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A VERY LOW FREQUENCY, HIGH POWER, POWER SUPPLY

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ABSTRACT:

In this work, an equation for the relaxation oscillation of a direct current motor generator set is derived. This equation is solved numerically and plotted by the digital computer. From the plots the motor current as well as the period of oscillation are derived. The comparison between these values and those obtained experimentally shows good agreement.

The reason for the small deviation between the experimental and the computed values are discussed. The method of varying the frequency of oscillation of the system is also discussed and illustrated graphically.

INTRODUCTION

A D.C series generator driven at constant speed has a negative resistance characteristic. If this generator is connected to a separately excited motor the system will be an electro-mechanical one, that contains electrical and mechanical elements. It is possible by analogies to transform the system to a pure electrical equivalent one, in which the inductance and capacitance are constants, while the resistance is variable. The resistance is a current dependent variable., that leads to an oscillatory system.

Initially, when the current is zero, the resistance will have a high negative value, which leads to a negative damping, hence initiating oscillations. As the current increase, the negative value of the resistance reduces and reaches a positive value when current passes to its maximum, the oscillation then ceases and the current reduces to invert the resistance again to a negative value and so on.

The relaxation oscillation in triode tubes (1) has similar equations, but with pure electric constants. The first who noticed this phenomena in electrical machine was Janette (2)

In some applications a low frequency power supply is required. Of these applications the speed control of induction motors by injecting certain voltage in the rotor circuit.

The RC oscillators (3) fails to achieve these requirements for two reasons.

i) The value of both R and C must be abnormally high, that can be practically achieved for such very low frequency.

ii) The RC oscillators have limited output power, that cannot satisfy these requirements.

The system also may find an application in spin motions as in washing machines and likes. The system may be reduced to only one machine, if the primemover and the series generator are replaced by an electronic circuit that has similar characteristics as the series generator.

MATHEMATICAL DERIVATION

The combination consists of a direct current series generator driven at constant speed, and a direct current separately excited motor operating at no load. The set is connected as shown in fig.(1)

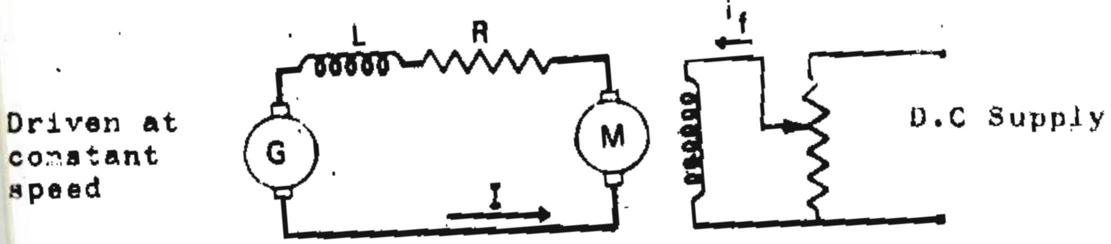


Fig.(1)

The characteristic of the series generator at constant speed, fig(2) can be written in the form,

$$E_g = K_1 I - K_2 I^3 \quad \text{volt} \quad ..(1)$$

where K_1 and K_2 are positive constants. The cubic term represents the influence of saturation. (4)

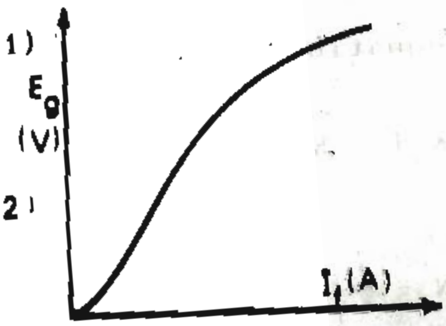


Fig (2)

The instantaneous current 'i' at any instant is given by the following differential equation:-

$$E_g = IR + L\frac{dI}{dt} + E_m \quad \text{volt} \quad (2)$$

where:

R = the total series resistance of the set in ohms.

L = the totil series inductance of the set in henry

E_m = the motor armature back e.m f., since the motor is separately excited, the motor back e.m.f E_m is proportional to the motor speed N.

then: $E_m = K_3 N$ .... volt (3)

where:

K_3 = constant depends on the field current

The developed torque in the motor armature Td is given by

$$Td = K_4 I \quad \text{newton-meter} \quad \ldots (4)$$

the constants K_3, K_4 depend on the field flux and hence the field current.

This developed torque is absorbed in the constant friction torque in the motor K_5', and in all the viscous friction torque and other torques which are proportional to the speed $K_6' N$, and in the dynamic torque which accelerate the motor $(K_7' \frac{dN}{dt})$.

then we get,

$$Td = K_5' + K_6' N + K_7' \frac{dN}{dt} \quad \ldots \quad (5)$$

From equations (4), (5) and dividing by K_4 we get,

$$I = K_5 + K_6 N + K_7 \frac{dN}{dt} \quad \ldots \quad (6)$$

From equation (1), (2), (3) we get,

$$K_1 I - K_2 I^3 = IR + L \frac{dI}{dt} + K_3 N$$

then

$$N = \frac{1}{K_3} \left[(K_1 - R) I - K_2 I^3 - L \frac{dI}{dt} \right] \quad \ldots (7)$$

and

$$\frac{dN}{dt} = \frac{1}{K_3} \left[(K_1 - R) \frac{dI}{dt} - 3 K_2 I^2 \frac{dI}{dt} - L \frac{d^2 I}{dt^2} \right] \quad \ldots (8)$$

elimination of N and $\frac{dN}{dt}$ from equation (6), (7), (8), leads to ;

$$\frac{d^2 I}{dt^2} = (a - bI^2) \frac{dI}{dt} - C^2 I - dI^3 + e \quad \ldots (9)$$

where:

$$a = \frac{K_1 - R}{L} - \frac{K_6}{K_7}$$

$$b = \frac{3 K_2}{L}$$

$$\left.\begin{aligned} C^2 &= \frac{1}{L\,K_7}\left[K_3 - K_6(K_1 - R)\right] \\ d &= \frac{K_2\,K_6}{K_7} \\ e &= \frac{K_3\,K_5}{K_7} \end{aligned}\right\} \quad \ldots\ldots\ldots (10)$$

Let $I = \sqrt{\frac{a}{b}}\, y$ .... (11)

equation (9) will be,

$$\frac{d^2y}{dt^2} = a(1-y^2)\frac{dy}{dt} - C^2 y - \frac{ady^3}{b} + e\sqrt{\frac{b}{a}} \quad \ldots\ldots (12)$$

Let $x = Ct$ ... (13)

equation (12) will be

$$\frac{d^2y}{dx^2} = \mu(1-y^2)\frac{dy}{dx} - y - py^3 - Q \quad \ldots (14)$$

where

$$\left.\begin{aligned} \mu &= \frac{a}{C} \\ P &= \frac{ad}{bC^2} \\ Q &= \frac{e}{C^2}\sqrt{\frac{b}{a}} \end{aligned}\right. \quad \ldots (15)$$

equation (14) is similar to that given by Vanderpol, except in the terms P and Q which are due to the consideration of the motor friction.

