SPECTRAL AVERAGING TECHNIQUES FOR UNSYNCHRONIZED WAVEFORMS

تقنيات لتقدير المتوسط الطيفى لموجات غير متزامنة

Fatma E. Z. Abou-Chadi

Faculty of Engineering - Mansoura University

الخلامة ـ يعرض هذا البحث طريقتين لحاب المتوسط الطيفي لمجموعة من
الموجات الغر متزامنة : طريقة افراد طيف الطور وطريقة متجه الطور . ويقارن
البحث بين أداء الطريقتين من حيث تحين نبة الاشارة ـ الى ـ الموضاء
والحصول على شكل نمط متوسط لمجموعة الموجات الداخلة في حاب المتوسط . وقد
أظهرت النتائج أن الطريقة الثانية أحن في الأداء وأنب للحصول على تقدير
غير مشوه لشكل الموجة الأصلية .

ABSTRACT-Two algorithms for averaging unsynchronized waveforms are considered; the phase unwrapping and the phase vector techniques. The two techniques employ Fourier analysis and are applied to simulated signals contaminated with random noise. They both achieve a reasonable signal-to-noise ratio improvement and yield an average waveform that is properly positioned close to the ensemble mean delay. However, the phase vector method is shown to have a better performance and to be more convenient for producing an undistorted estimate of the original waveform.

I. INTRODUCTION

Signal averaging is often employed to recover a waveform in the presence of noise. In many cases, it is desired to obtain a representative pattern for an ensemble of waveforms embedded in noise or to determine the frequency content associated with a particular waveform once the effects of noise have been eliminated. When this averaging is taken in the time domain on an ensemble of waveforms which are synchronized with respect to a time reference, the result will be a faithful signal average. In this process, the coherence between signal components will tend to enhance the signal during accumulation while the random noise components will tend to cancel one another. Generally, the signal-to-noise ratio will improve at a rate proportional to the square root of the number of waveforms averaged. This is known as coherent averaging [1].

A usual limitation imposed on obtaining a meaningful averaging in the time domain for an ensemble of waveforms is the lack of a synchronizing pulse. Alternative cross-correlation techniques; template matching and template updating techniques are usually employed with unsynchronized data. These often depend on an initial guess of the waveshape [2]. Making a good initial estimate may be difficult if there is a low signal-to-noise ratio and especially if the signal and noise have similar spectral densities.

A more desirable technique for averaging waveforms in the absence of an external synchronizing signal would be one which averages the frequency spectra for the ensemble, for both phase as well as magnitude. Motivated by the relationship between phase and delay, Cheung [3] have developed a spectral averaging technique which yields an estimate of the signal at its mean delay. The technique involves Fourier transformation follwed by an averaging scheme whereby all harmonics contained within the amplitude and phase spectra are averaged. Transformation back to the time domain is then performed. This technique has been widely used in situations where it is difficult to achieve synchronization for an ensemble of waveforms, like for instance, in evoked electroencephalographic (EEG) potential signals [4,5].

This paper presents two algorithms for performing spectral averaging namely; the phase unwrapping and the phase vector methods. The capability of improving the signal-to-noise ratio for each algorithm is determined through examples of simulated signals containing noise levels comparable to those encountered in biological signals. The efficiency of the two algorithms to yield an estimate of the signal which is positioned close to the ensemble mean delay is also compared using an ensemble of simulated compound action potential (CAP) waveforms.

II. EMSEMBLE AVERAGE AND THE JITTER PROBLEM

If a deterministic signal s(t) is corrupted by additive noise n(t) and there is an ensemble of waveforms $\{r_i(t)\}$,

$$r_i(t) = s(t) + n_i(t), \quad i=0,\dots,M, \quad 0 \leqslant t \leqslant T \qquad (1)$$

then the average of the ensemble of waveforms can be used as an estimate of the signal

$$y(t) = \frac{1}{M} \sum_{i=1}^{M} r_i(t), \qquad 0 \leqslant t \leqslant T \qquad (2)$$

Applications of this technique in biological signal analysis are numerous and include the estimation of an average locomotor electromyographic pattern [6], electrogastographic waveforms [7] and electrocardiographic (ECG) beats [8].

