution $x^* = B^{-1}b$.

Step (1) :

Put $\eta = \eta_{k-1}$ in (2.4), $\eta \in (\underline{\eta}.\overline{\eta})$. Then we apply the compact simplex method to get the end table for $\eta = \eta_{k-1}$

If there is one s $(0 \leq s \leq n)$ with $k_s(\eta_{k-1}) < 0$ and $\hat{c}_B y_j - \hat{c}_j \leq 0$ $(0 \leq j \leq n)$ go to step (5), otherwise if $z_j - c_j \geq 0$, $\hat{c}_B^T y_j - \hat{c}_j < 0$. Then

$$\overline{\eta}_{k-1} = min \left[\overline{\eta}\, ;\, inf_j \left\{-\frac{z_j - c_j}{\hat{c}_B^T y_j - \hat{c}_j} \mid \hat{c}_B^T y_j - \hat{c}_j < 0\right\}\right]$$

Step (2) :

If $\overline{\eta}_{k-1} = \overline{\eta}$ go to step (4), otherwise substitute by $\overline{\eta}_{k-1}$ in (2.4), go to step (1) and if $\hat{c}_B^T y_j - \hat{c}_j = 0$, go to step (4) .

Step (3) :

Calculate another table (optimum) which is a similar, where $z_j^* - c_j^* \geq 0$, $\hat{c}_B^{*T} y_j - \hat{c}_j^* < 0$. Then

$$\overline{\eta}_{k-1}^* = min \left[\overline{\eta}\, ;\, inf \left\{-\frac{z_j^* - c_j^*}{\hat{c}_B^{*T} y_j - \hat{c}_j^*} \mid \hat{c}_B^{*T} y_j - \hat{c}_j^* < 0\right\}\right]$$

If $x^k = y$, $\overline{\eta}_{k-1} = \overline{\eta}_{k-1}^*$, go to step (2)

Step (4) :

The vector x^k is optimum for all $\eta \in (\underline{\eta}_{k-1}, \overline{\eta}_{k-1})$. Put $\eta_k = \eta_{k-1}$, k = k+1 if $\hat{c}_B^{*T} y_j - \hat{c}_j^* = 0$, go to step (6), or if $x^k = y$, $\overline{\eta}_{k-1} = \overline{\eta}_{k-1}^*$, go to step (2) .

Step (5) :

If there is one $s^*(0 \leq s^* \leq 0)$ with $k_s^*(\eta_{k-s}) < 0$, and $\hat{c}_{B\,s}^T * y_j^* - \hat{c}_{js} * \leq 0$, go to step (6), otherwise, we find

$$\eta^* = min \left[\overline{\eta} \, , \, sup \left\{ -\frac{z_j - c_j}{\hat{c}_B^T y_j - \hat{c}_j} \mid \hat{c}_B^T y_j - \hat{c}_j > 0 \right. \right. ,$$

$$\left. \left. k_s^*(\eta_{k-1}) < 0 \right\} \right]$$

If $\eta^* = \overline{\eta}$, go to step (6), or if $\eta^* = \eta_{k-1}$, then go to step (1).