The solution of equation (14) can be programmed and ploted using digital computer curve tracer. From the ploted curves, the output current oscillations, period and amplitude can be obtained.

NUMERICAL RESULTS

The numerical solution of the non linear differential equation (14), using Rong Kotta 2 Method, is programmed in appendix II . The solution is ploted for various values of field current. From the plots the period and the amplitude of oscillation x, y are determined. The actual period T, and hence the actual frequency $f = 1/T$, and the current amplitude I_o are calculated from equations (11), (13), then

$$f = 1/T = x_o/C$$

$$I_o = \sqrt{a/b} \quad , \quad I_o = A_1 \, y_o$$

To limit the armature current, a resistance r is connected in series with the armature circuit.

Case 1

For series resistance $r = 10$ ohm

Then the total series resistance = 10 + 3.044 = 13.044 ohm

(The value of K_1, K_2 are evaluated as shown in appendix I)

$$K_1 - R = 7.411$$

$$(K_1 - R)/L = 7.554536$$

The computer ploting is shown in fig. (3). The effect of the motor field current upon the frequency of oscillation is shown in fig. (4). From the plotes it is found that the amplitude of oscillation $y_o = 2$, and from the numerical solution, the values of A_1 is nearly constant and equal to 8.685, then the current amplitude

$$I_o = 2 \times 8,685 = 17.370 \text{ A}$$

Case 2

For series resistance $r = 15$ Ohms. Then the total series resistance = 16 + 3.054 = 18.044 ohms.

$$K_1 - R = 2.411$$

$$(K_1 - R)/L = 2.457696$$

The results are shown in fig (5, 6)

$$I_o = 2 \times 4.955 = 9.91 \text{ A}$$

Experimental Results

1 The system is connected as shown in fig.(7)

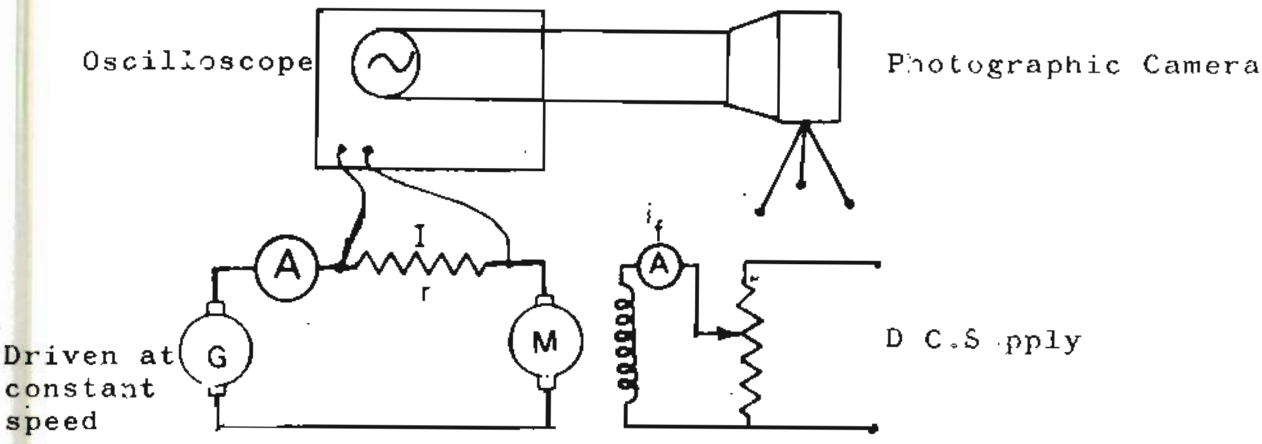


fig.(7)

2. The generator is driven at constant speed 1500 r.p.m
3. For various values of motor field current i_f , the frequency and the amplitude of oscillation are measured.

An oscilloscope is connected across the added series resistance for oscillographing the motor armature current oscillations. The load to be supplied by low frequnncy may be represented by this resistance or connected across it.

The oscilloscope is calibrated in amplitude for predicting the maximum voltage across the series resistance, and hence for the maximum circuit current, for each position of the attenuator knop. The maximum current is also measured by a moving coil ammeter of which the natural frequency is lower than the frequency of oscillation of the system. The period of oscillation is also calculated from the oscillation of the meter pointer. The time base of the oscilloscope is adjusted to scan at least two oscillations.The above steps are repeated for different values of motor field current i_f .

Results:

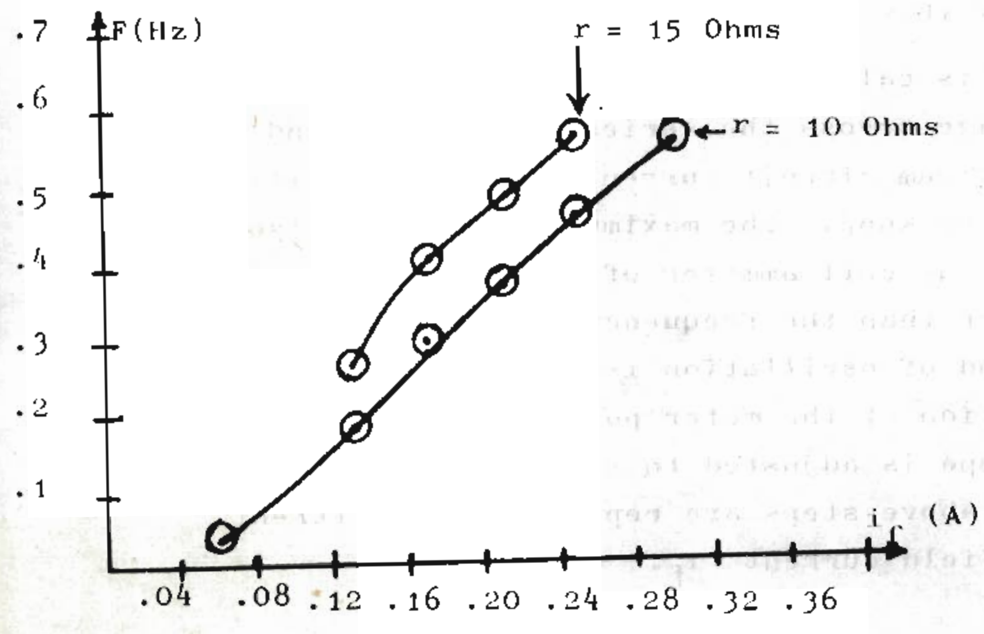
Case 1. For added series resistance = 10 ohms

i_f(A)	0.05	0.06	0 13	0.17	0.21	0.25	0.3	0.36
F Hz	one direction	0.035	0.175	0.286	0.37	0.455	0.55	stop
I (A)	14.5	14 2	14.	17.4	11.2	10.2	10	10.1

Case 2. For added series resistance - 15 ohm

i_f (A)	0 05	0.06	0.13	0.17	0.21	0 25	0.3	0.36
F Hz	one direction		0.25	0.40	0.48	0.55	stop	
I (A)	8.5	8.4	8.3	8.1	7 8	7.3	7	7 2

For both cases the shape of oscillations are shown by the set of photographs figs.(3,5). Curves showing the effect of both the added ressstance in the armatures circuit as well as the motor field current, on the oscillation frequency are illustrated in figs(4,6,8), (only two cases are illustrated).