It can be shown [1] that if n(t) is a zero-mean stationary process, uncorrelated from waveform to another and uncorrelated with s(t), then the ensemble average forms a consistent signal estimator, i.e.

$$E\left\{y(t)\right\} = s(t)$$
$$\mathrm{Var}\left\{y(t)\right\} = \frac{1}{M}\,\mathrm{var}\left\{n(t)\right\} \qquad (3)$$

where E{y(t)} is the expected value and var{y(t)} is the variance of y at instant t. Hence, it may be concluded that the signal-to-noise ratio improves with a factor $M^{1/2}$.

Ensemble averaging can also be evaluated from a filtering standpoint. Rompelman and Ross [1] have reported that the equivalent filter transfer function is

$$H(f) = \frac{1}{M} \frac{\sin M\pi fT}{\sin \pi fT} \qquad (4)$$

where T is the waveform interval.

However, the estimation of the signal by ensemble averaging is affected by variations in signal delay; ensemble average in the presence of signal jitter results in a distortion that can also be interpreted from a filtering viewpoint.

Consider a signal waveform s(t) which is subject to a random jitter δ_i. For simplicity, let us assume δ_i to be uniformly distributed between $(-\delta, \delta)$. By ensemble averaging, the signal is

$$\tilde{s}(t) = \frac{1}{M} \sum_{i=1}^{M} s(t + \delta_i) \qquad (5)$$

As M tends to infinity, this is equivalent to convolving s(t) with the distribution function $P(\delta)$. This is the same as multiplying the spectrum of s(t) with that of $P(\delta)$ and hence causes smoothing. It is obvious that the wider the distribution function $P(\delta)$ (that is, the larger time jitter), the sharper the filtering effect becomes.

If in addition to the jitter, there is a significant amount of noise, a difficult alignment problem arises. Some researchers have used cross-correlation techniques [2,9] where at each stage, an updated signal estimate is used as a template to realign the ensemble. The convergence of these techniques depends on the nature of the signal, signal-to-noise, and on the initial signal estimate. An alternative approach adopted by some researchers utilizes the FFT transformation to determine a good estimate of the signal at its mean delay in the ensemble of waveforms. This is known as spectral averaging.

III. SPECTRAL AVERAGING

A simple time delay manifests itself as an additive linear phase in the frequency domain. Using this relationship between phase and time delay, a frequency domain averaging has been developed. Given an ensemble of noisy waveforms in which the signal has a variable delay, spectral averaging yields an estimate of the signal at its mean delay. Two different methods are followed to implement this averaging: the phase-unwrapping method and the phase vector method.

III.1 Phase-Unwrapping Method

Consider an ensemble of (noise-free) waveforms

$$r_i(t) = s(t - D_i), \quad i=0,\ldots,M, \quad 0 \leqslant t \leqslant T \qquad (6)$$

where D_i is the delay. The Fourier transform of $r_i(t)$ is

$$R_i(\omega) = \left|R_i(\omega)\right| \exp(j\phi_{ri}(\omega)), \qquad i=1,\ldots,M \qquad (7a)$$

where $$\left|R_i(\omega)\right| = \left|s(\omega)\right| \qquad (7b)$$

and $$\phi_{ri}(\omega) = \phi_s(\omega) - \omega D_i \qquad (7c)$$

and where ϕ represents the Fourier phase. If we could now use these phases , $\phi_{ri}(\omega)$, to compute $\phi_{s\bar{D}}$ where

$$\phi_{s\bar{D}}(\omega) = \phi_s(\omega) - \omega \bar{D} \qquad (8a)$$

and $$\bar{D} = \frac{1}{M}\sum_{i=1}^{M} D_i \qquad (8b)$$

we could then reconstruct the signal at its mean delay [10],

$$s(t - \bar{D}) = F^{-1}\left\{ s(\omega) \exp(j\phi_{s\bar{D}}(\omega))\right\} \qquad (9)$$

where F^{-1} stands for the inverse Fourier transform of the quantity in brackets.