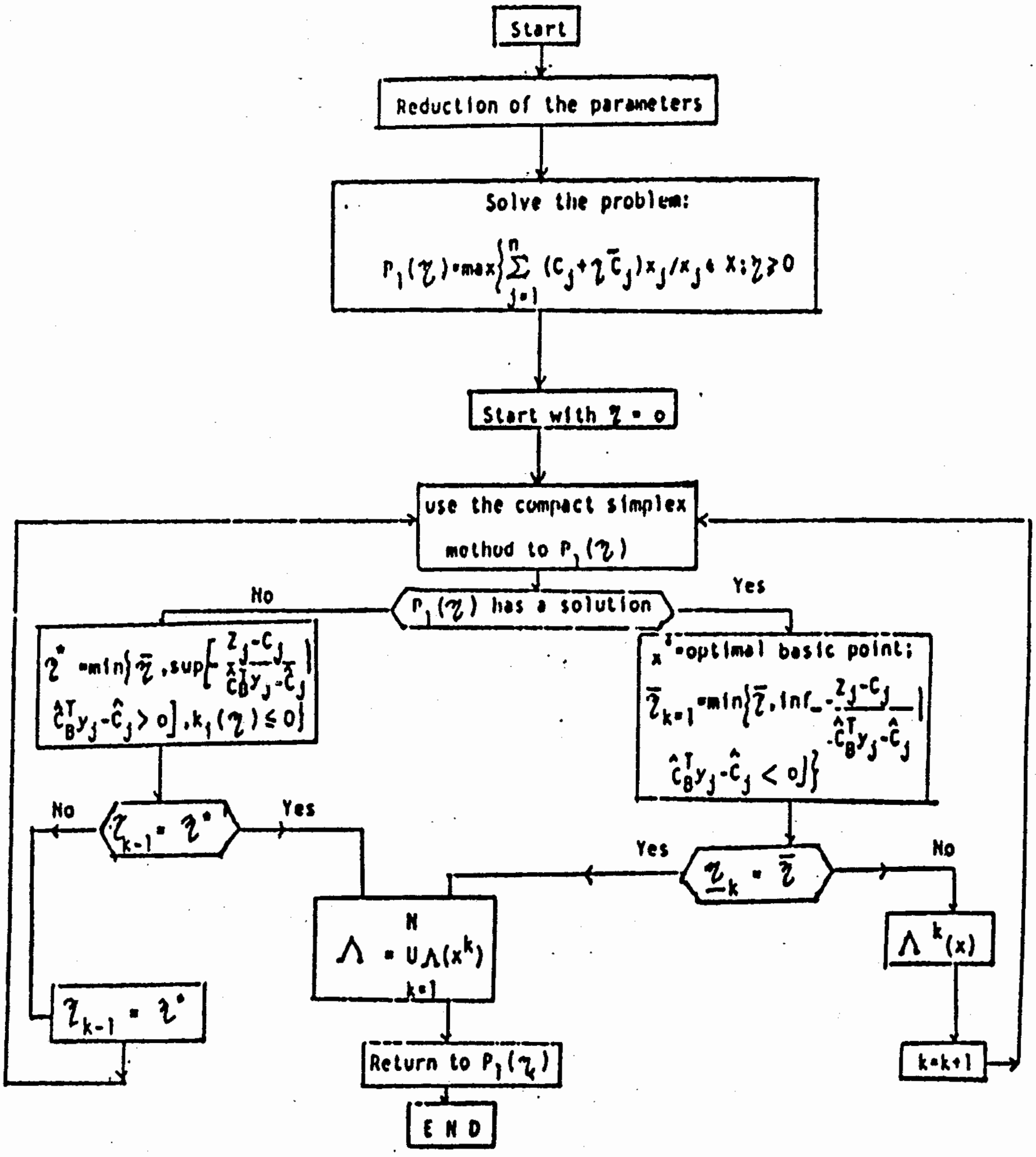
Step (6) :

The problem (2.4) for all $\eta \in (\underline{\eta} \, , \, \overline{\eta})$ is unsolvable .

Step (7) : End.

With this algorithm, we obtain a unique determined subdivision of $(\underline{\eta} \, , \, \overline{\eta})$, in a finite number of subintervals and to each subinterval $(\eta_{k-1} \, , \, \eta_k)$ there is only one optimal point, otherwise the problem is unsolvable in the corresponding interval .

The above algorithm can be summarized in the following flow chart


Start
Reduction of the parameters
Solve the problem:
P1(η)=max{Σ (cj+ηc̄j)xj/xj∈X;η≥0}
Start with η = o
use the compact simplex method to P1(η)
P1(η) has a solution
No
Yes
x* = optimal basic point;
No
Yes
Yes
No
Λ^k(x)
Return to P1(η)
k=k+1
E N D

Flow Chart (1)

Generalization Algorithm.

To demonstrate algorithm I, consider the following example :

<u>**Example (2.1)**</u> :

Let

$$P_1(\lambda) = \max \ \{\lambda_o f_o(x) + \lambda_1 f_1(x) + \lambda_2 f_2(x) | \lambda_i > 0 \ , \\ (i = 0, 1, 2) \ , \ \ x \in M \ ; \sum_{j=0}^{2} \lambda_j = 1 \Big\}$$

where; $f_o(x) = 4x_1 + x_2 + 2x_3$, $f_1(x) = x_1 + x_2 - x_3$, $f_2(x) = -x_1 + x_2 + 4x_3$

subject to M :

$x_1 + x_2 + x_3 \leq 3$, $2x_1 + 2x_2 + x_3 \leq 4$, $x_1 - x_2 \leq 0$, $x_i \geq 0$ $(i = 1, 2, 3)$.