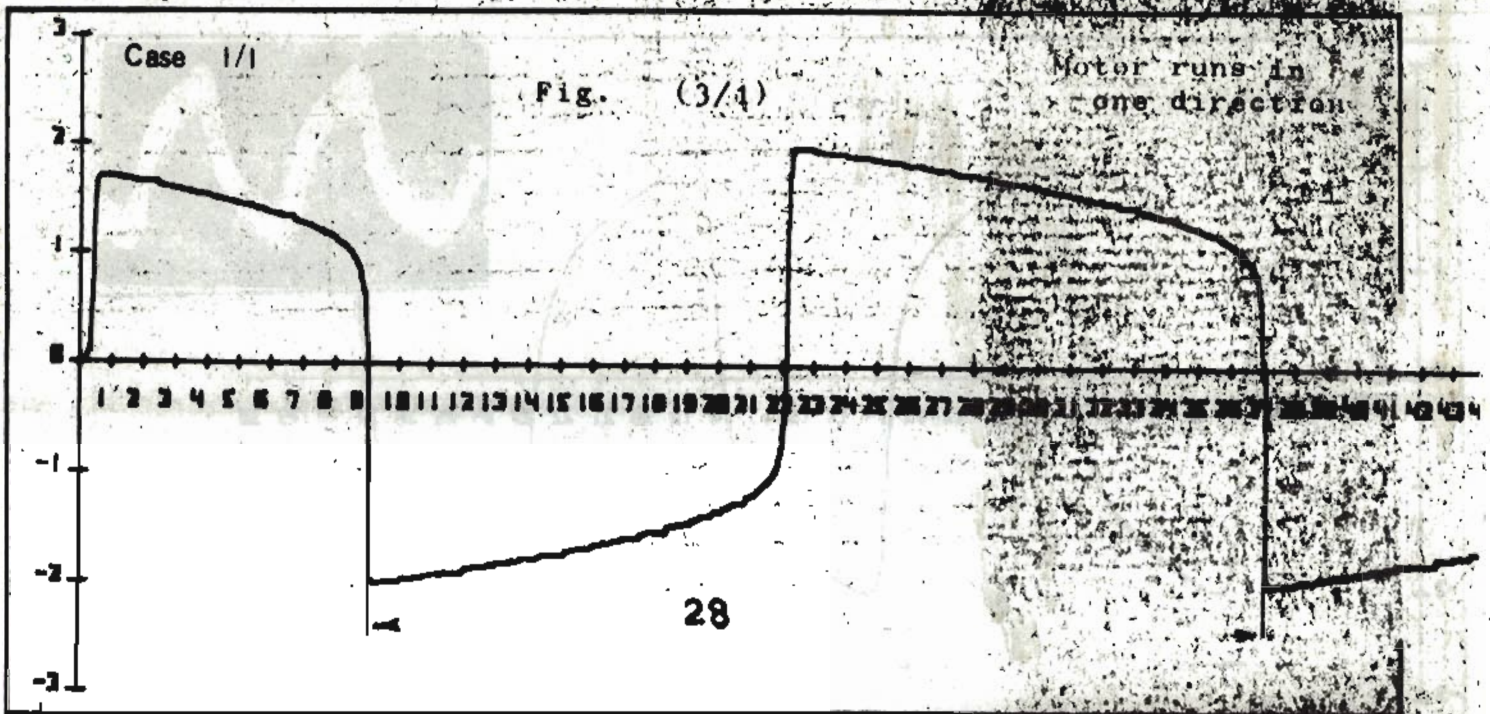


Fig. (3/1)

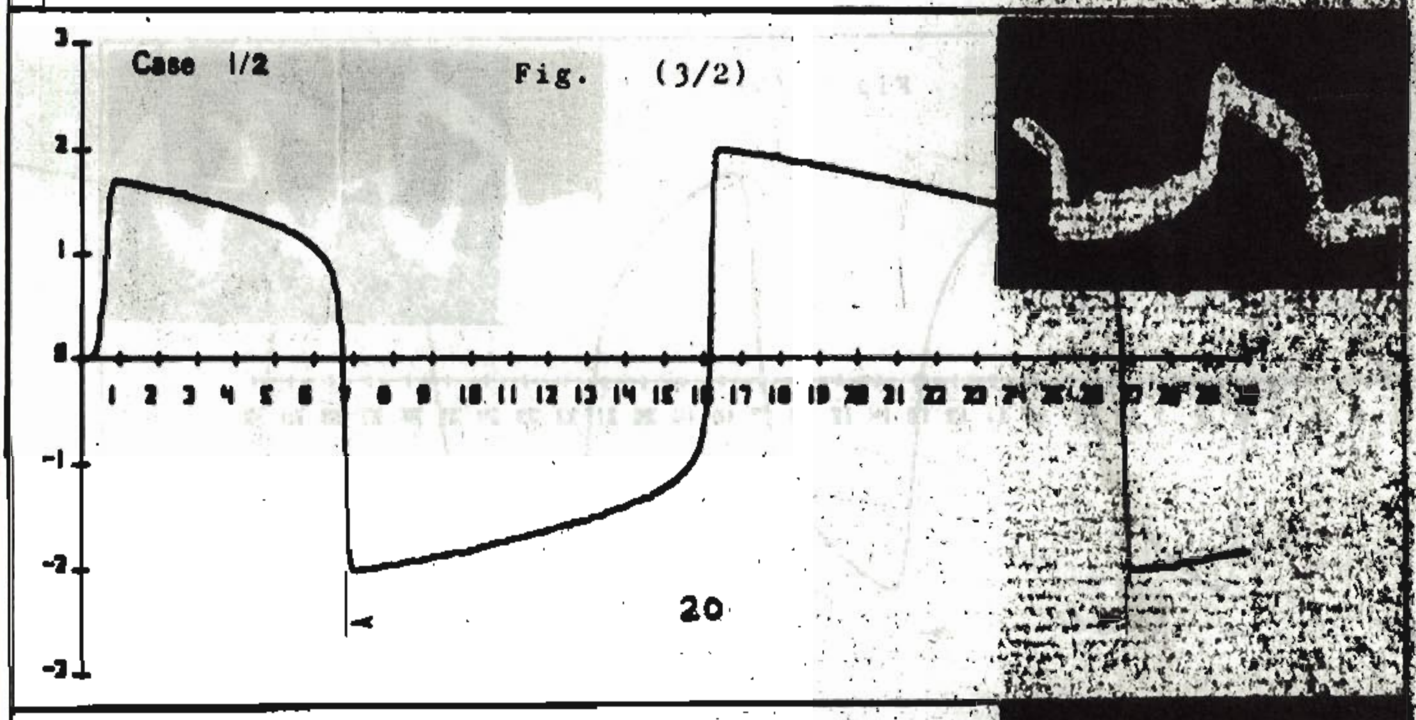


Fig. (3/2)