However, we can not simply average the principal phase values of the measurements $\phi_{ri}(\omega)$ to produce $\phi_{s\bar{D}}(\omega)$ since, in general, averaging principal values gives a biased estimate of the principal value of the average; although the phase components calculated from the discrete Fourier transform can have values in the range $[-\infty,\infty]$, the values of $\phi_{ri}(f)$ are all mapped in the region $[-\pi,\pi]$. This is due to the periodic character of the arctg function. Therefore, we must use the unwrapped phase, a function which is reliably obtained for the discrete Fourier transform (DFT) by a procedure due to Cheung [3]. The procedure consists of adding or subtracting multiples of 2π to the phase values at appropriate frequencies in order to make all the differences of two consecutive phase spectral values less than π. The removal of the folds in the phase spectrum reveals any trend which may be present. This trend represents the displacement of the main signal features from their temporally most-symmetrical position. Therefore, the unwrapping procedure must be followed by a linear regression and a trend removal. The whole process is known as "Regression Spinning" [11].

Thus, with ϕ_{ri} now representing the unwrapped detrended Fourier phase, we can write

$$\phi_{s\bar{D}}(\omega) = \frac{1}{M} \sum_{i=1}^{M} \phi_{ri}(\omega) \tag{10}$$

which represents the average phase value that can be used in reconstructing the spectral average using Eq.(9).

III.2 The Phase Vector Method

An alternative phase averaging procedure has been adopted by Sayers et al. [12]. Consider a discrete signal x(n) and its discrete Fourier transform X(k). X(k) is a complex quantity having real and imaginary parts R(k) and I(k)

$$X(k) = R(k) + I(k) \tag{11}$$

The amplitude A(k) and phase $\phi(k)$ of each spectral component are defined as:

$$A(k) = \sqrt{R^2(k) + I^2(k)} \tag{12}$$

$$\phi(k) = \text{arctg} \left\{ \frac{I(k)}{R(k)} \right\} \tag{13}$$

The average Fourier amplitude is computed as

$$\bar{A}(k) = \frac{1}{M} \sum_{i=1}^{M} A_i(k) \tag{14}$$

The average phase is formed by the normalized vector method. A normalised phase vector is defined as:

$$X(k) = \left\{ \frac{R(k) + I(k)}{A(k)} \right\} = C(k) + j\ S(k) \tag{15}$$

The average C(k) and S(k) of real and imaginary parts of the M-various phase vectors are:

$$\bar{C}(k) = \frac{1}{M} \sum_{i=1}^{M} C_i(k), \qquad \bar{S}(k) = \frac{1}{M} \sum_{i=1}^{M} S_i(k) \tag{16}$$

Then the mean phase is obtained as the arctangent of the ratio $\bar{S}(k)$ to $\bar{C}(k)$

$$\bar{\phi}(k) = \text{arctg} \left\{ \frac{\bar{S}(k)}{\bar{C}(k)} \right\} \tag{17}$$

At the end of the averaging process, the average amplitude is combined with the average phase angle. Each harmonic is considered independently and the result of an inverse discrete Fourier transform would yield the spectral average.

IV. SIMULATIONS

Two types of simulated signals have been used to compare between the above averaging algorithms. The first type consists of two hundred identical single cycle discrete sine signals of length 128 samples. Each sequence is given a random delay (phase shift) derived from uniform distribution of 64 units wide and centered at time unit 64. Random numbers uniformly distributed between 0 and 1 have been generated using a random generator. A Gaussian distributed sequence has then been generated according to an algorithm suggested in [13]. The composite signals $r_i(t)$ given in Eq.(1) are formed by adding the Gaussian white noise to signal s(t) such that the signal-to-noise ratio (SNR) is

$$SNR = A / \sigma \tag{18}$$

where A is the peak of the sine wave and σ is the standard deviation of the noise. Fig.1 shows examples of the signal $r_i(t)$ with SNR of 5. Fig.2 shows a typical record with its corresponding amplitude and phase spectra.