Or, we have :

$$P_1(\lambda) = \max \ \left\{ f_o(x) + \frac{\lambda_1}{\lambda_o} f_1(x) + \frac{\lambda_2}{\lambda_o} f_2(x) \ |\lambda_i > 0 \ , \ (i = 0,1,2) \ ; \ x \in M \right\}$$

put $\frac{\lambda_1}{\lambda_o} = \mu_1$, $\frac{\lambda_2}{\lambda_o} = \mu_2$

since

$$\frac{\sum_{i=1}^{n} \lambda_i}{\lambda_o} = n \rightarrow \sum_{s=1}^{2} \mu_s = 2 \rightarrow \mu_1 + \mu_2 = 2 \ ,$$

$$or \quad \mu_1 = 2 - \mu_2$$

put $\mu_2 = \eta$ implies that $\mu_1 = 2 - \eta$

i.e. our problem becomes :

$$P_1'(\eta) = \lambda_o \max \ \{f_o(x) + (2-\eta) f_1(x) + \eta f_2(x) | \ \eta > 0 \ , \ x \in M\}$$

or

$$P_1(\eta) = \max \ \{(6 - 2\eta)x_1 + (7 - 2\eta)x_2 + 5\eta x_3 \mid \eta > 0 \ , \ x \in M\}$$

subject to : $x_1 + x_2 + x_3 + x_4 = 3$, $2x_1 + 2x_2 + x_3 + x_5 = 4$, $x_1 - x_2 + x_6 = 0$,

$x_r \geq 0$ $(r = 1, 2, ..., 6)$.

so we have a linear parametric programming with one parameter in the objective function, then we can solve it by algorithm I as follows :

put $\eta = 0$

$P_1(\eta) = \ max \ \{6x_1 + 7x_2\}$,

or $P_1(\eta) = 6x_1 - 7x_2 = 0$;

subject to : $x_1 + x_2 + x_3 + x_4 = 3$, $2x_1 + 2x_2 + x_3 + x_5 = 4$, $x_1 - x_2 + x_6 = 0$,

$x_r \geq 0 \ \ (r = 1, 2, ..., 6)$.

The optimal basic feasible solution is found to be $x_B = (0,2,0)$ in the optimum simplex table, in which $z_1 - c_1 = 1$, $z_3 - c_3 = 7/2$, $z_5 - c_5 = 7/2$

When $\eta \neq 0$:

It is required to perform an initial sensitivity analysis when c is changed from (6,7,0) to $[(6 - 2\eta), (7 - 2\eta), 5\eta]$, where η is unknown . Let us write $c^* = c + \eta c = (6, 7, 0) + \eta(-2 , -2 , 5)$.

In order to perform the initial sensitivity analysis the following quantities are required :

$$\hat{c}_B = (0, 0, 0) \ ; \ and \ \hat{c}_j = (-2, -2, 5)$$

$$\therefore \ \hat{c}_B^T x_1 - \hat{c}_1 = 2 \ , \ \ \hat{c}_B^T x_3 - \hat{c}_3 = -5 \ , \ \ \hat{c}_B^T x_5 - \hat{c}_5 = 0$$

i.e. the first critical value of η is given by :

$$\eta_1 = \ min_j \ \left\{ -\frac{z_i - c_j}{\hat{c}_B^T x_j - \hat{c}_j} \mid \hat{c}_B^T x_j - \hat{c}_j < 0 \right\} = \left\{ \frac{-7/2}{-5} \right\} = 0.7$$

$$\therefore \ \eta^{(1)} = 0 + 0.7 = 0.7 \ ; \ \ and \ x^{(0)} = (0.2.0) \ at \ 0 \leq \eta \leq 0.7$$

at $\eta = 0.7$:

our problem becomes

$P_1(\eta) = \ max \ \{4.6x_1 + 5.6x_2 + 3.5x_3\}$, or $P_1(\eta) - 4.6x_1 - 5.6x_2 - 3.5x_3 = 0$;

subject to : $x_1 + x_2 + x_3 + x_4 = 3$, $2x_1 + 2x_2 + x_3 + x_5 = 4$, $x_1 - x_2 + x_6 = 0$,

$x_r \geq 0 \ \ (r = 1, 2, ..., 6)$.

