

THE EFFECT OF THREAD SWELLING PERCENT INSIDE WOVEN CLOTH ON
FABRIC DIMENSIONAL PROPERTIES UNDER DIFFERENT VALUES OF
INITIAL COVER FACTOR AND THREAD AVERAGE FLOAT RATIO

تأثير الزيادة النسبية بسبب تضخم الخيوط بعدالنسيج على الخواص الابعادية
للنسوج تحت ظروف مختلفة من نسب تشييف الخيوط ومعامل التغطية الابتدائى

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الخلاصة: يشمل هذا البحث استنتاج العلاقات الرياضية وحلها وتمثيلها بيانيا بامكانيات الحاسب الآلى بين الزيادة النسبية فى قطر الخيط داخل النسوج - نتيجة للتضخم اثناء عمليات التجهيز المبلله - وبين ظاهرة انكماش الاقمشة والتغير الحادث فى كثافة الخيوط فى وحدة الاطوال ووزن القماش لوحده المساحات والكثافة الحجمية للنسوج ونسبة التجمد للخيوط ومعامل التغطية الكلى وذلك للأقمشة المتزنة عند قيم مختلفة من معامل التغطية الابتدائى ومتوسط نسبة التشييف للخيوط. حيث أمكن فقط باستعمال الحاسب الآلى حل المعادلات الرياضية والوصول الى التوقع الدقيق لتأثير كل من التركيب النسجى وكثافة الخيوط الابتدائية على درجات الانكماش المتوقعة اثناء عمليات التجهيز المبلله واختيار أنسب الظروف الانشائية للحصول على خواص ابعادية ثابتة عند الفجوة الكبيرة بين التجارب العملية والنماذج الرياضية.

ABSTRACT: This investigation includes conclusion, solving and graphical representation of the mathematical relations between the relative increase of thread diameter caused by swelling due to wet finishing processes and relative changes in fabric shrinkage, thread density, crimp%, fabric weight/unit area, weight/unit volume and total cover factor for balanced fabrics under different values of initial cover factor and thread average float ratio. The results indicated that only by using computers these equations were possible to be solved giving the opportunity to expect the effect of fabric structure and thread density on shrinkage behavior and corosspondant fabric dimensional changes and giving the chance to choose the suitable fabric constructure and to fill the gap between theory and practice.

INTRODUCTION

Most of finishing process applied on grey fabrics are wet processes where fibers such as cotton and wool swells by the effect of water absorbtion causing thread cross sectional area to increase and tending to make yarns circular /2/, the finished fabric is again subjected to the same wet treatment during enduse in the form of garment due to frequent washing, both wet treatments cause the fabric to shrink mainly in length and bring a lot of dimensional changes in fabric properties and hence physical and mechanical, such changes found practically to be irreversible i.e. fabric does not regain its shape after drying.

Fabric shrinkage in particular has attracted the attention of cloth traders and garment makers mainly to maintain shape and size of already made garments during enduse and to ensure an acceptable

shrinkage limits to customers, thus shrinkage measurement became one of the standard quality tests although swelling of fibers and yarns affects thread density, crimp%, fabric thickness, cloth cover factor, weight/unit area and weight/unit volume. Practical experiment on shrinkage and number of washing times and type of raw material indicated that dimensional changes continue to change for sometime during enduse /4/ and with the gradual increase of quality standards, the necessity to antishrink or shrink-resistant fabrics began to be urgent and anti shrinking /2,6/, treatments and machines began to be used on industrial scale as an option according to dealers requirements, the idea of such treatment is based on drying the fabric between 250-300°C on curved rubber belts causing some compulsory degree of shrinkage in advance and whether fabric shrinkage is caused by thread swelling or fabric relation after stretching /2/ it seems to be an undesirable property, while in case of producing crepe /5/or mercerized fabrics shrinkage behaviour seems to be very required even with 25%.

The mathematical models or theoretical approach of the relations between swelling and shrinkage with other fabric properties seemed to be missing and the gap between practice and theory was very wide inspite of the capabilities of using computer to solve such problems. This work is intended to be beginning of a series of theoretical studies to quantify these relations and to give the chance to estimate the swelling effect or shrinkage effect on dimensional fabric properties.

I.1- The relation between crimped length, thread diameter and thread spacing for balanced plain weave.

Fig. (1) shows a balanced plain weave of circular cross-section - half a repeat is enough to represent such symmetrical geometry - where thread diameters in both direction were allowed to increase from their initial value D_0 to D resulting a decrease in their thread spacing from P_0 to P , to relate this decrease - practically known as shrinkage% - to diameter increase - practically swelling% - a mathematical equations must be derived considering the consistency of crimped length (L), and fully flexible threads.

Nomenclature

- D thread diameter after swelling
 - D_0 initial thread diameter
 - P_0 thread spacing
 - P initial thread spacing
 - L_0 thread crimped length for half repeat of plain fabric
 - θ wrap angle between warp-weft.
 - I_c initial fabric cover factor.
 - $(L_T)^{kg}$ total crimped length per repeat for fabric of average float ratio 1
 - F thread average float ratio.
- $L = ab + bd + dg$ measured along its neutral axes (abdg)
 $L = 2(ab + bc) \text{ as}$ $ab = dg = \text{arc length against angle } (\theta)$
 $bd = \text{straight length} = 2bc$