The second class of simulated waveforms is an ensemble of 100 waveforms simulating the waveforms of the compound action potential (CAP) of the auditory nerve response to click stimulus. This signal is used in electrocochleography (ECochG) for otological and audiological examination. It represents responses from individual primary auditory neurons due to a stimulus.

The CAP waveforms are modeled by a linear segmental approximation; linear segments linking the extremum points in each waveform. The duration and slope of the linear segmental representation are evaluated. The signal has been simulated using the information derived from the duration and slope distributions of the original records [14]. It is possible to generate series of random numbers derived from a gaussian distribution using the mean value and twice the value of the corresponding standard deviation of the durations and slopes. For each specific slope, a duration of random value is assigned until an interval of 64 samples (10 msec) has been accumulated to produce one pseudo-record bearing the major features of the original CAP patterns. The resultant signal is then smoothed using a Hanning filter in order to obtain the nearest shape (Fig.3). It is clear that the original waveforms have different latencies, i.e. unsynchronized.

V. RESULTS

Fig.4 shows the ensemble average waveform for the 200 simulated synchronized waveforms (SNR = 5 and $\phi_i = 0$). As can be seen, in coherent averaging all signals are symmetrically added, whereas the random noise components are summed and tend to be reduced as the number of waveforms is increased. The figure also illustrates the amplitude and unwrapped phase spectra of the synchronized ensemble average. It is clear that the noise has been eliminated to a large extent.

The results of ensemble averaging of the unsynchronized waveforms with uniformly distributed delay are shown in Fig.5.a. It is clear that the signal jitter is severe enough to obscure much of the signal details (i.e. waveform distortion). The amplitude of the frequency harmonics is decreased but the phase is still unchanged. Fig.5.b shows the ensemble averaged CAP's. It is clear that the pattern main features has been altered due to the variable latencies of the original waveforms.

Fig.6.(a,b) shows the spectral average pattern using the phase unwrapping method. Magnitude and phase estimates were computed as in (11) and (12) and signal recovery performed as in (13). The procedure allows waveforms to be positioned close to certain mean delay but does not improve the SNR as much as the coherent average (Fig.4).

The results of the phase vector method are shown in Fig.7.(a,b). It is clear that SNR is higher than that obtained from the phase unwrapping method. Moreover, the CAP waveform is positioned closer to the mean delay of the ensemble.

The above results have been validated and confirmed by calculating the correlation coefficient (CC) between the average obtained from each algorithm and each waveform of the corresponding ensemble. As the original waveforms have different latencies, the sum of the calculated CC's are expected to be zero if the resultant average is exactly positioned at the mean delay of the ensemble of waveforms. The CC's is found to have an average value of 0.0088 for the phase unwrapping method and 0.00094 for the unit phasor method in case of the sine waveforms (Fig.1). The corresponding values of the CAP waveforms are: -0.342 for the phase unwrapping method and 0.00518 for the unit phasor technique. These values of the correletion cofficients confirm that the unit phasor method gives more accurate average pattern than that obtained from the phase unwrapping technique.

VI. CONCLUSION

Two algorithms for signal averaging of unsynchronized data have been investigated in the present work; the phase unwrapping and the phase vector methods. The two techniques have improved the signal-to noise ratio of repetitave signals buried in noise. They both employ the Fourier transform and phase averaging to achieve synchronization. However, the phase vector algorithm achieves better SNR improvement and yields an average pattern that is positioned closer to the ensemble mean latency. The results obtained are important for the evaluation of an average pattern for an ensemble of noisy waveforms when it is difficult to achieve synchronization.

REFERENCES

[1] O. Rompelman and H.H.Ross, "Coherent averaging technique: A tutorial review," J. Biomed. Eng., vol.8, pp.24-29,1986.