By a similar manner, as explained in the algorithm, continue for

$$\eta = 0.9 \ , \ \ \eta = 1.4 \ , \ \ \eta = 2.4 \ .$$

Hence we have the following optimum solution to given optimal problem as :

$x^{(0)} = (0,2,0)$	at	0	$\leq \eta < 0.7$
$x^{(1)} = (0,1,2)$	at	0.7	$\leq \eta < 0.9$
$x^{(2)} = (0,1,2)$	at	0.9	$\leq \eta < 1.4$
$x^{(3)} = (0,0,3)$	at	1.4	$\leq \eta < 2.4$
$x^{(4)} = (0,0,3)$	at	2.4	$\leq \eta_o$

$x^{(0)} = (0,2,0)$ is optimal for all $\lambda \in \Lambda(x^o)$. Therefore

$0 \leq \frac{\lambda_2}{\lambda_o} < 0.7$	$1.3 \leq \frac{\lambda_1}{\lambda_o} < 2$
$-\frac{\lambda_2}{\lambda_o} \geq 0 \rightarrow \lambda_2 \geq 0$;	$\frac{\lambda_2}{\lambda_o} < \frac{7}{10} \rightarrow \lambda_2 - 10 > 0$
$\frac{\lambda_1}{\lambda_o} \geq \frac{13}{10} \rightarrow 10\lambda_1 - 13$	$\lambda_o \geq 0$; $\frac{\lambda_1}{\lambda_o} < 2 \rightarrow 2\lambda_o - \lambda_1 > 0$

i.e. $x^{(0)} = (0,2,0)$ is optimal for all $\eta \in \Lambda(x^o)$; where

$\Lambda(x^o) = \{\lambda \mid 7\lambda_o - 10\lambda_2 > 0 \ , \ 10\lambda_1 - 13\lambda_o \geq 0 \ , \ 2\lambda_o - \lambda_1 > 0$;

$\lambda_j \geq 0 \ \ (j = 0, 1, 2) \}$.

Also $x^{(1)} = (0,1,2)$ is optimal for all $\lambda \in \Lambda(x^1)$, where

$\Lambda(x^1) = \{\lambda \mid 10\lambda_2 - 7\lambda_o \geq 0 \ , \ 7\lambda_o - 5\lambda_2 > 0 \ , \ 5\lambda_1 - 3\lambda_o > 0$,

$13\lambda_o - 10\lambda_1 \geq 0$; $\lambda_j \geq 0 \ \ (j = 0, 1, 2) \}$.

$x^{(2)} = (0,0,3)$ is optimal for all $\lambda \in \Lambda(x^2)$, where

$\Lambda(x^2) = \{\lambda \mid 5\lambda_2 - 7\lambda_o \geq 0 \ , \ 12\lambda_o - 5\lambda_2 \geq 0 \ , \ 5\lambda_1 + 2\lambda_o \geq 0 \ ,$

$3\lambda_o - 5\lambda_1 \geq 0 \ ; \ \lambda_j \geq 0 \ \ (j = 0, 1, 2) \} \ .$

Thus, the set of all efficient points is defined as :

$$E_{\varepsilon x} = \{ \ x^{(0)} \ , \ x^{(1)} \ , \ x^{(2)} \} \ .$$

The set of optimal parameters $\Lambda(x^j)$ which associated with x^j. is defined as :

$$\Lambda = \{ \ \Lambda(x^o) \ \cup \ \Lambda(x^1) \ \cup \ \Lambda(x^2) \} \ . \ \text{(The solvability set).}$$

3. Solving The Multi Parametric Linear Programming Problem (The Parameters in the Constraints) :

Consider the primal problem :

$$P(\lambda) = \ max \ \left\{ \sum_{j=0}^{n} \lambda_j f_j(x) \mid x \in X \ , \ \lambda_j \geq 0 \ ; \ \sum_{j=0}^{n} \lambda_j = 1 \right\} \ ,$$

Subject to :

$$M = \{x \in X \mid Ax \leq b \ , \ x \geq 0\} \ . \tag{3.1}$$

rewriting problem (3.1), using the reduction of parameters as:

$$P_1(\eta) = \ max \ \left\{ \sum_{j=0}^{n} \bar{c}_j x_j \mid x_j \in X \ , \ x_j \geq 0 \right\} \ , \tag{3.2}$$

where $\bar{c}_j = c_j + \eta \hat{c}_j$; $\eta \in (\underline{\eta} \ , \ \overline{\eta})$ be scalar parameter subject to

$$M = \{x_j \in X \mid ax_j \leq b \ , \ x_j \geq 0 \ \ (j = 0, 1, ..., n)\}$$

Now using the duality theorem, we can obtain a new parametric programming problem in one parameter (the dual problem) . This dual problem takes the form

$$D_1(\eta) = \ min \ \left\{ \sum_{r=0}^{m} b_r y_r \mid y_r \in X \ , \ y_r \geq 0 \ , \ and \ b_r \geq 0 \right\} \ , \tag{3.3}$$

subject to :

$$N = \{y_r \in X \mid Dy_r \geq \bar{c}^* \ , \ where\ \bar{c}^* = c + \eta\hat{c} \ , \ \eta \geq 0\}$$

To solve the dual problem (3.3) we use a new algorithm similar to algorithm(I) called algorithm (II) ($D_1(\eta)$ algorithm) .

