EVALUATING ACCURACY OF EACH OBSERVED DIRECTION AT TRIANGULATION POINTS WHEN USING THE METHOD OF ROUNDS FOR ANGULAR MEASUREMENTS

"تقدير الدقة لكل إتجاه مرصود على حده لنقط شبكات المثلثات باستخدام طريقة الدوران لقياس الزوايا"

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الخلاصة:

حتى الوقت القريب لم نستطع بأى معادلة حساب الخطأ التربيعى المتوسط لكل إتجاه مرصود أثناء قياس الزوايا تشبكات مثلثات الدرجة الأولى و الثانية. فكانت العادة فى فرض ان القيمة المضبوطة هى عباره عن أخذ القيمة المتوسطة المحسوبة من إتجهات عددها م. فى هذا البحث تم إستنتاج معادلة تمكننا من حساب الخطأ التربيعى المتوسط لكل اتجاه مصحح للنقط المعلومة

و يوصى باستخدام هذه المعادلة فى المجال العملى.

ABSTRACT

Until the recent time, there was no formula enabling to compute the mean square errors for each observed direction for angular measurements in 1st and 2nd order triangulation. It is usually supposed that adjusted values of these directions are equal to the average value calculated from m rounds.

In this paper, there was a deduction of formula enabling to compute the mean square error for each adjusted direction at the reference points. This formula is recommended for the practical use.

INTRODUCTION

For angular measurements in 1st and 2nd order triangulation, there are used methods enabling to represent the results of measurements and their adjustment on each reference point as a series of observed directions having the same weight. It is usually supposed that adjusted values of these directions are equal to average values calculated from m rounds, have the same weight $P_j = m$.

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This assumption does not take into account different impacts of various factors on observed directions, such as changes in refraction, oscillations of signals, etc. That is why this formula does not give sufficiently complete information on real weights of each individually taken adjusted direction. An objective judgement on accuracy of each adjusted direction may be obtained only on the basis of real results of measurements. For that, with the use of these results it is necessary to calculate mean square errors for each observed direction and not their averaged value for all sighted directions, as it is usually being done in practice. This last remark could be explained by the fact that until recent time there was no formula enabling to compute these errors. In 1989 there was proposed one of such formulas [2] but without its deduction, nor theoretical substantiation. That is why it seems to be expedient to fill in this gap.

DEDUCTION OF FORMULA ENABLING TO COMPUTE MEAN SQUARE ERRORS OF ADJUSTED DIRECTIONS OBSERVED AT REFERENCE POINTS:

In this paper for the first time in geodetic publications, there will be given a deduction of formula enabling to compute mean square errors for each adjusted direction observed at the reference points.

Let us suppose that there are n directions to be sighted at a reference point having numbers j=1,2,3 ....n and that they were observed in a single group (without dividing them in groups) by m rounds. In each round directions are reduced to the initial (first) direction. Designating by (1.j) the angles measured in i-th round, computed from initial direction, and by [1.j] their adjusted values, calculated as:

$$[1.j] = \frac{1}{m}\sum_{i=1}^{m} (1.j)_i \qquad (1)$$

Then, the results of observations and their adjustment at a control point could be represented as a series of directions, their values being given in the following table:

Table (1):

Direction No. j	Adusted value	Error M_j
1	0 00 00, 00	M_1
2	[1.2]	M_2
3	[1.3]	M_3
4	[1.4]	M_4
....	....	...
n	[1.n]	M_n

It is known that if a result of adjustment of angular measurements carried out at a control point is represented as a series of directions observed (as it is shown in Table 1), then the inverse weight of an angle j.l, calculated as difference of readings done for directions l and j, can be found as follows [1]:

$$\frac{1}{p_{j.l}} = \frac{1}{p_j} + \frac{1}{p_l} \qquad (2)$$

or, what is the same, by means of:

$$Q_{j.l} = Q_j + Q_l \qquad (3)$$

where Q_j and Q_l are the inverse weights of directions, forming angle j.l.