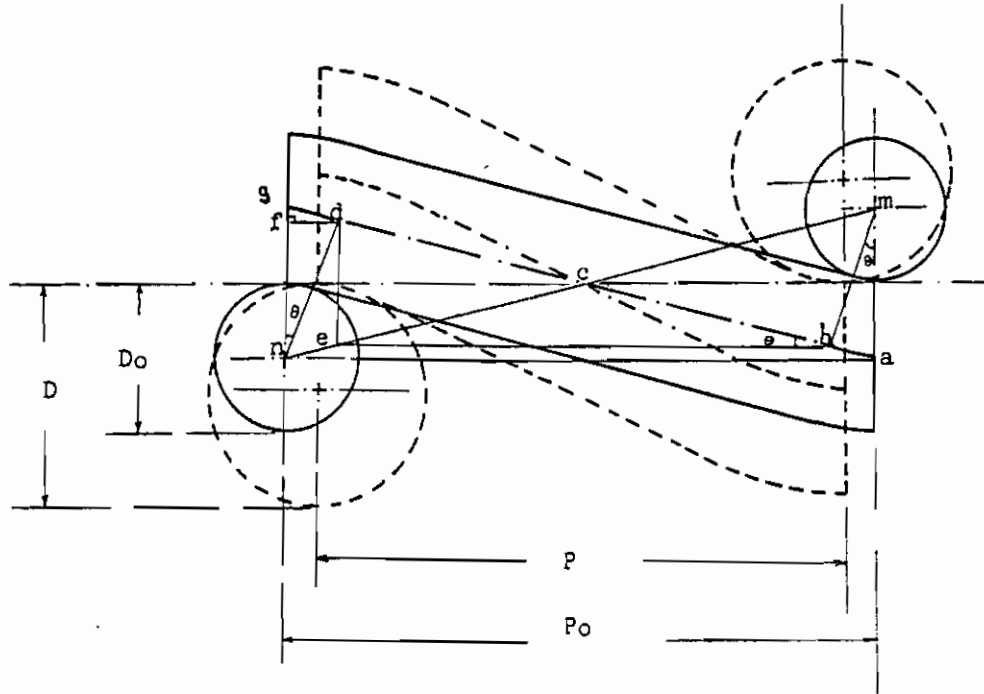


Fig.(1) Balanced plain weave cross-section under swelling effect.

the straight length (bd) could be calculated as follows:

$$\begin{aligned}
 2 \text{ bc} &= 2 \sqrt{(m \text{ c})^2 - D_o^2} && \text{(from } \Delta \text{ m bc)} \\
 &= 2 \sqrt{\frac{1}{2} (m \text{ n})^2 - D_o^2} && \text{(mc} = \frac{1}{2} \text{ mn)} \\
 &= 2 \sqrt{\frac{1}{2} (P_o^2 + D_o^2) - D_o^2} && \text{(as (mn)}^2 = P_o^2 + D_o^2 \text{ from } \Delta \text{ mna)} \\
 \therefore \text{ bd} &= \sqrt{P_o^2 - 3 D_o^2} && \dots(1)
 \end{aligned}$$

the arc length of (ab) estimated by calculating first the wrap angle (θ) as follows:

$$\begin{aligned}
 \text{bd} \cdot \sin \theta &= D_o - 2 D_o (1 - \cos \theta) && \text{from } \Delta \text{ bde and } \Delta \text{ dnf} \\
 (\text{bd} \cdot \sin \theta + D_o)^2 &= (2 D_o \cos \theta)^2 \\
 \overline{\text{bd}}^2 \sin^2 \theta + 2(\text{bd}) \cdot D_o \cdot \sin \theta + D_o^2 - 4 D_o^2 (1 - \sin^2 \theta) &= 0 \\
 (\overline{\text{bd}}^2 + 4 D_o^2) \sin^2 \theta + 2 D_o \cdot (\text{bd}) \cdot \sin \theta - 3 D_o^2 &= 0 \\
 \sin \theta &= \frac{-2 D_o (\text{bd}) + \sqrt{(2(\text{bd}) \cdot D_o)^2 + 4 ((\text{bd})^2 + 4 D_o^2) \cdot 3 D_o^2}}{2 ((\text{bd})^2 + 4 D_o^2)}
 \end{aligned}$$

neglecting the other root of equation as wrap angle is not allowed to be $> 180^\circ$.

Using equation (1)

$$\therefore \theta = \arcsin \frac{-D_o \sqrt{P_o^2 - 3 D_o^2} + 2 D_o P_o}{P_o^2 + D_o^2} \dots(2)$$

$$L = \sqrt{P_o^2 - 3 D_o^2} + 2 D_o \arcsin \frac{-D_o \sqrt{P_o^2 - 3 D_o^2} + 2 D_o P_o}{P_o^2 + D_o^2} \dots(3)$$

$$L/D_o = \sqrt{(P_o/D_o)^2 - 3} + 2 \arcsin \frac{-\sqrt{(P_o/D_o)^2 - 3} + 2 P_o/D_o}{(P_o/D_o)^2 + 1}$$

if $G = P_o/D_o$, $Z = L/D_o$

$$\therefore Z = \sqrt{G^2 - 3} + 2 \arcsin \frac{2G - \sqrt{G^2 - 3}}{G^2 + 1} \dots(4)$$

hence the constant crimped length (L) related to the initial diameter (D_o) could be calculated from initial condition of P_o/D_o (practically reciprocal of the cover fractional ratio).

Equation (3) could be then written after diameter increase and thread spacing decrease in a general formula of (P) and (D) and dividing both sides by D_o and substituting $P/D_o = Y$, $D/D_o = X$.

The General formula could be written as follows:

$$Z = \sqrt{Y^2 - 3 X^2} + Z X \arcsin \frac{2 X Y - X \sqrt{Y^2 - 3 X^2}}{Y^2 + X^2} \dots(5)$$

where Z represents the constant crimped length which could be calculated by initial condition.

Y the thread spacing related to initial thread diameter.

X the thread diameter related to initial thread diameter.

As shown from equation (5) it is difficult to estimate the effect of (X) as an independent variable on Y as dependant variable at constant values of Z and only by computation such problem could be solved by assuming an initial values for Y for given value of X and Z then by looping method the correct value of Y could be estimated and from (X) & (Y) the relation between thread swelling and thread spacing could be illustrated as in Fig.(2).

1.2- The effect of thread swelling percent on shrinkage

By taking the basic structural unit of plain weave the fabric shrinkage 5% could be calculated as follows:

$$\text{fabric shrinkage } 5\% = \frac{\text{initial thread spacing} - \text{final thread spacing}}{\text{initial thread spacing}} \times 100$$

$$\begin{aligned} \therefore S\% &= (1 - P/P_0) \times 100 \\ &= (1 - Y/G) \times 100 \end{aligned} \quad \dots(6)$$

By using equations(5) and (6) the relation between S% and $(X=D/D_0)$ could be obtained at different values of G as in Fig.(3) where X is represented as swelling percent (D/D_0-1) and G is represented as initial cloth cover factor IKc.

$$\begin{aligned} \text{where IKc} &= \frac{28}{G} + \frac{28}{G} - \frac{28}{G} \cdot \frac{28}{G} \cdot \frac{1}{28} \\ &= \frac{28}{G} \left(2 - \frac{1}{G}\right) \end{aligned} \quad \dots(7)$$

Areal fabric shrinkage % could also be calculated as;

$$((1 - S)^2 - 1) \times 100 = (S^2 - 2S) \times 100$$

1.3- The relation between warp and weft crimp and swelling percent

By taking the same basic unit of plain weave:

$$\begin{aligned} \text{The Crimp \%} &= (L - P)/P \times 100 \\ &= (L/P - 1) \times 100 \\ &= ((L/D_0)/(P/D_0) - 1) \times 100 \\ &= (Z/Y - 1) \times 100 \end{aligned} \quad \dots(8)$$

as (Z) is a constant depends on value of G then the relation between C% and swelling percent $(X - 1)$ could be estimated using equations(8) and (5) as in Fig. (4) for different values of IKc.