Algorithm (II) :

Step (0) :

Put $\eta = 0$, and solve the linear programming problem with the compact dual simplex method to have an optimal basic feasible solution $y^* = D^{-1}c$.

Step (1) :

Put $\eta = \eta_{k-1}$ in (3.3), where $\eta \in (\underline{\eta}.\bar{\eta})$. Then we apply the compact dual simplex method to get the end table for $\eta = \eta_{k-1}$. If there is one s $(0 \leq s \leq n)$, such that $\theta_s = D^{-1}\hat{c}_s^T \leq 0$ go to step (5), otherwise if $y_D \geq 0$, and $\theta = D^{-1}\hat{c}^T \leq 0$. Then

$$\bar{\eta}_{k-1} = min_j \left[\bar{\eta} \ , \ inf\left\{-\frac{y^T D_j}{\theta_j} \mid \theta_j < 0\right\}\right]$$

Step (2) :

If $\bar{\eta}_{k-1} = \bar{\eta}$ go to step (4), otherwise substitute $\bar{\eta}_{k-1}$ instead of η_{k-1} in (3.3), go to step (1) and if $\theta = D^{-1}\hat{c}^T = 0$, go to step (4) .

Step (3) :

Calculate the optimum table, where $y_D^* \geq 0$, $\theta_j^* = D^{-1}\hat{c}_j^T < 0$. Then

$$\bar{\eta}_{k-1} = min_j \left[\bar{\eta} \ ; \ inf\left\{-\frac{y^{*T} D_j}{\theta_j^*}\theta_j^* < 0\right\}\right] ,$$

If $y^r = X$, $\bar{\eta}_{k-1} = \bar{\eta}_{k-1}^*$, go to step (2)

Step (4) :

The vector y^k is optimum for all $\eta \in (\underline{\eta}_{k-1}, \overline{\eta}_{k-1})$. Put $\underline{\eta}_k = \overline{\eta}_{k-1}$, k $=$ k+1 , and if $\theta_j^* = D^{-1}\hat{c}_j^T = 0$, go to step (6), otherwise, $y^r = X$, $\overline{\eta}_{k-1} = \overline{\eta}_{k-1}^*$, go to step (2) .

Step (5) :

If there is one $s^*(0 \leq s^* \leq m-n)$ with $\theta_s^* = D^{-1}\hat{c}_j^T < 0$, go to step (6), otherwise, we have :

$$\eta^* = min_j \left[\overline{\eta} , \; sup \left\{ -\frac{y^{*T}D_j}{\theta_{s^*j}} \mid \theta_{s^*j} > 0 \; , \; Z_{s^*} - c_{s^*} < 0 \right\}\right]$$

If $\eta^* = \overline{\eta}$, go to step (6), or if $\eta_{k-1} = \eta^*$, go to step (1).

Step (6) :

The problem (3.3) for all $\eta \in (\underline{\eta} , \overline{\eta})$ is unsolvable .