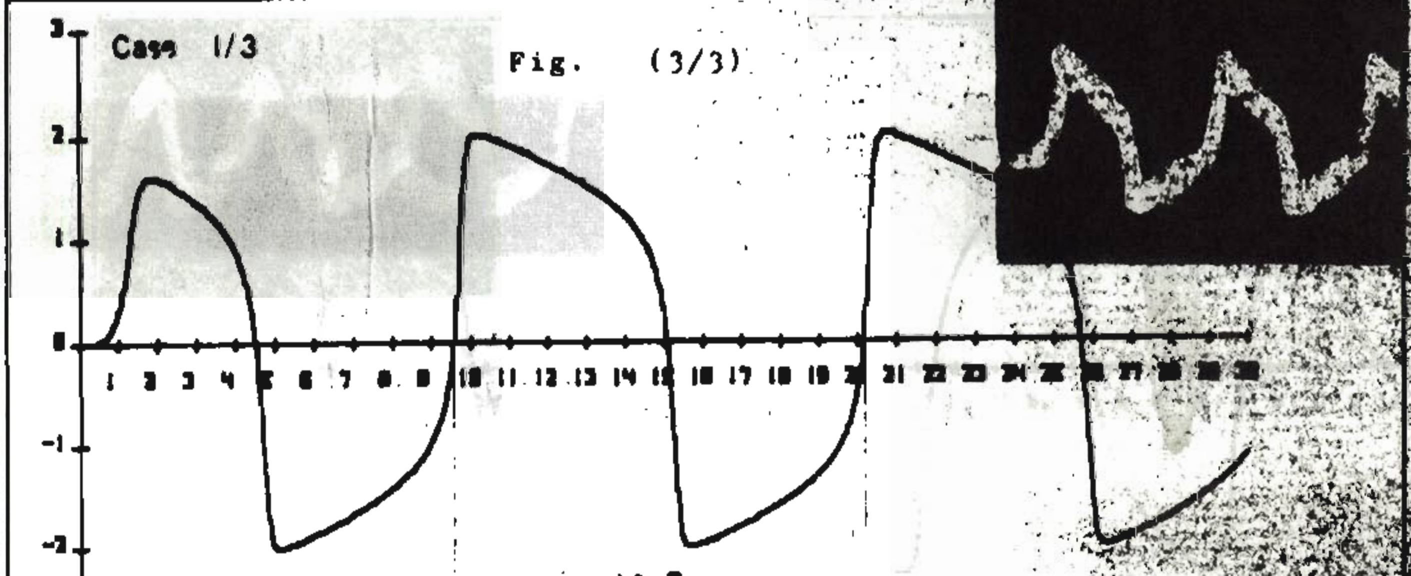


Fig. (3/3)

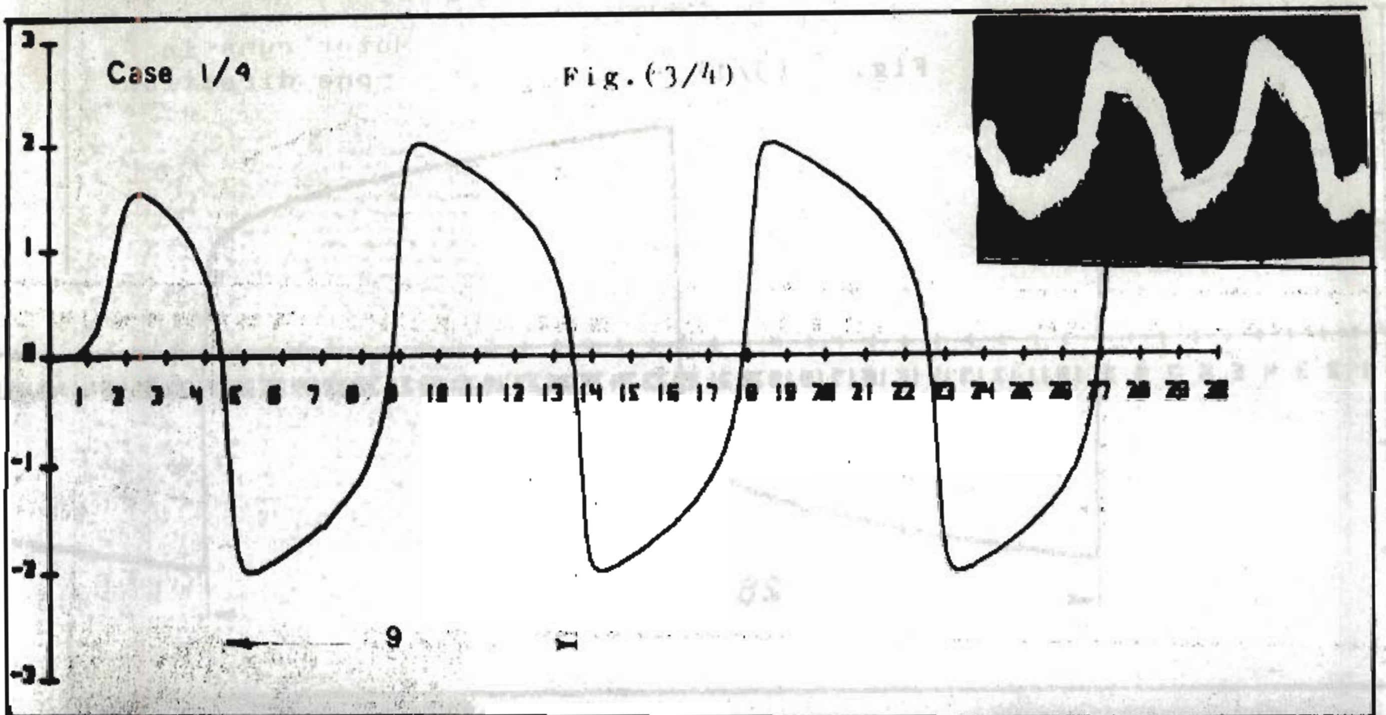


Fig.(3/4)