[2] F.E.Z. Abou-Chadi, "Coherent averaging in the presence of noise," Mansoura Eng. Journal, vol.15, no.2, pp.1-8,1990.

[3] Y-S. Cheung. A feasibility study of the spatio-temporal analysis of cardiac precordial vibrations. Ph.D. Thesis, Imperial College, Univ. of London, 1977.

[4] M.S. Perlman and V.H. Auerbach, "A phase-locking technique for estimating the ensemble average of time-series data," IEEE Trans. Acoustics, speech and signal processing, vol.ASSP-25, no.4, pp.295-299, 1977.

[5] S. Foster. Signal analysis of the auditory evoked EEG potential. Ph.D. Thesis, Imperial College, University of London, 1982.

[6] R. Shiavi and N.Green, "Ensemble averaging of locomotor electromyographic patterns," Med. & Biol. Eng. & Comput., vol.21, pp-573-578, 1983.

[7] A.C.W. Volkers, E.J. Van der Schee and J.L. Grashuis, "Electrogastrography in the dog: waveform analysis by a coherent averaging technique," Med. & Biol. Eng. & Comput., vol.21, pp.56-64, 1983.

[8] H.T. Le, W.C. Van Arsdel, A.M. Makowki, E.W. Pottala, and J.J. Bailey, "Automated analysis of rodent three channel electrocardiograms and vectorcardiograms" IEEE Trans. Biomed. Eng., vol.BME-3, no.1, pp.43-52, 1985.

[9] F.E.Z. Abou-Chadi, A.I. Abdel-Fattah and M. El-Gayar, "Pattern analysis of gastrointestinal postneonatal daily death counts in Dakahlia (1981-1987)," Proc. of the 25th annual conf. on statist., comp., and op.research, vol.25, pt.3, pp.1-16, 1990.

[10] M.A. Rodriguez, R.H. Williams and T.J. Carlow, "Signal delay and waveform estimation using unwrapped phase averaging," IEEE Trans. Acoust., speech and Signal Process., vol.ASSP-29, no.3, pp.508-513, 1981.

[11] P.R.,Sutton. (1976). Regression Spinning and the standardisation of phase spectra. Ph.D. thesis, Imperial College, University of London.

[12] B.McA., Sayers, L.R., Cicchiello, E.B., Raftery, S., Mann, and H., Green (1982), "The assessment of continuous ambulatory blood pressure records," Med. Inform., vol.7, No.2: 93-108.

[13] B.S. Gottfried. Elements of stochastic process simulation. Printice Hall, 1984.

[14] Z. Bahrami. Signal analysis of human elctrocochleography with special reference to the assessment of cochlear function. Ph.D. Thesis, Imperial College, University of London, 1984.

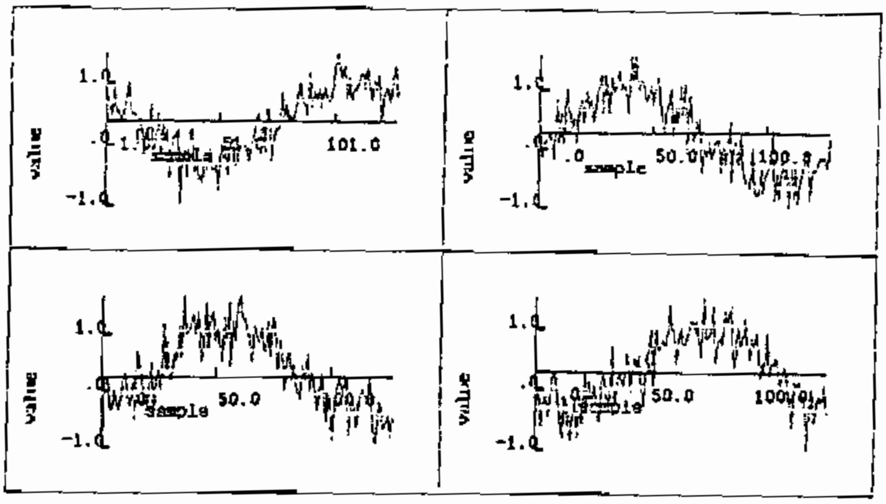


Fig.1 Examples of simulated noisy signals (SNR =5)