1.4- The relation between fabric cover factor and thread swelling%.

As the general formula of fabric cover factor Kc equals:

$$= \frac{\text{warp cover factor} + \text{weft cover factor} - 1/28}{(\text{warp cover factor} \times \text{weft cover factor})}$$

$$\begin{aligned} \therefore Kc &= 2 \times 28 (D/P) - 1/28 (D^2/P^2) \dots \text{in case of balanced weave} \\ &= 56 (D/D_0)/(P/P_0) - 1/28 (D/D_0)^2/(P/P_0)^2 \\ &= 56 X/Y - 1/28 X^2/Y^2 \end{aligned} \quad \dots(9)$$

The relation between Kc and $(X-1)$ for different IKc values could be obtained as in Fig.(5).

1.5- The relation between relative fabric weight/unit area (W^*) and swelling%.

$$\begin{aligned} \text{As } (W^*) &= \frac{\text{fabric weight/unit area after shrinkage-fabric initial weight/unit area}}{\text{Initial fabric weight/unit area}} \times 100 \\ &= (W/W_0 - 1) \times 100 \end{aligned}$$

$$\begin{aligned}
 \text{as } W &= 2 \cdot \frac{1 \cdot (1/P) \cdot (1+C)}{N_e \cdot S}, & W_o &= 2 \cdot \frac{1 \cdot (1/P_o) \cdot (1+C_o)}{N_e \cdot S} \\
 W^* \% &= \left(\frac{P_o(1+C)}{P(1+C_o)} - 1 \right) \times 100 \\
 &= \left(\frac{P_o/D_o(1+C)}{P/D_o(1+C_o)} - 1 \right) \times 100 \\
 W^* &= \left(\frac{G(1+C)}{Y(1+C_o)} - 1 \right) \times 100 \quad \dots(10)
 \end{aligned}$$

As (G) and C % are constants the relation between W*% and swelling percent ((X-1) x 100) could be estimated from the corresponding values of Y and C% at different values of IKc as in Fig. (6).

1.6- The relation between volumetric density ratio and thread swelling percent.

Assuming a piece of fabric of unit area and fixed weight Q and initial thickness 2 D_o:

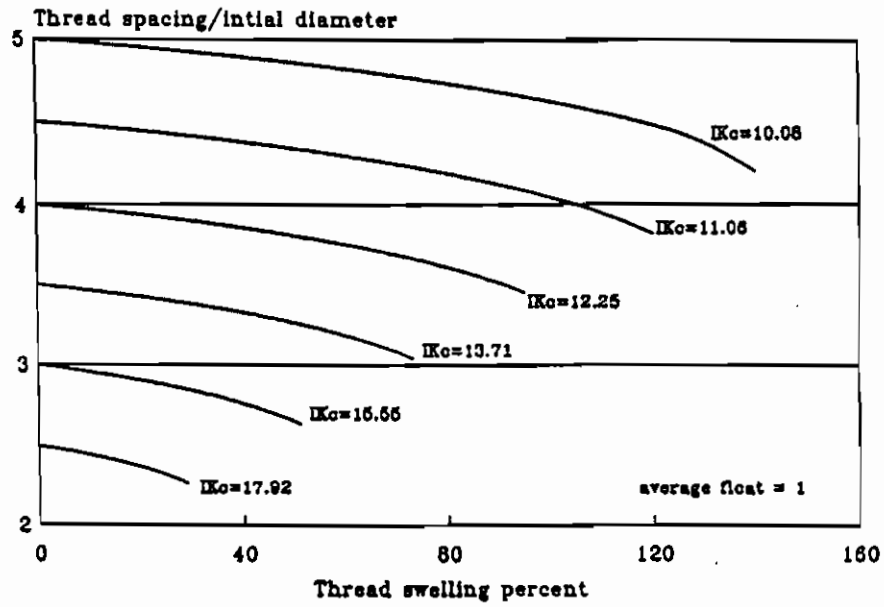
$$\begin{aligned}
 \text{Initial fabric weight/unit volume} &= \frac{Q}{1.1.2 D_o} \\
 \text{Final fabric weight/unit volume} &= \frac{Q}{(1-S)(1-S) \cdot 2D} \\
 \text{Volumetric density ratio} &= \frac{\text{final fabric weight/unit volume}}{\text{initial fabric weight/unit volume}} \\
 &= \frac{1}{(1-S)^2} \cdot \frac{D_o}{D} \\
 &= \frac{1}{X(1-S)^2} \quad \dots(11)
 \end{aligned}$$

Fig.(7) shows the relation between swelling percent and volumetric density ratio at different values of IKc.

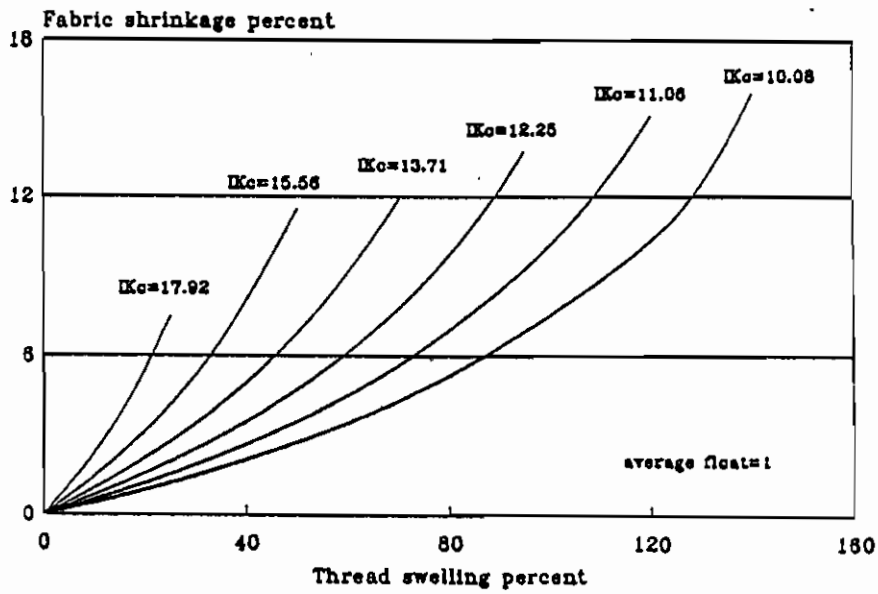
2.1- The mathematical relation between the crimped thread length per unit repeat and average fabric float ratio.