The directions sighted at a control point (see Table 1) could be grouped to form all possible angle combinations:

1.2	1.3	1.4	...	1.n	
	2.3	2.4	......	2.n	
		3.4	...	3.n	(4)
......	.....	...	.....	....	
				n-1.n	

The number of measured angles r (4) is equal to the number of combinations of n by 2, i.e.

$$r = C_n^2 = \frac{n(n-1)}{2} \qquad (5)$$

Using (3) for adjusted angles of the first row (4) we could write the following:

$$\begin{aligned} Q_{1.2} &= Q_1 + Q_2; \\ Q_{1.3} &= Q_1 + Q_3; \\ Q_{1.4} &= Q_1 + Q_4; \\ &\ldots\ldots \\ Q_{1.n} &= Q_1 + Q_n, \end{aligned} \qquad (A)$$

for angles of the second row:

$$\begin{aligned} Q_{2.3} &= Q_2 + Q_3; \\ Q_{2.4} &= Q_2 + Q_4; \\ &\ldots\ldots \\ Q_{2.n} &= Q_2 + Q_n; \end{aligned} \qquad (B)$$

for angles of the third row:

$$
\begin{aligned}
Q_{3.4} &= Q_3 + Q_4. \\
Q_{3.5} &= Q_3 + Q_5; \\
&\ldots\ldots\ldots\ldots \\
Q_{3.n} &= Q_3 + Q_n.
\end{aligned} \qquad \text{(C)}
$$

In similar way one can obtain corresponding equations for all other angles (4).

Let us find the inverse weight of the first adjusted direction. Summing up first two equations of the group (A) term by term, and subtracting the first expression of the group (B) from this sum, we obtain:

$$Q_{1.2} + Q_{1.3} - Q_{2.3} = 2Q_1,$$

from that the first value of inverse weight is found as.

$$Q_1^{(1)} = \frac{1}{2}(Q_{1.2} + Q_{1.3} - Q_{2.3})$$

Summing up the first and third expressions of the group (A) and subtracting the second one of the group (B) from the result, we could find the second value of the inverse weight:

$$Q_1^{(2)} = \frac{1}{2}(Q_{1.2} + Q_{1.4} - Q_{2.4}).$$

Using a similar approach for inverse weights of other angles, indicated in the first and second rows of (4), we obtain the following expressions for inverse weight of the first direction.

$$Q_1^{(3)} = \frac{1}{2}(Q_{1.2} + Q_{1.5} - Q_{2.5}).$$

$$Q_1^{(3)} = \frac{1}{2}(Q_{1.2} + Q_{1.5} - Q_{2.5}).$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$Q_1^{(i)} = \frac{1}{2}(Q_{1.2} + Q_{1.n} - Q_{2.n}).$$

Then, summing up the second expression in group (A) with each of subsequent equations and subtracting corresponding equations of the group (C) from results obtained, we could write that:

$$Q_1^{(i+1)} = \frac{1}{2}(Q_{1.3} + Q_{1.4} - Q_{3.4})$$

$$Q_1^{(i+2)} = \frac{1}{2}(Q_{1.3} - Q_{1.5} - Q_{3.5}).$$

....

$$Q_1^{(i+k)} = \frac{1}{2}(Q_{1.3} + Q_{1.n} - Q_{3.n}).$$

In similar way, all the other equations of (A) and the groups following the group (C) enable to obtain $\frac{1}{2}(n-1)(n-2)$ values of inverse weights $Q_1^{(i)}$ for the first adjusted direction, where n is the number of rays sighted. The final value of inverse weight of the first adjusted direction is equal to the mean of all computed values $Q_1^{(i)}$ which could be found by means of following generalized formula:

$$Q_1 = \frac{(n-2)\Sigma Q_{j.1} - \Sigma Q_{i.k}}{(n-1)(n-2)}. \qquad (6)$$

where i, k ≠ 1.