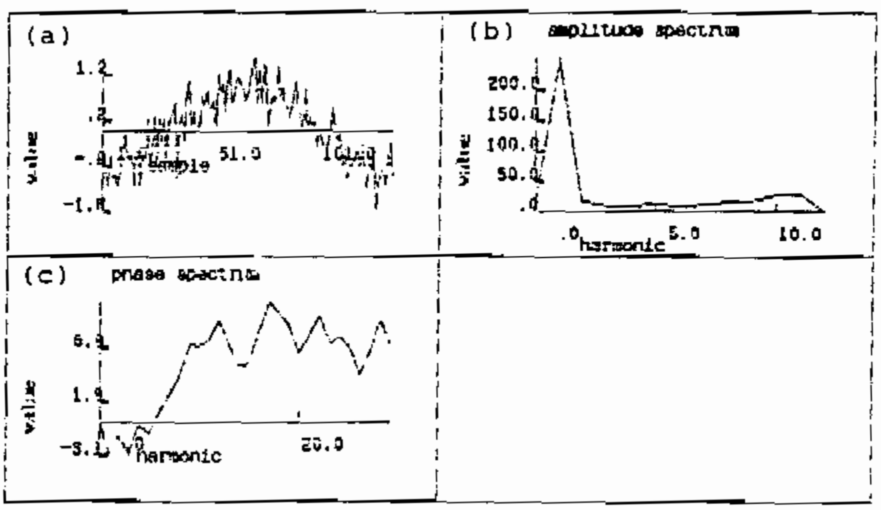


Fig.2.a- A typical simulated waveform
b- its amplitude spectrum
c- the corresponding unwrapped phase spectrum