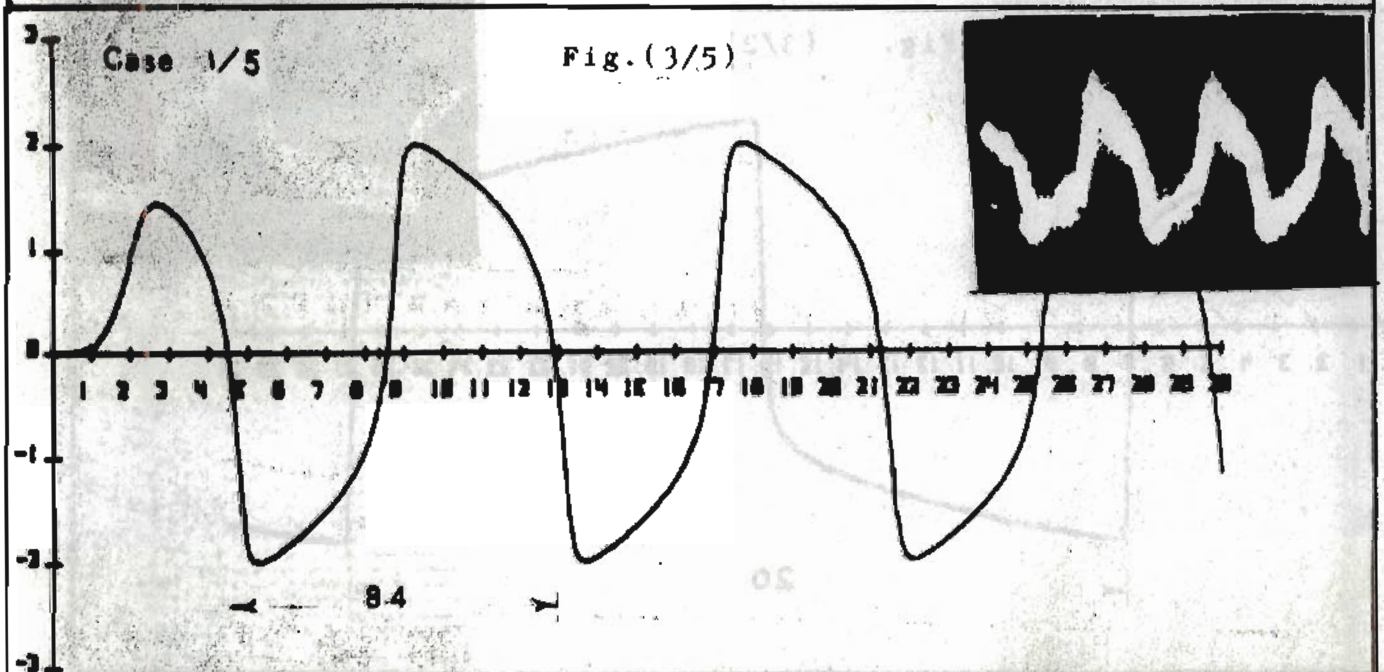


Fig.(3/5)

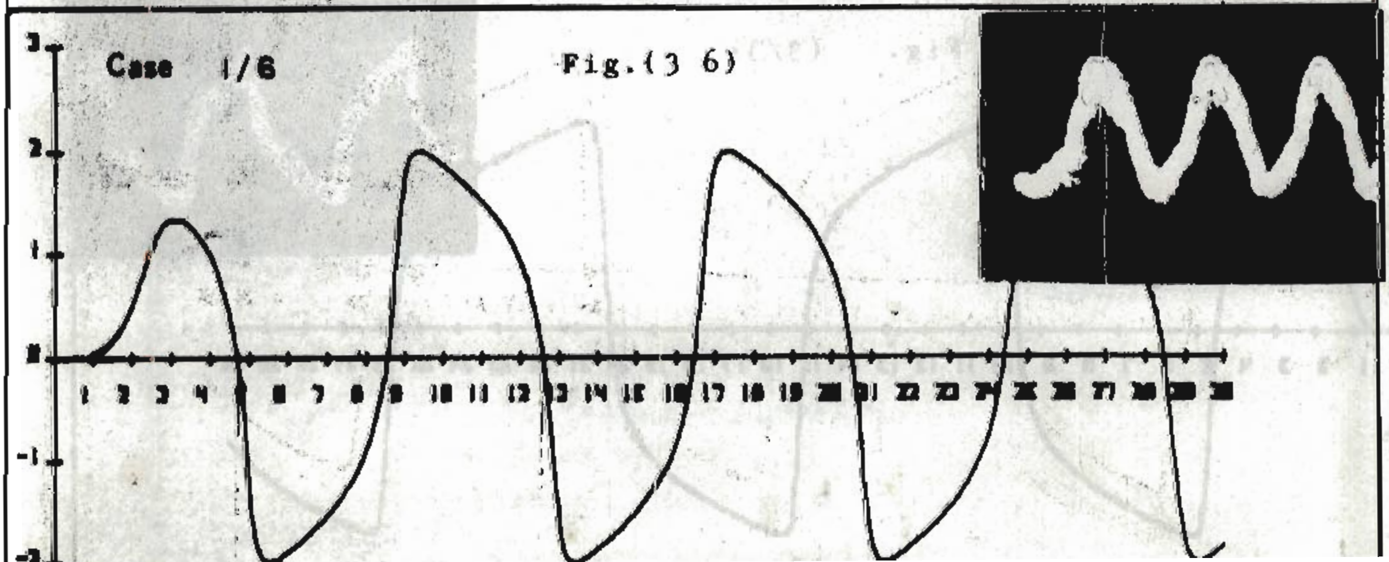


Fig.(3 6)