The first element in the nominator is related to those angles of (4) which are connected with the direction number one, i.e. to the angles 1.j, the second element is related to other angles of the group (4), which are not connected with the first direction, i.e. to angles i.k.

Then, taking the expressions of the group (B) as a basis and performing similar manipulations, we find inverse weight of the second adjusted direction:

$$Q_2 = \frac{(n-2)\Sigma Q_{2.1} - \Sigma Q_{i.k}}{(n-1)(n-2)} \qquad (7)$$

where i, k ≠ 2.

The inverse weight Q_3 of the third adjusted direction

$$Q_3 = \frac{(n-2)\Sigma Q_{3,1} - \Sigma Q_{i,k}}{(n-1)(n-2)}. \qquad (8)$$

where i, k ≠ 3, and then the same could be done for the fourth and other directions. Expressions (6) - (8) show that the inverse weight of any adjusted direction is computed by means of the same rule. In order to compute the inverse weight of the direction umber j, one should divide all the angles (4) into two sub-groups. One of them should include the angles j,l which contained the directionj to be evaluated, the second one should include all the other angles i,k which do not contain the direction j.

Then the formula for the computing of any adjusted (averaged from m rounds) value of a direction j could be written as follows:

$$Q_j = \frac{(n-2)\Sigma Q_{j,1} - \Sigma Q_{i,k}}{(n-1)(n-2)}. \qquad (9)$$

Let us determine now the inverse weight $Q_{I,K}$ of all the angles (4). The adjusted value of any angle [I K] measured by rounds method is computed as the mean value of m rounds:

$$[I.K] = \frac{1}{m}\sum_{1}^{m}(I.K).(I = K = 1,2,\ldots,n)$$

In this case, the inverse weight $Q_{I,K}$ will be equal to the squared RMSE $M_{I,K}$ of an angle:

$$Q_{I,K} = M^2_{I,K}. \qquad (10)$$

which is calculated as follows:

$$M_{I,K} = \sqrt{\frac{\Sigma V^2_{I,K}}{m(m-1)}}, \qquad (11)$$

where $V_{I,K}$ = (I.K) - [I.K] is the difference between the value of the angle I.K obtained in a round and the average value of this angle obtained from m rounds; m is the number of rounds.

Formula (10) allowed for (11) transforms into:

$$Q_{i.K} = \frac{\Sigma V_{i.K}^2}{m(m-1)} \quad . \qquad (12)$$

When using the method of rounds the values of adjusted directions are equal to their average values calculated from m rounds. That is why the inverse weight Qj of adjusted value of a direction is equal to its squared RMSE M_j^2 computed by means of the formula (11), i.e.:

$$Q_j = M_j^2 \quad . \qquad (13)$$

From this

$$M_j = \sqrt{Q_j} \qquad (14)$$

Replacing Qj by its value (9) and allowing for the fact that the inverse weights $Q_{j.I}$ and $Q_{i.K}$ could be determined with the use of the formula (12), we obtain the final formula for the computing of RMSE for each adjusted direction:

$$M_j = \sqrt{\frac{(n-2)\Sigma V_{j.I}^2 - \Sigma V_{i.k}^2}{m(m-1)(n-1)(n-2)}} \quad . \qquad (15)$$

where j is the number of a direction:

$V_{j.I}$ = (j.I) - [j.i] and $V_{i.k}$ = (i.k) - [i.k] are the deviations of angles measured in rounds from their mean values:

m is the number of rounds.

The summing up of deviations $V_{j.I}$ and $V_{i.k}$ is performed for all the rounds taken together.

In Tables 2 and 3, there is given an example of computation of mean square errors Mj for all the directions observed at the second order triangulation point.