Let for example a balanced fabric of 1/2 as a pattern of thread interlacing in both warp and weft directions-which is practically used as a type of twill or irregular matt weave-, a full repeat is shown in Fig.(8). The total crimped length (L_T) per unit repeat could be calculated as follows:

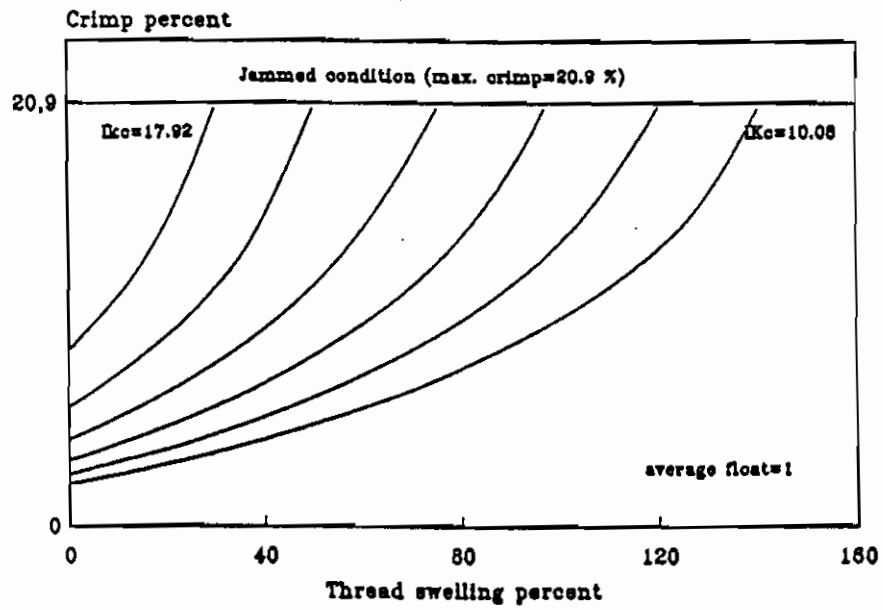
$$\begin{aligned}
 L_T &= ab + bc + cd + de + ef + fg + gh + hi + ij \\
 \text{as } ab &= de = ef = hi = \text{arc length against angle } \theta \\
 \text{and } bc &= cd = fg = gh
 \end{aligned}$$



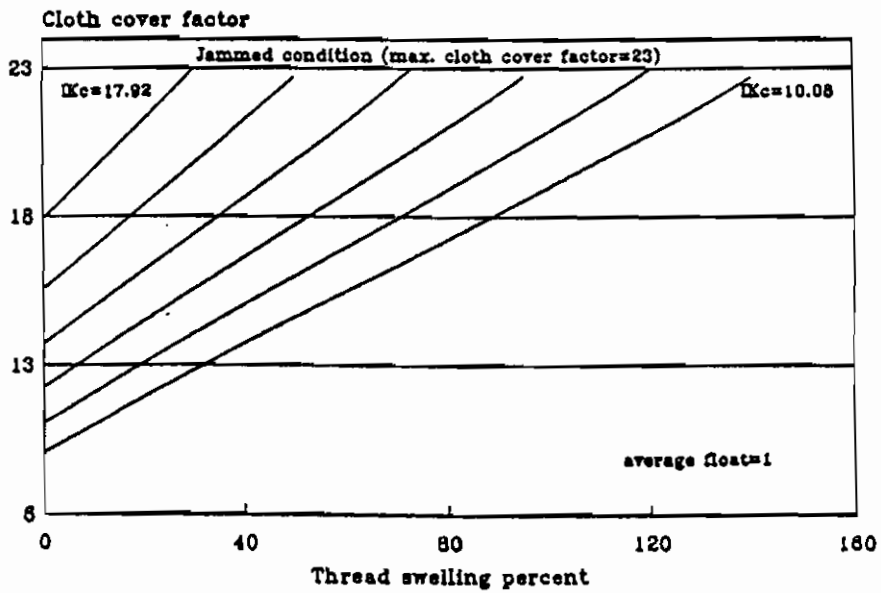
Fig(2) Thread swelling % - Rel.thread spacing at different initial cloth cover factor



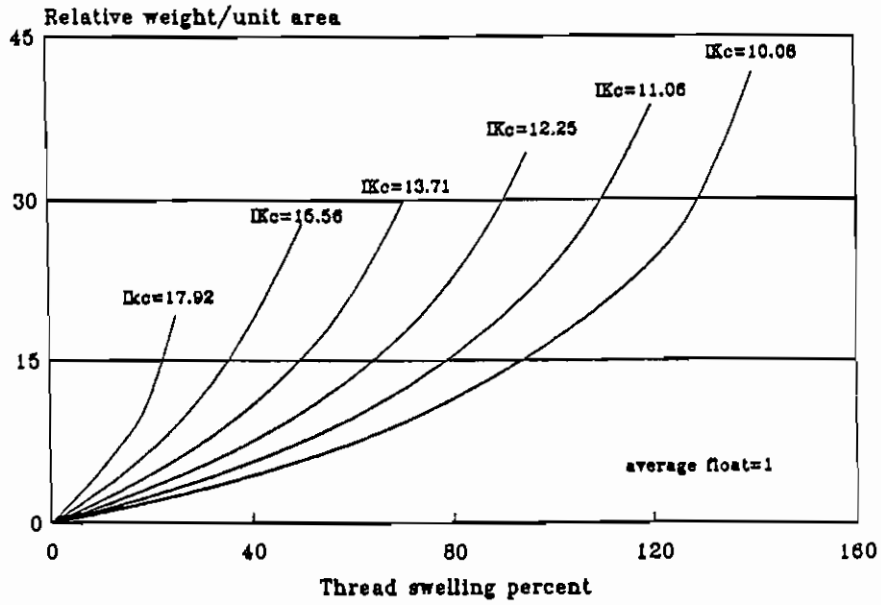
Fig(3) Thread Swelling % - Shrinkage % at different initial cloth cover factor