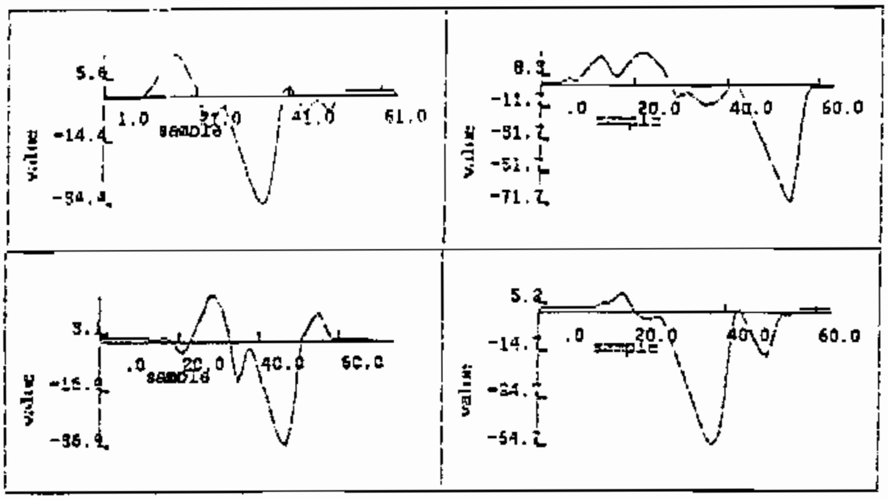


Fig.3 Examples of CAP waveforms

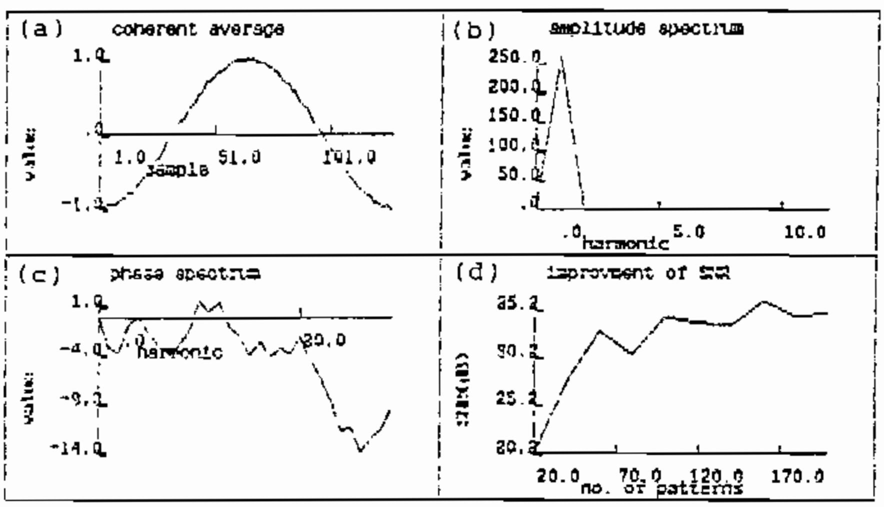


Fig.4 The resultant average of 200 synchronized signals
a- ensemble average
b- amplitude spectrum
c- unwrapped phase spectrum
d- SNR versus the number of averaged patterns

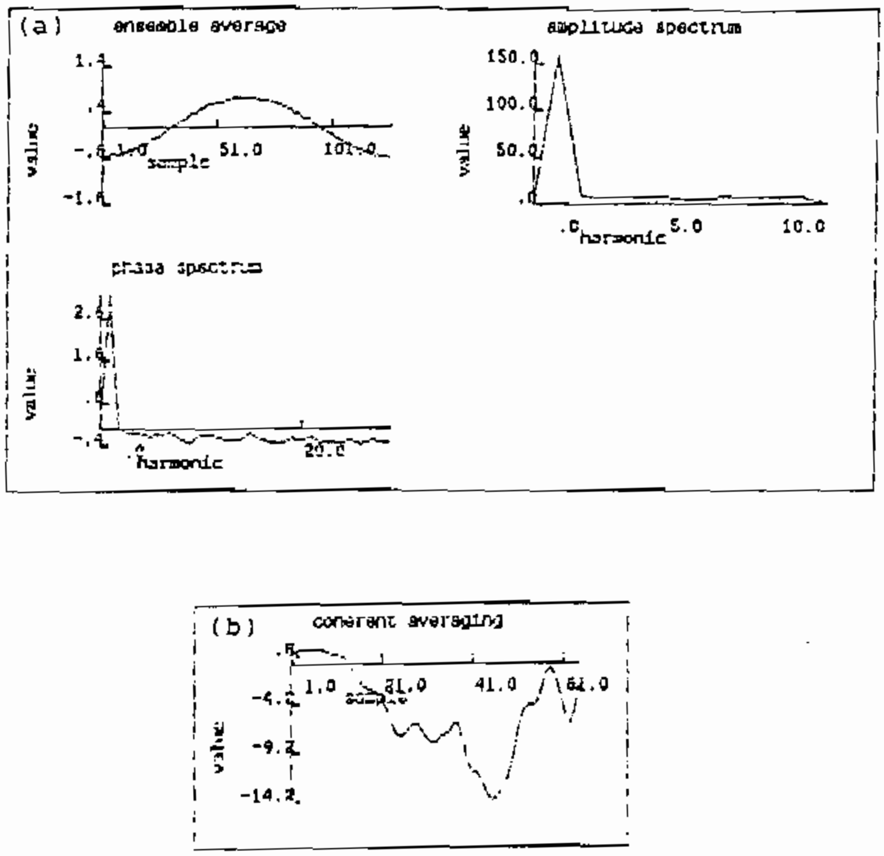


Fig.5 Ensemble average of:
a- 200 unsynchronized noisy sinusoidal signals
b- simulated CAP signals