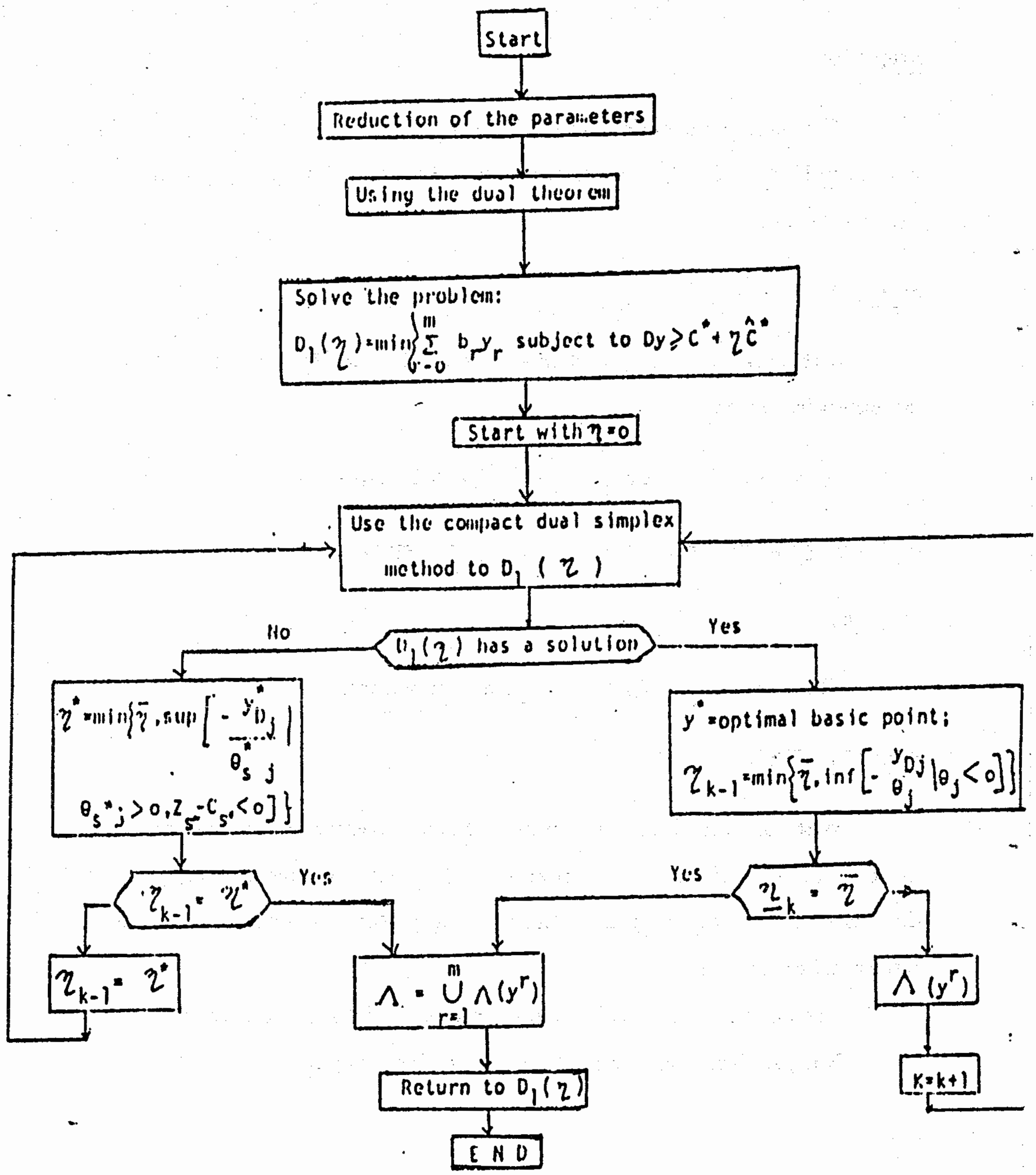
Step (7) : End.

With this algorithm, we obtain a unique determined subdiviation of $(\underline{\eta} , \overline{\eta})$, in a finite number of sub-intervals and to each sub-interval (η_{k-1} , η_k) there is only one optimal point, otherwise the problem is unsolvable in the corresponding interval .

The algorithm (II) can be summarized in the following flow chart

Now, applying algorithm (II) to the previous example 2.1.

Generalization Algorithm.



Flow Chart (II)

Example (3.1) :

consider the problem

$$P_1(\lambda) = \; max \; \{\lambda_o f_o(x) + \lambda_1 f_1(x) + \lambda_2 f_2(x) \mid x \in M \; , \; \lambda_j \geq 0 \; , \; and \; \sum_{j=0}^{2} \lambda_j = 1 \Bigg\} \; ,$$

where

$f_o(x) = 4x_1 + x_2 + 2x_3$,

$f_1(x) = x_1 + 3x_2 - x_3$,

$f_2(x) = -x_1 + x_2 + 4x_3$

subject to M :

$x_1 + x_2 + x_3 \leq 3$,

$2x_1 + 2x_2 + x_3 \leq 4$,

$x_1 - x_2 \leq 0$, $x_i \geq 0$ $(i = 1, 2, 3)$.