Fig(4) Thread swelling % - Crimp %
at different initial cloth cover factor



Fig(5) Thread swelling % - Cloth cover
at different initial cloth cover factor



Fig(6) Thread swelling % - Rel.weight/unit area at different initial cloth cover factor

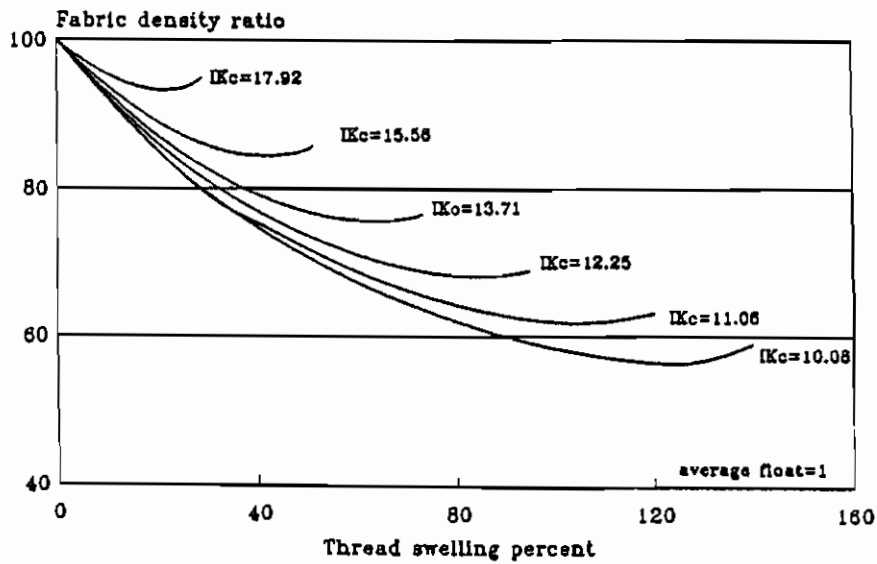


Fig (7) Thread swelling % - Fabric density ratio at different initial cloth cover factor

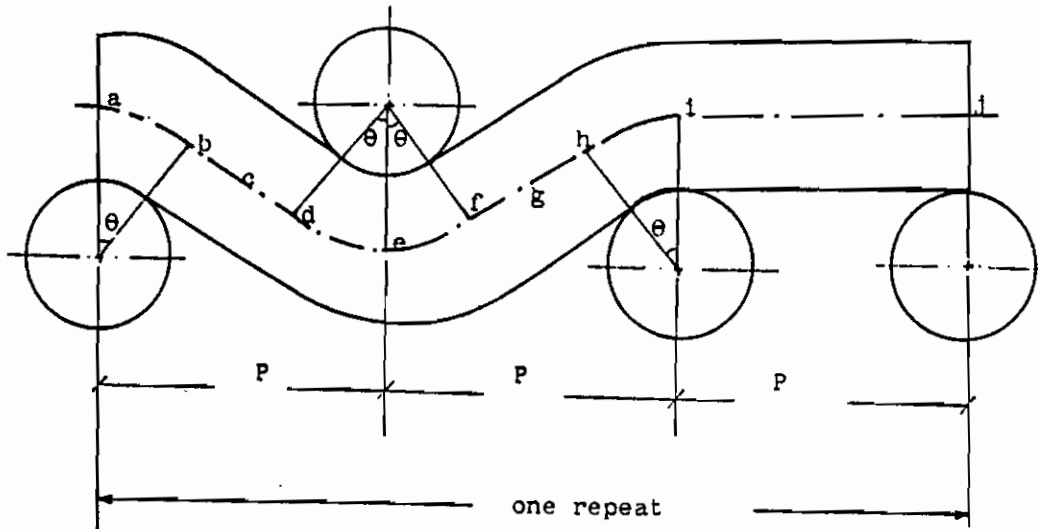


Fig.(8) Balanced fabric of 1/2 thread interlacing.

$$\begin{aligned} \text{Therefore } L_T &= 2(2 ab + 2 bc + \frac{1}{2} ij) \\ &= 2(L + \frac{1}{2} p). \end{aligned}$$

The average thread float ratio by definition (F):

$$\begin{aligned} F &= \frac{\text{total number of ends/repeat}}{\text{number of interlacing/repeat}} \\ &= \frac{1+2}{2} = 1.5 \quad (\text{in case of } 1/2 \text{ thread interlacing}) \\ L_T &= 2(L + (F-1)P) \quad \dots(12) \end{aligned}$$

Equation (12) could be used as general formula to calculate the total crimped length (L_T) for any average float ratio.

$$\begin{aligned} \therefore L_T/D_0 &= 2L/D_0 + 2(F-1).P/D_0 \\ \therefore Z_T &= 2Z + 2(F-1).Y \\ &= 2\sqrt{Y^2 - 3X^2} + 4X \cdot \text{arc sin } \frac{2XY - X\sqrt{Y^2 - 3X^2}}{Y^2 + X^2} + 2(F-1).Y \quad \dots(13) \end{aligned}$$

To illustrate graphically such relation where a new independent variable (F) have been added it is necessary to fix a certain value for (Z) by means of Choosing a certain value for P_0/D_0 which corresponds to intisl fabric cover float IKc, the value 10.88 for IKc was chosen to relate between the value (X-1) and (Y) by computing at different values of (F). Fig.(9) shows the relation between (X) and (Y) at different values for average thread float.

2.2- Effect of thread swelling percent on crimp % at different values of average thread float ratio.

$$C \% = \frac{L_T - 3P}{3P} \quad (\text{in case of } 1/2 \text{ thread interlacing})$$

$$C \% = \frac{2L + P - 3P}{3P}$$

$$= \frac{2}{3} (L/P - 1)$$

∴ general formula for crimp could be described as:

$$C \% = \frac{1}{F} (L/P - 1) = \frac{1}{F} (Z/Y - 1) \quad \dots(14)$$

Using equations (13) and (14) the relation between C % and swelling percent could be estimated as in Fig. (10).

2.3- Effect of swelling % on fabric cover factor and shrinkage%.

Using equations (6) and (9) with the relation between X, Y from equation (13) the fabric cover factor and shrinkage % could be estimated as in Figs. (11) and (12) respectively.

2.4- Effect of thread swelling % on relative weight per unit area (W^*).

Using equation (10) with the correspondent values Y from equation (13) and correspondent values of crimp eq. (14) the relation between swelling percent and relative weight/unit area could be estimated as in Fig. (13).