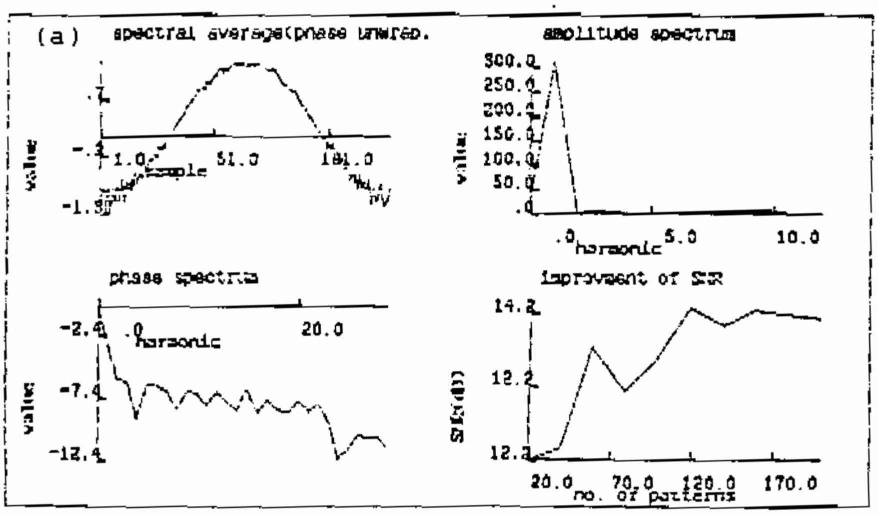


Fig.6 Spectral average using the phase unwrapping method:
a- noisy sinusoidal signals
b- simulated CAP signals

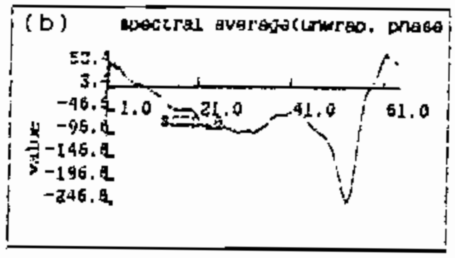
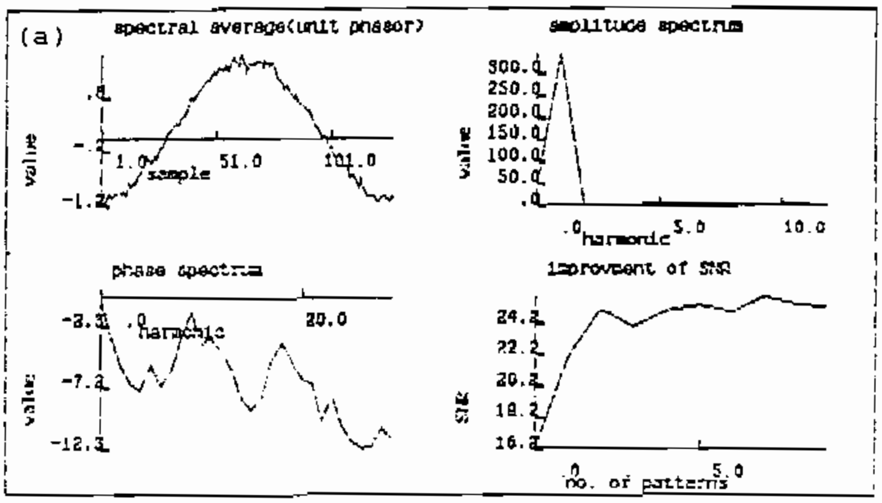


Fig.6 Cont.



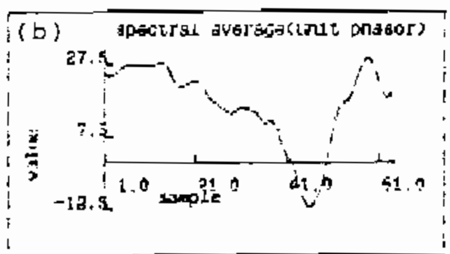


Fig.7 Spectral average pattern using phase vector technique:
a- noisy sinusoidal signals
b- simulated CAP signals