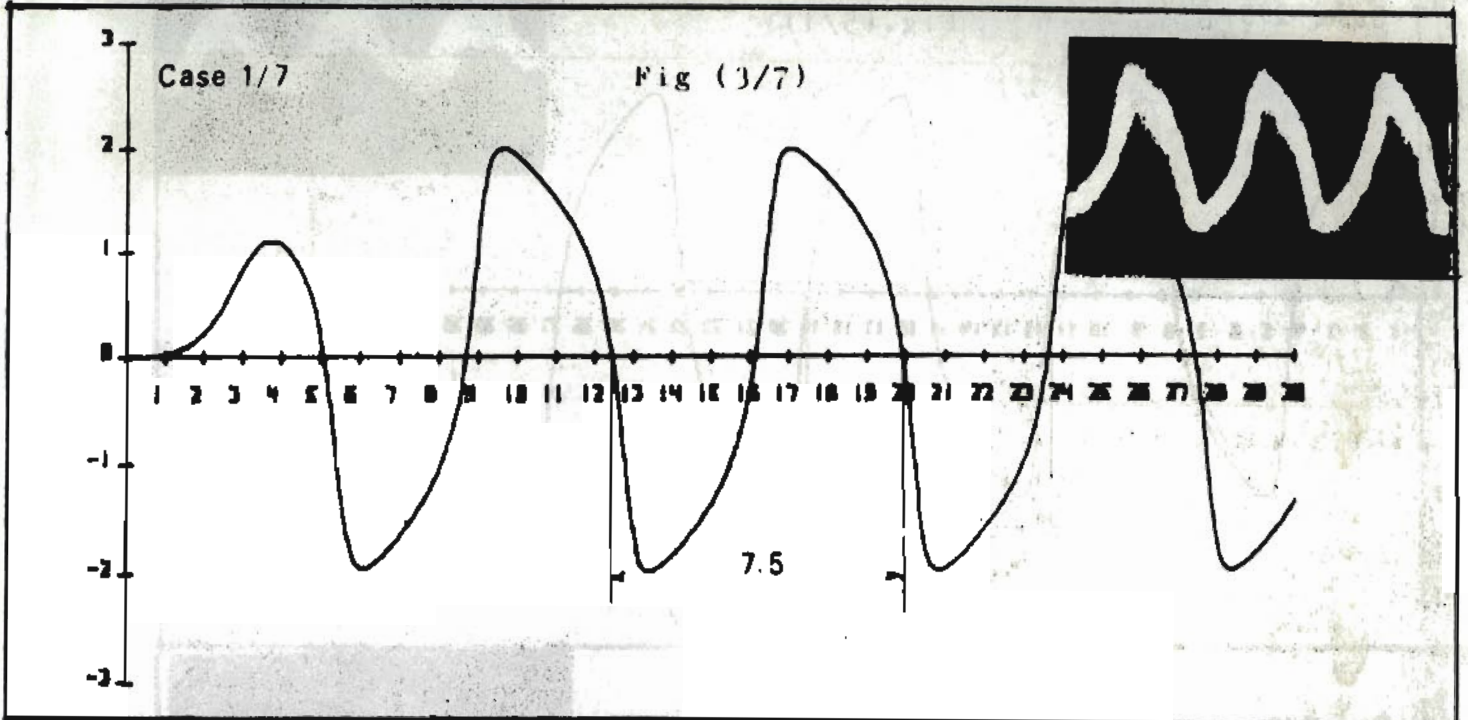


Fig (3/7)

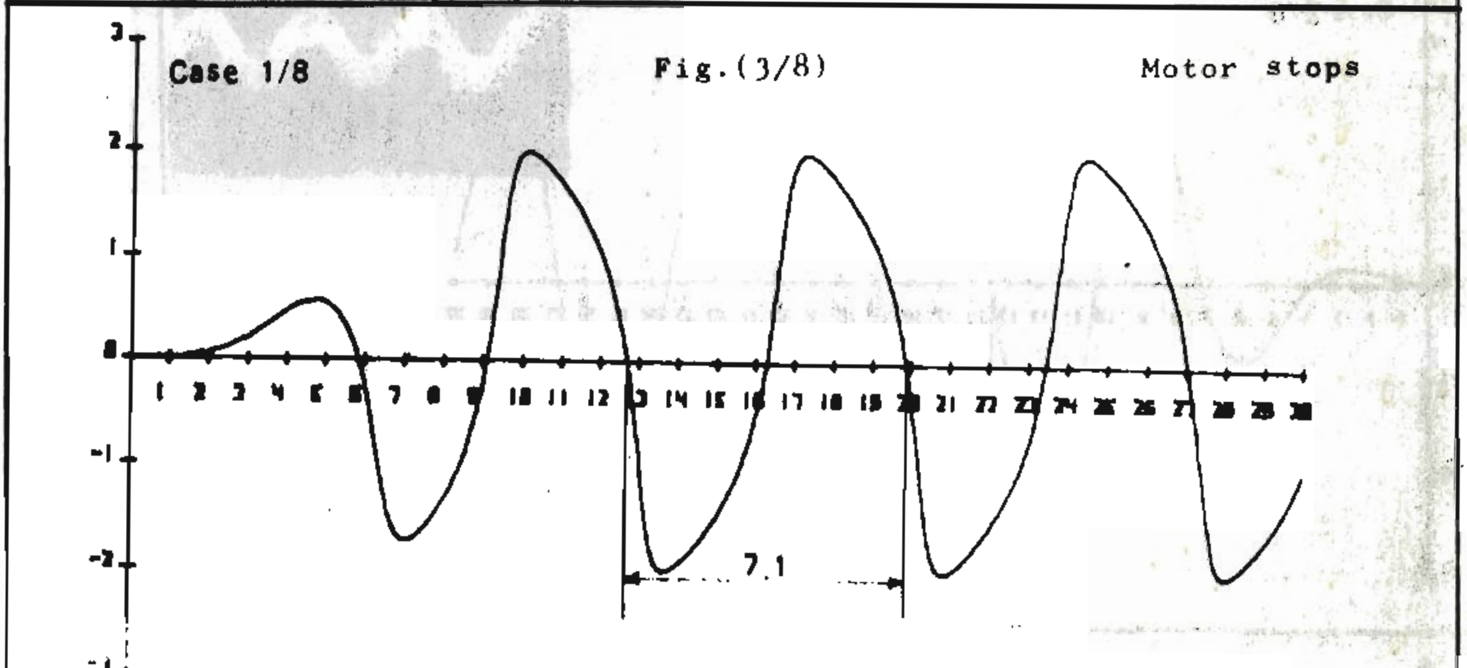


Fig.(3/8)

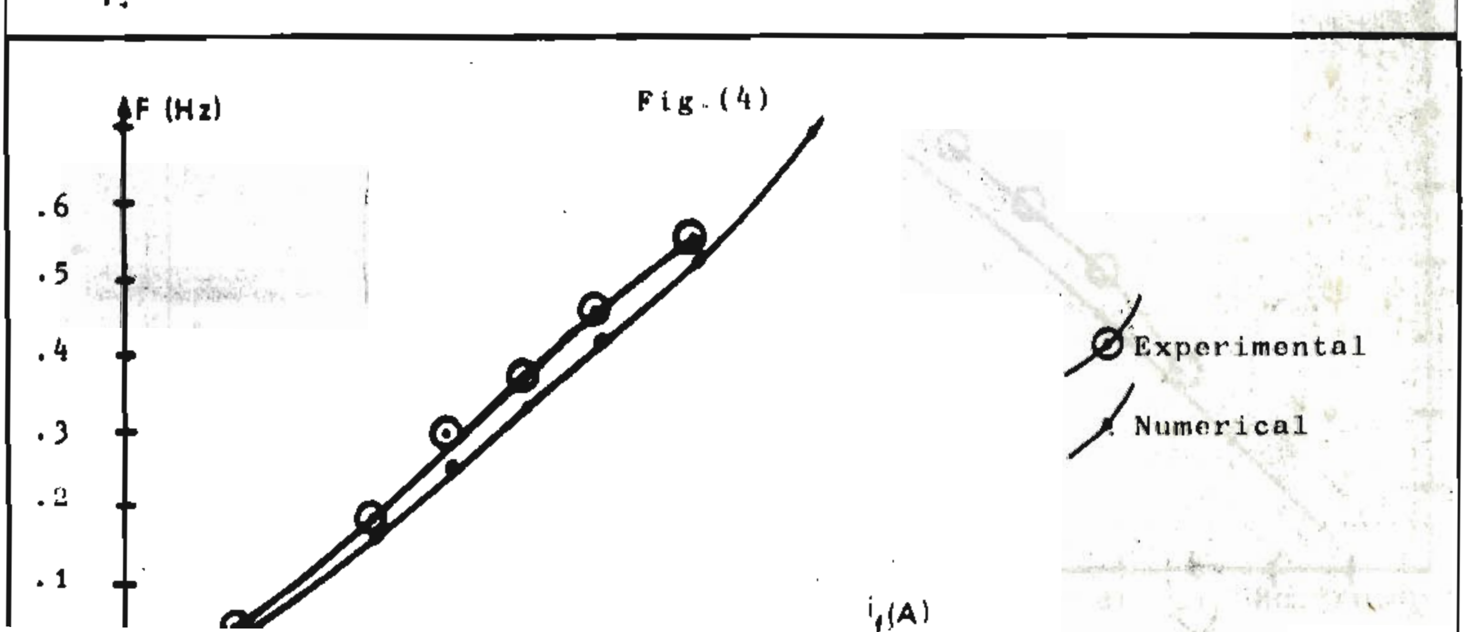


Fig.(4)