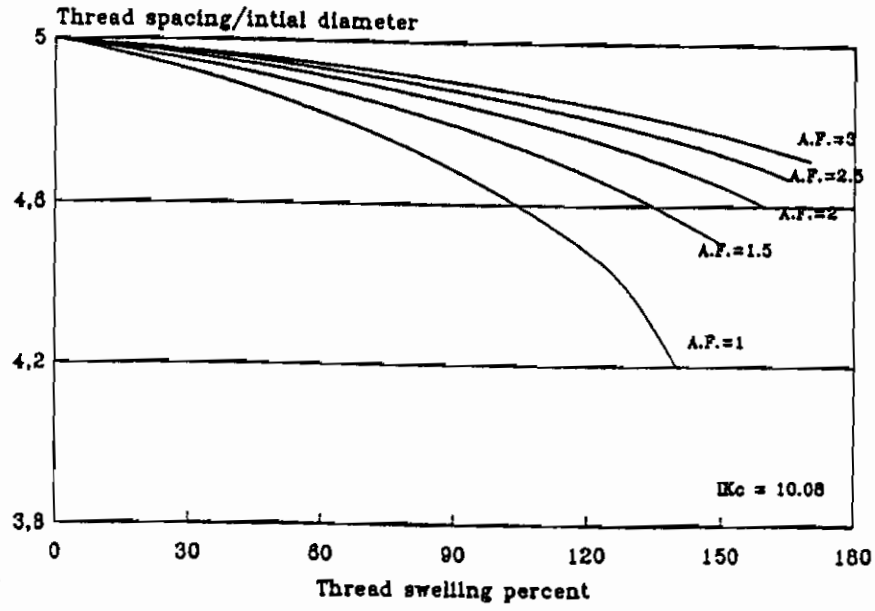
2.5- Effect of swelling percent on volumetric density.

Using equation (11) with the correspondent values of shrinkage percent the relation between swelling percent and the ratio of volumetric density could be illustrated as in Fig. (14).

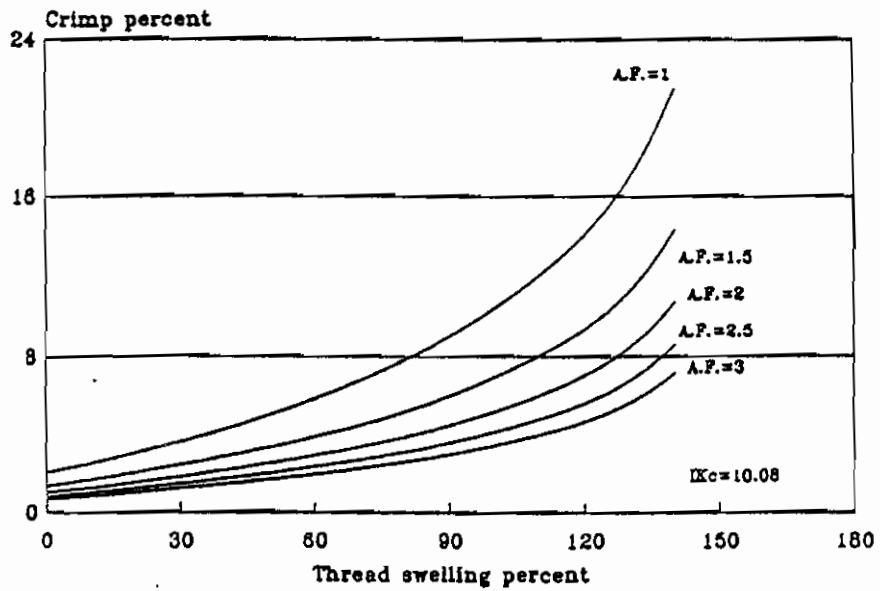
DISCUSSION AND CONCLUSION

It is obvious that the mathematical relations between the various fabric parameters affected by thread swelling were genuine and successful, featured by the difficulty of finding a proper mathematical solution as seen from equations (5) and (13), and only by the use of computer it was possible to overcome such difficulty to represent and analyze these relations.

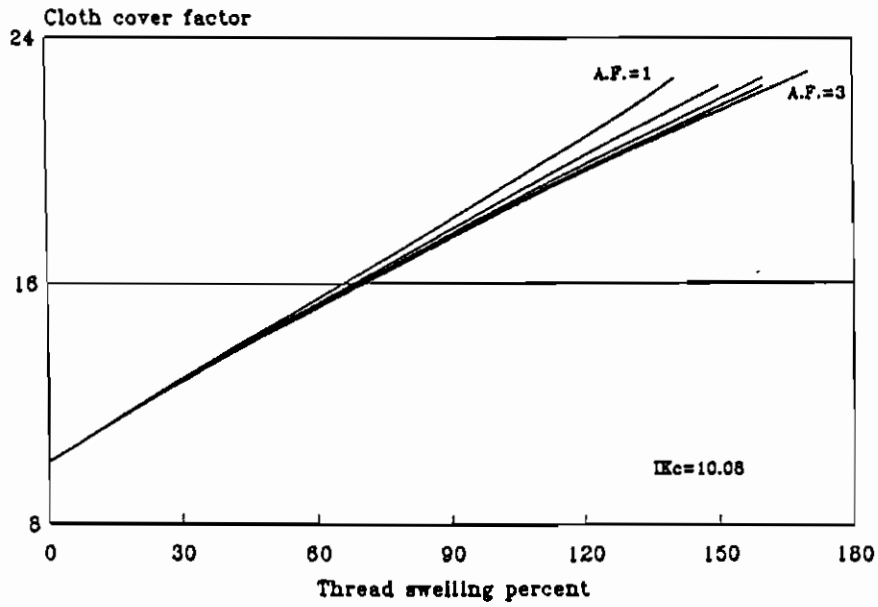
The assumption of maintaining the crimped length (L) and (L_T) as constants was very effective and proved to be reasonable and probably very near from actual conditions. If flattening is to be added as an initial shape of thread cross section the model could be extended further. It is also obvious from the graphical representation of the derived equations that the following conclusions could be reached.