This problem can be reduced by using the reduction of parameters

$$P_1(\eta) = \; max \; \{(6 - 2\eta)x_1 + (7 - 2\eta)x_2 + 5\eta x_3 \mid \eta \geq 0 \; , \; x \in M\} \; ,$$

subject to :

$$x_1 + x_2 + x_3 \leq 3 \; ,$$

$$2x_1 + 2x_2 + x_3 \leq 4 \; , \; x_1 - x_2 \leq 0 \; , \; x_i \geq 0 \; \; (r = 1, 2, 3) \; .$$

Using the duality theorem to convert the problem with the parameters in the right hand side of the constraints i.e.

$$D_1(\eta) = \; min \; \{3y_1 + 4y_2 \mid y \in N\} \; ,$$

subject to n :

$$y_1 + 2y_2 + y_3 \geq (6 - 2\eta) \; , \; y_1 + 2y_2 - y_3 \geq (7 - 2\eta) \; ,$$

$$y_1 + y_2 \geq 5\eta \; , \; y_i \geq 0 \; \; (i = 1, 2, 3) \; .$$

This problem can be rewriten in the form :

$$D_1(\eta) = \ min \ \{3y_1 + 4y_2 \mid y \in N\} \ ,$$

subject to N :

$$-y_1 - 2y_2 - y_3 + y_4 = (2\eta - 6) \ , \ -y_1 - 2y_2 + y_3 + y_5 = (2\eta - 7) \ ,$$

$$-y_1 - y_2 + y_6 = -5\eta \ , \ y_s \geq 0 \ \ (s = 1, 2, ..., 6) \ .$$

To solve this problem $D_1(\eta)$, we can use algorithm-II, and for getting the o.b we shall use the dual simplex method .

put $\eta = 0$

$$D_1(\eta) - 3y_1 - 4y_2 = 0 \ , \ -y_1 - 2y_2 - y_3 + y_4 = -6 \ ,$$

$$-y_1 - 2y_2 + y_3 + y_5 = -7 \ , \ -y_1 - y_2 + y_6 = 0 \ ,$$

$$y_s \geq 0 \ \ (s = 1, 2, ..., 6) \ .$$

Since all $z_j^* - c_j^* \leq 0$, i.e. min $D_1(\eta) = 14$ and the o.b.f.s. is (0,3,5,0) .

Now, if $\eta \neq 0$, let us write :

$$\bar{c} = c + \eta c = (-6, -7, 0) + \eta(2\,, 2\,, -5) \ .$$

In order to perform the initial sensitivity analysis, we need to find :

$$D^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -0.5 & 0 \\ 0 & -0.5 & 1 \end{bmatrix} \ ; \ \hat{c}^T = \begin{bmatrix} 2 \\ 2 \\ -5 \end{bmatrix}$$

i.e., $\theta = D^{-1}\hat{c}^T = (0, -1, -7)$, and $Y_D^T = (1, 3.5, 3.5)$.

The first critical value of η is given by :

$$\eta = \ min \ \left\{ -\frac{y_{Dj}^T}{\theta_j} \ \mid \theta_j < 0 \right\}$$

i.e.

$$\eta_1 = \ min \ \left\{ \frac{-3.5}{-1} \, , \ \frac{-3.5}{-7} \right\} = \ min \ \{3.5 \ , \ 0.5\} = 0.5$$

$$.. \ \eta^{(1)} = \eta_1 + 0 = 0.5 \ ; \ \ and \ y^{(0)} = (0, 3.5, 0) \ at \ 0 \leq \eta \leq 0.5$$

Put $\eta = 0.5$:

our problem becomes

$D_1(\eta) - 3y_1 - 4y_2 = 0$, $-y_1 - 2y_2 - y_3 + y_4 = -5$,

$-y_1 - 2y_2 + y_3 + y_5 = -6$, $-y_1 - y_2 + y_6 = -2.5$,

$y_s \geq 0 \;\; (s = 1, 2, ..., 6)$.

By a similar way, for $\eta = 0.6$, $\eta = 0.97$, and $\eta = 1$.

Hence we have shown that an optimum solution to the given last problem is :

$y^{(0)} = (0, 3.5, 0)$	at	0	$\leq \eta < 0.5$
$y^{(1)} = (0, 3, 2)$	at	0.5	$\leq \eta < 0.6$
$y^{(2)} = (0.2, 2.8, 2)$	at	0.6	$\leq \eta < 0.97$
$y^{(3)} = (4.64, 0.21, 0)$	at	0.9	$\leq \eta < 1$
$y^{(4)} = (5, 0, 0)$	at	1	$\leq \eta$