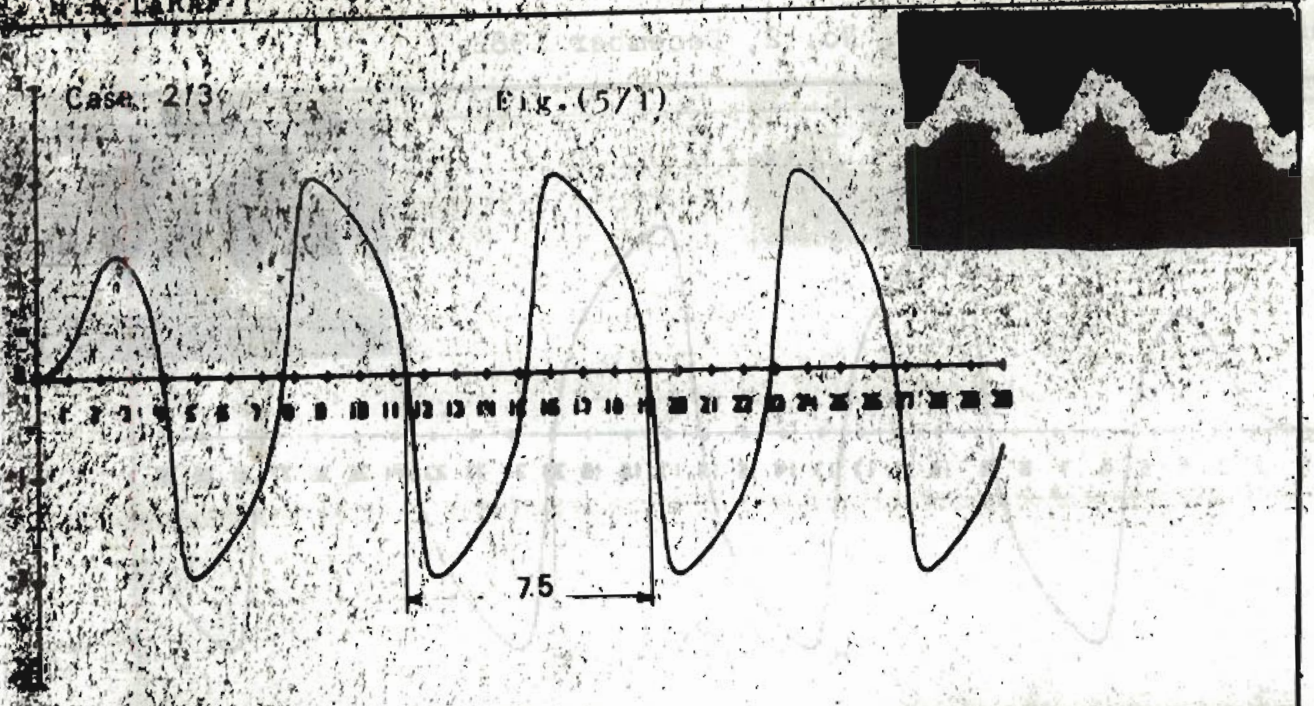


Fig. (5/1)

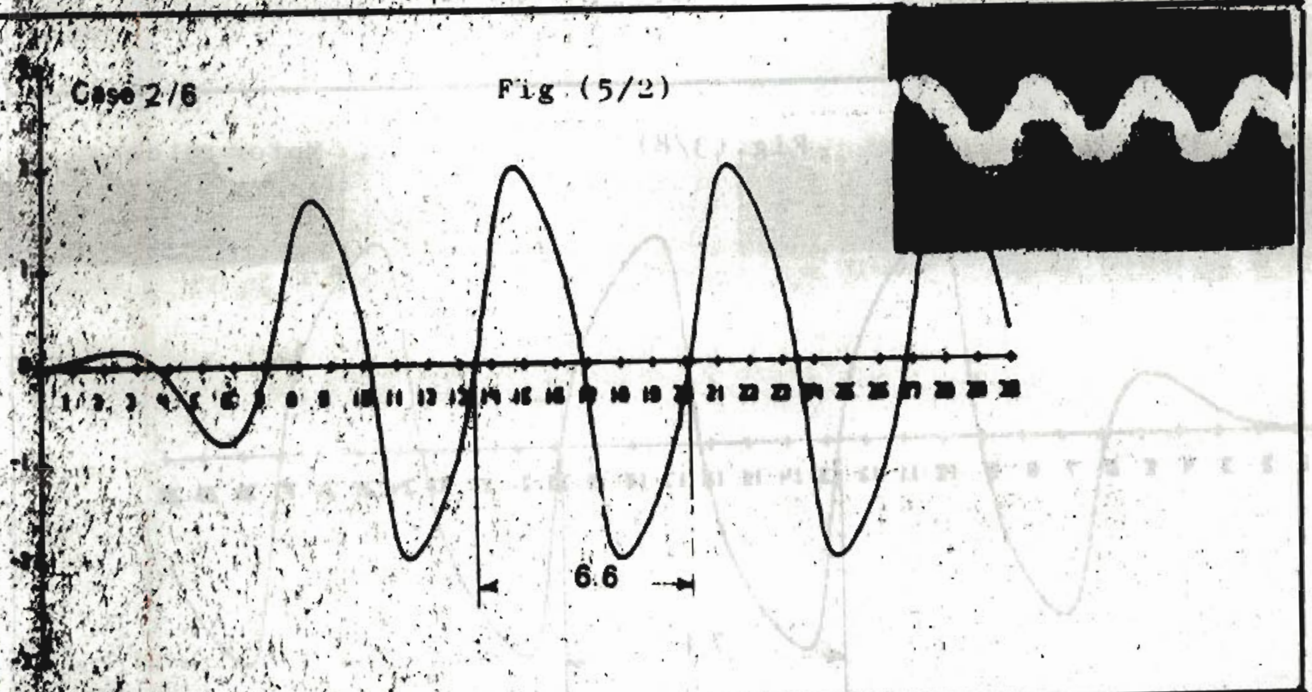


Fig. (5/2)