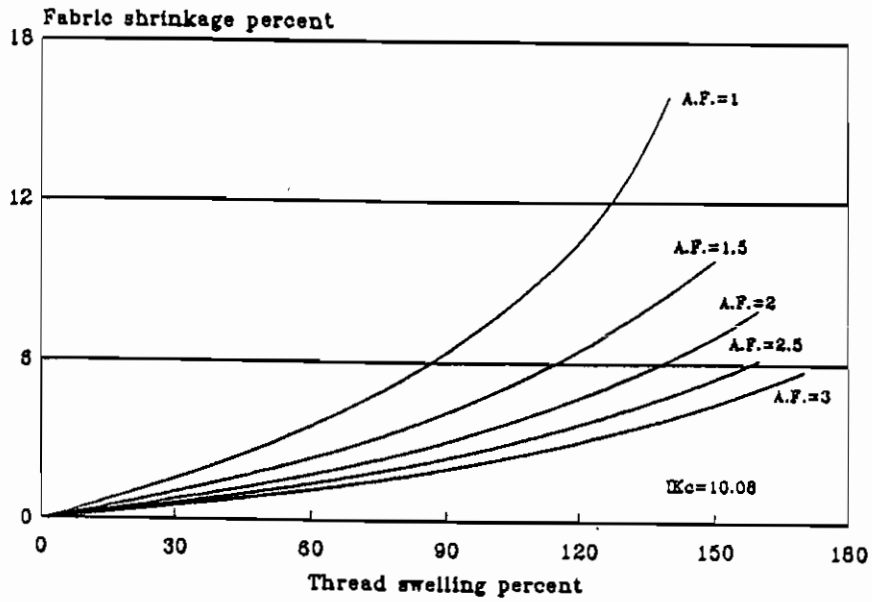
Fig(9) Thread swelling % - Rel.thread spacing at different average float



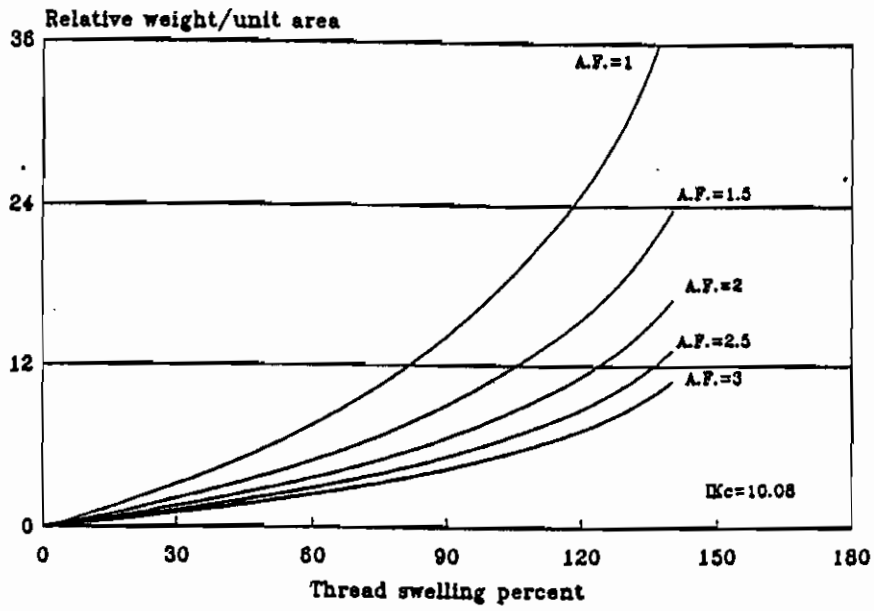
Fig(10) Thread swelling % - Crimp % at different average float



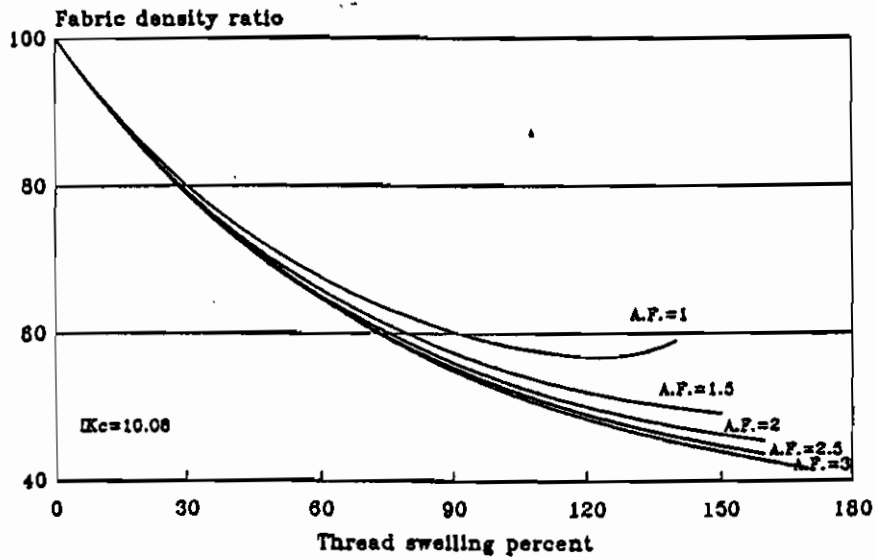
Fig(11) Thread swelling % - Cloth cover at different average float



Fig(12) Thread swelling % - Shrinkage % at different average float



Fig(13) Thread swelling % - Rel.Weight/unit area at different average float



Fig(14) Thread swelling % - Fabric density ratio at different average float

By the increase of thread swelling % which is indicated by relative increase of thread diameter, the relative thread spacing decreases i.e. increases of thread density per unit length and the increase of initial cloth cover factor decreases the allowed range for thread spacing decrease, while the increase of thread average float ratio tends to reduce such rate of decrease.

Shrinkage % is considered to be the practical form of thread spacing decrease and it is affected by both initial cover factor and average float ratio, e.g. at 25% swelling the shrinkage reaches 8% at initial cover factor of 18 app. while reaches only 1% at initial cover factor of 10 app. At the same time the max. shrinkage % in the first case reaches only 10% while it can reach easily 16% in the second case and when the average float ratio was increased from 1 to 3 the shrinkage decreased from 16% to 3.5%.