Table 2: Adjustment and accuracy evaluation of directions observed by the methods of rounds at the 2nd order triangulation point:

Round #	Initial direction 1	1.2		1.3		1.4		$V_{2.3} = V_{1.3}-V_{1.2}$	$V_{2.4} = V_{1.4}-V_{1.2}$	$V_{3.4} = V_{1.4}-V_{1.3}$
		63°15'	$V_{1.2}$	109°17'	$V_{1.3}$	186°11'	$V_{1.4}$			
1	0° 00' 00.00"	44.0"	-1.32"	24.1"	-0.12"	47.2"	-1.86"	+1.20"	-0.54"	-1.74"
2	0° 00' 00.00"	45.8	+0.48	26.7	+2.48	51.5	+2.44	+2.00	+1.96	-0.04
3	0° 00' 00.00"	44.3	-1.02	22.4	-1.82	50.1	+1.04	-0.80	+2.06	+2.86
4	0° 00' 00.00"	43.0	-2.32	24.8	+0.58	45.9	-3.16	+2.90	-0.84	-3.74
5	0° 00' 00.00"	46.7	+1.38	27.3	+3.08	50.8	+1.74-	+1.70	+0.36	-1.34
6	0° 00' 00.00"	44.0	-1.32	22.5	-1.72	48.2	0.86	-0.40	+0.46	+0.86
7	0° 00' 00.00"	44.6	-0.72	23.9	-0.32	49.4	+0.31	+0.10	+1.06	+0.66
8	0° 00' 00.00"	46.0	+0.68	23.0	-1.22	48.2	-0.86	-1.90	-1.54	+0.36
9	0° 00' 00.00"	46.4	+1.08	23.3	-0.92	49.7	+0.64	-2.00	-0.44	+1.56
10	0° 00' 00.00"	47.8	+2.48	25.4	+1.18	49.9	+0.84+	-1.30	-1.64	-0.34
11	0° 00' 00.00"	44.9	-0.42	21.0	-0.22	50.2	1.14	+0.20	+1.56	+1.36
12	0° 00' 00.00"	46.4	+1.08	23.2	-1.02	47.6	-1.46	-2.10	-2.54	-0.44
Average	[1 j]=									
0°00'00.00"		45.32"		24.22"		49.06"				
Control	sum $\sum V$		+0.06"		-0.01"		-0.02"	-0.10"	-0.08"	+0.02"
	$[V^2]$		21.68		27.18		29.65	31.45	24.69	32.89

Table 3:

Direction No. j	Adjusted value of observed direction	$[v_{j,I}^2]$	$[v_{i,k}^2]$	M_j
1	0° 00' 00".00	78.51	89.03	0.29"
2	63 15 45.32	77.82	89.72	0.29"
3	109 47 24.22	91.52	76.02	0.37"
4	186 34 49.06	87.23	80.31	0.34"

CONCLUSIONS

Formula (15) deduced in this paper is strict and could be recommended for practical use. In order to verify this formulas numerically, let us compute twice the average value of RMSE M_N for all observed directions taken together. According to the data of Table 3 we have:

$$M_N = \sqrt{\frac{\Sigma M_j^2}{n}} = \sqrt{\frac{0.4207}{4}} = 0.324''. \quad (16)$$

Using well known formulas [2], we also could find this value:

$$M_N = \sqrt{\frac{n\Sigma[v_{i,j}^2] - \Sigma[V_{i,j}]^2}{mn(m-1)(n-1)}} = \sqrt{\frac{\Sigma[V^2]}{mn(m-1)(n-1)}}. \quad (17)$$

According to the data of Table 2, we have $\sum[V^2] = 167.54$. Taking into account that m = 12, n = 4, the following results is obtained with the use of the second expression of (16):

$$M_N = \sqrt{\frac{167.54}{1584}} = 0.325''.$$

As one could see, these values obtained by means of totally different formulas, are the same, thus confirming the reliability of the formula (15) deduced in this paper.

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