Since, y = (0 , 3.5 , 0) is optimal for all $\lambda \in \Lambda(y)$, therefore

$$0 \leq \frac{\lambda_2}{\lambda_o} < \frac{1}{2} \rightarrow \frac{3}{2} < \frac{\lambda_1}{\lambda_o} \leq 2$$

i.e.

$$\Lambda^*(y^o) = \{\lambda \mid \lambda_o - 2\lambda_o > \;\lambda_1 - 3\lambda_o > 0 \;,\; 2\lambda_o - \lambda_1 \geq 0 \;;$$

$$\lambda_j \geq 0 \;\; (j = 0, 1, 2)\,\}$$

Also $y^{(1)} = (0, 3, 0)$ is optimal for all $\lambda \in \Lambda^*(y^1)$, where

$\Lambda^*(y^1) = \{\lambda \mid 2\lambda_2 - \lambda_o \geq 0 \;,\; 3\lambda_o - 5\lambda_2 > 0 \;,\; 5\lambda_1 - 7\lambda_o > 0$,

$3\lambda_o - 2\lambda_1 \geq 0 \;,\; \lambda_j \geq 0 \;\; (j = 0, 1, 2)\,\}$,

$y^{(2)} = (0.2 \;,\; 2.8 \;,\; 0)$ is optimal for all $\lambda \in \Lambda^*(y^2)$, where

$\Lambda^*(y^2) = \{\lambda \mid 5\lambda_2 - 3\lambda_o \geq 0 \;,\; 97\lambda_o - 100\lambda_2 > 0 \;,\; 100\lambda_1 - 103\lambda_o > 0$,

$7\lambda_o - 5\lambda_1 \geq 0 \;;\; \lambda_j \geq 0 \;\; (j = 0, 1, 2)\,\}$,

$y^{(3)} = (4.66 \ , \ 0.21 \ , \ 0)$ is optimal for all $\lambda \in \Lambda(y^3)$, where

$\Lambda^*(y^3) = \{\lambda \mid 100\lambda_2 - 97\lambda_o \geq 0 \ , \ \lambda_o - \lambda_2 > 0 \ , \ \lambda_1 - \lambda_o > 0 \ ,$

$103\lambda_o - 100\lambda_1 \geq 0 \ ; \ \lambda_j \geq 0 \ \ (j = 0, 1, 2) \ \} \ ,$

$y^{(4)} = (5 \ , \ 0 \ , \ 0)$ is optimal for all $\lambda \in \Lambda^*(y^4)$, where

$\Lambda^*(y^4) = \{\lambda \mid \lambda_2 - \lambda_o \geq 0 \ , \ \lambda_o - \lambda_1 \geq 0 \ ; \ \lambda_j \geq 0 \ \ (j = 0, 1, 2) \ \} \ .$

Therefore, the set of all efficient points is defined as :

$$E^*_{ox} = \{ \ y^{(0)} \ , \ y^{(1)} \ , \ y^{(2)} \ , \ y^{(3)} \ , \ y^{(4)} \} \ .$$

The set of optimal parameters $\Lambda(y)$ where y, is defined as :

$$\Lambda^* = \{ \ \Lambda^*(y^o) \cup \Lambda^*(y^1) \cup \Lambda^*(y^2) \cup \Lambda^*(y^3) \cup \Lambda^*(y^4) \} \ .$$

(The solvability set).

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تعميم منهج لحل مسائل البرمجة الخطية المتعددة البارامتر

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فى هذا البحث تعتبر مسألة البرمجة الخطية المتعددة البارامتر كمسألة ايجاد مجوعة جميع الحلول المثلى بالنسبة الى المخروط السائد المعلوم وكذلك مجوعة البارامتر المثلى.

والمسألة الأصلية حيث البارامتر موجودة فى دالة الهدف يمكن حلها بالبرنامج I . بينما المسألة التعاكسية حيث البارامتر موجودة فى الطرف الأيمن من القيود يمكن حلها بالبرنامج II.

طريقة السمبلكس ماهى الا طريقة جبرية لحل مسألة البرمجة الخطية المتعددة البارامتر فقط فى عدد محدود من الخطوات. هذا المنهج يعطينا مجموعة الحلول المثلى لأية مسألة برمجة خطية متعددة دالة الهدف ومن ثم مجموعة الحول (The sovability set) (أى مجموعة البارامتر المثلى). وهنا حلول المسألة تعنى أن نقط نهايات المخروط لمجموعة مثلى معلومة بالنسبة الى مخروط محدب معلوم و التى تخصص الهيكل السائد لمتخذى القرار.