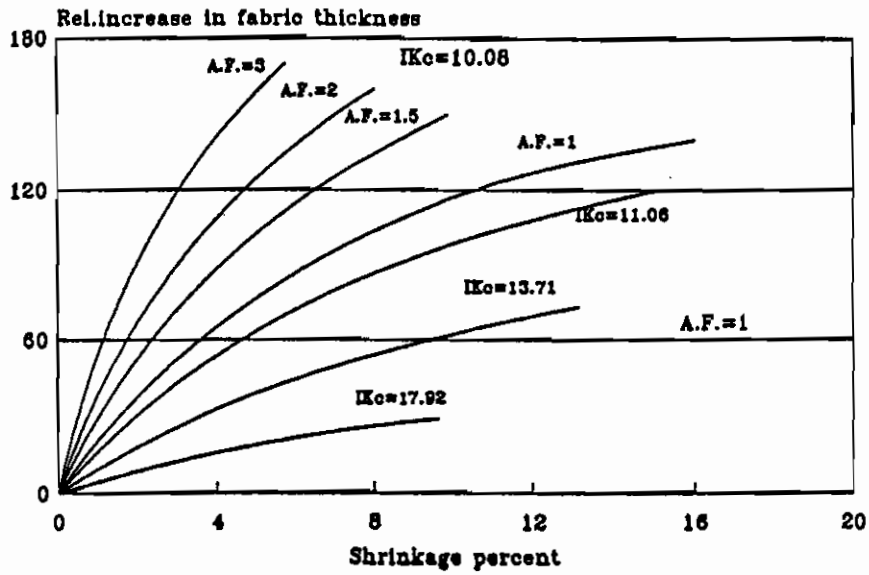
Crimp percent is obviously increases by the increase of swelling% initially with different rates and finally with approximately equal rates under the effect of initial cover factor ending with 20.8% for all values of cover factor at jammed condition and when increasing the average float ratio from 1 to 3 the crimp % decreased from 12% to 2% at 100% swelling.

The cloth cover factor is mutually increased by both increase of crimp % and decrease of thread spacing which are both initiated by swelling. The equal linear relation for different initial cover factor which ends with the max. cloth cover factor of 23 for plain weave is obvious while the effect of the increase of average float ratio is very less noticeable.

The fabric weight per unit area seems to be increasing exponentially by the increase of thread swelling % through its effect on increasing thread density and crimp%, such increase seems to be of equal rates under the effect of initial cloth cover factor with exception of smaller values of swelling%, while the increase of thread average float ratio tends to reduce noticeably these rates.

The fabric volumetric density is affected by two contradicting factors which are the decrease of fabric area due to shrinkage and the increase of fabric thickness as both initiated by thread swelling the results indicates such contradicting behaviour under the effect of initial cloth cover factor showing that the dominant effect is decreasing the volumetric density, which is also activated by average float ratio increase.

In practice it is easier to measure fabric length before and after wet treatments i.e. shrinkage % rather than thread diameters and hence swelling%, therefore it would be of great benefit practically to plot some of the fabric properties directly on their corresponding shrinkage% as shown in Fig.(15) which relates the increase of fabric thickness with fabric shrinkage under different conditions of initial cloth cover factor and thread average float ratio. It is also possible to plot crimp% and fabric cover factor against shrinkage % firstly under the different values of initial



Fig(15) Shrinkage % - Rel.increase in fabric thickness at different intial cloth cover factor and different average float

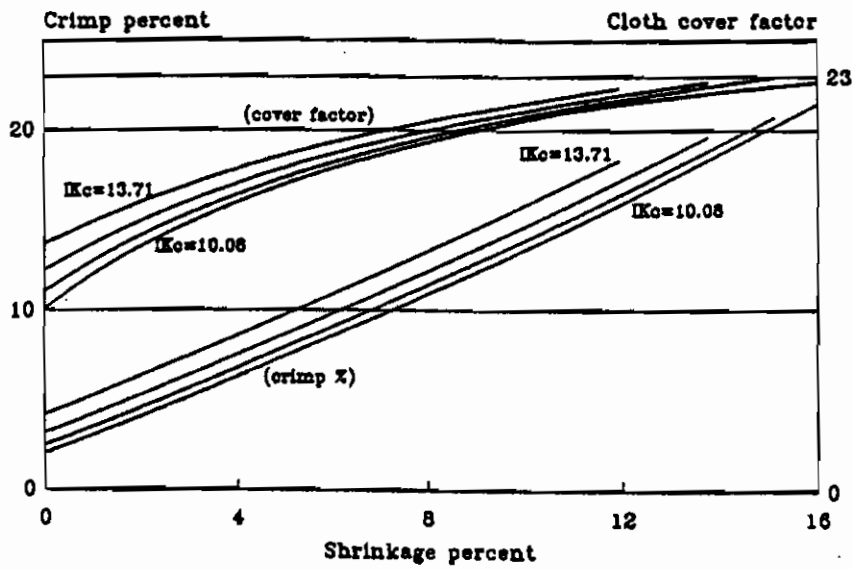
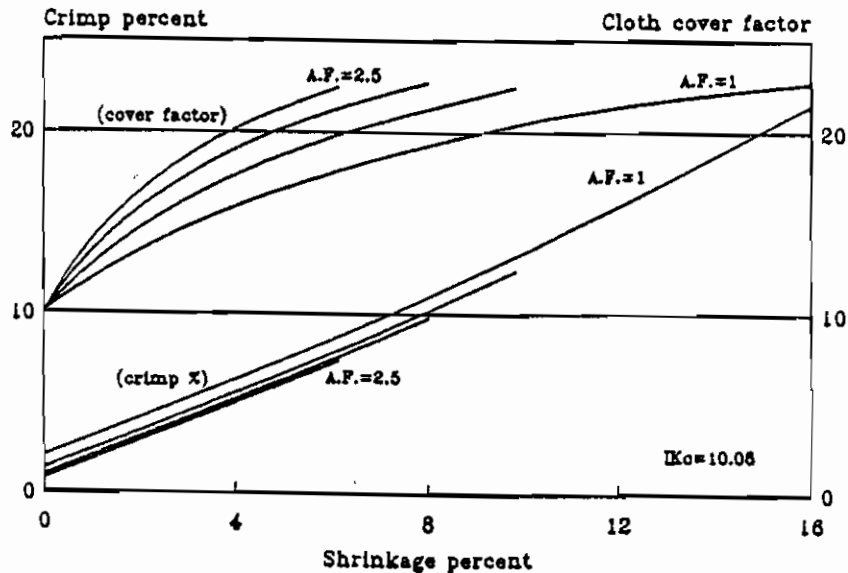


Fig (16) Fabric shrinkage % & Crimp % plotted against fabric shrinkage % at different intial cloth cover factor

cloth cover factor as in Fig.(16) and secondly under different values of thread average float as in Fig.(17). Finally this work is considered to be a real means of linking the use of computer to solve some of the mathematical problems in fabric geometry and modeling to fill the gap between theory and practice in some fields of study in textiles.



Fig(17) Fabric shrinkage % & Crimp % plotted against fabric shrinkage % at different average float

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