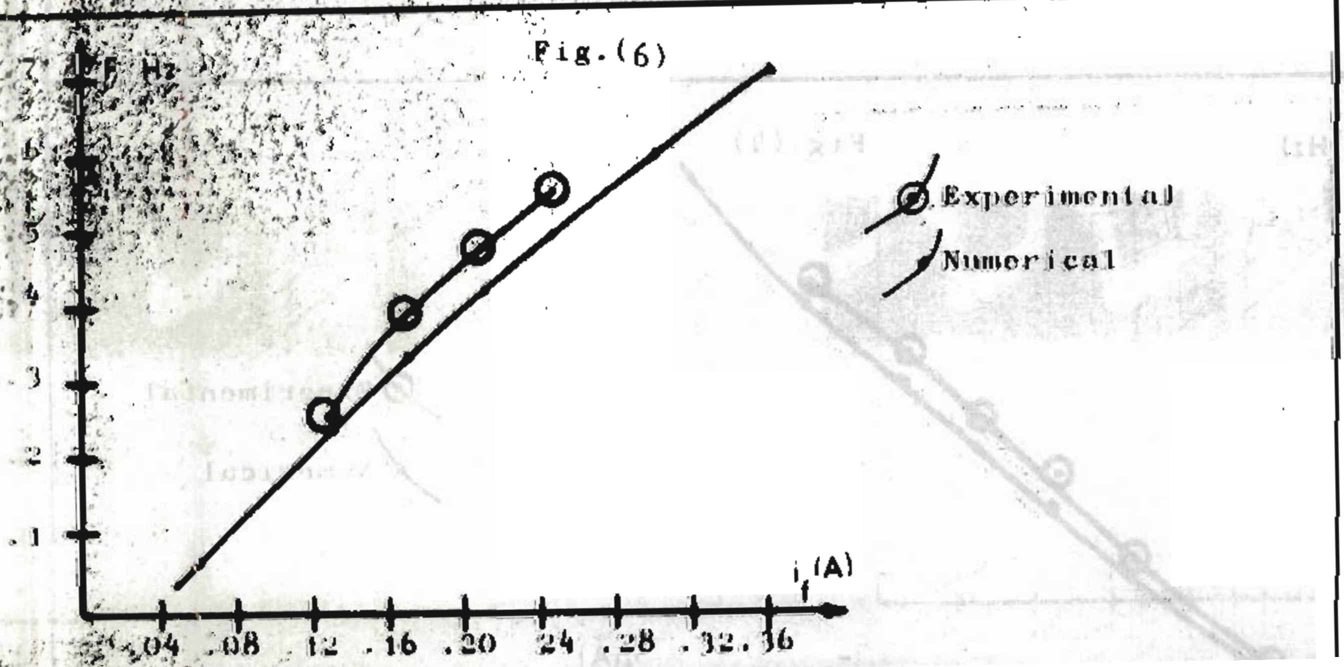


Fig. (6)

Comparison between numerical and experimental results:

Comparison between the wave form obtained from the plots of the numerical solution and that obtained for C.R oscillograph is given in figs.(3,5), this comparison shows that the experimental and theoretical results are in good agreement.

Comparison between the theoretical and the experimental frequency of oscillations for both cases of the added resistance is given in figs.(4,6), from which it is found that the difference is small.

In each case of added resistance, the amplitude of oscillations in the numerical solution is nearly constant, and independant on the motor field current.

The amplitude of oscillations obtained from the experimental results are low than that obtained from the numerical solutions, this deviation may result from the effect of both motor and generator armature reaction which is not considered in the numerical solution.

The effect of the added resistance upon the frequency of oscillation is shown in fig(8), from which it is found that, for the same value of motor field current, by increasing the value of the added resistance, the frequency increase but reduce the range of oscillation.

Conclusions:

The system provide a high power, extremely low frequency, a.c. supply. A power of 1 Kw may be reached with the studed system. The value of the added resistance is highly affecting the armature current, while its effect, on the period of oscillation is limited as shown in fig(8). The motor field current is mainly the factor that controls the system frequency. There are limits of field current for which the system is oscillatory. Outside these limits the systems is either continuously running in one direction, or no motion at all. For example with the added resistance 10 ohms, oscillations occur with the field current varying between 0.06 to 0.3 amperes. Small values of field current leads to a continuous running in one direction.

Increasing the field current increases the system frequency and the armature current wave form approaches the sinusoidal.

Determination of the conditions that must be satisfied for the system to be oscillatory is a suggestion for a future study. This must necessitate an analitical solution of equation (14). It is also interesting to study the harmonic content in the armature current when varying the field current.

<u>References:</u>

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Multimode Oscillations in a Modified Van Der Pole Oscillator.
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APPENDICES

Appendix 1:

Determination of Machine Constants.

Two identical D.C machines are used for motor and generator,

Rating:- 3 KW 220 V 1500 r.p.m

1. Using the universal bridge, the following measurements are obtained.
   a) Total series resistance, includes motor and generator, armatures, and generator series field - 3 044 ohm
   b) Total series inductance, includes motor and generator armatures and generator series field = 0.981 henry.
2. By retardation test, the motor moment of inertia is found to be J = 0.171 Kg mt^2
3. Generator constants K_1, K_2 are determined by running the series generator at constant speed 1500 r p.m separately excited as shown in fig.(1)

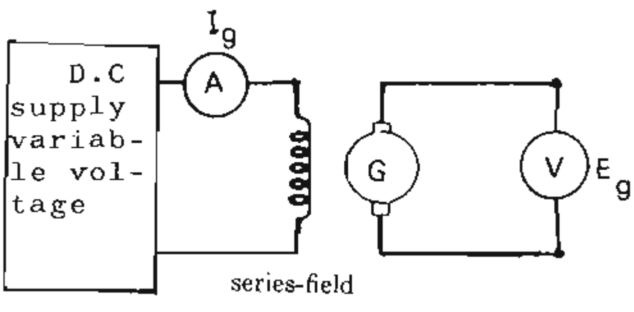


Fig.(1)

E_g (V)	0	20	26	33	43	63	76	100	117
I(A)	0	1.2	1.4	1.7	2.2	3.2	3.8	5.1	6

E_g (V)	129	139	153	163	170	177	185	190	197
I(A)	6.6	7.4	8.4	9.1	10	10.7	11.6	12.3	13.4

By using the Least Square method for interpolation and with programme No.1 using basic language with Hewlett Packard 9830 computer the values of K_1 and K_2 are, K_1 20.455 and K_2 = 0.0327

Curve 1 shows the computer ploting for the observed values and the estimated values, which are in a good agreement.

4. Motor e.m.f/speed constant K_3

This constant can be determined by running the motor as a separate excited generator. for different values of field current, varying the speed N and measuring the induced voltage E, K_3 is the slope of E/N curves as shown in the following table

I_f (A)	0.05	0.06	0.13	0.17	0.21	0.25	0.3	0.36
K_3	0.02	0.03	0.075	0.1	0.12	0.14	0.17	0.20

5. Motor current/torque constants K_5, K_6, K_7.

a) The constants K_5, K_6 can be determined by running the motor as a separate excited D.C Motor. Varying the supply voltage, fig(2), at steady state measure the motor speed N and the armature current I for different values of field current, from the ploted curves fig (3) we get

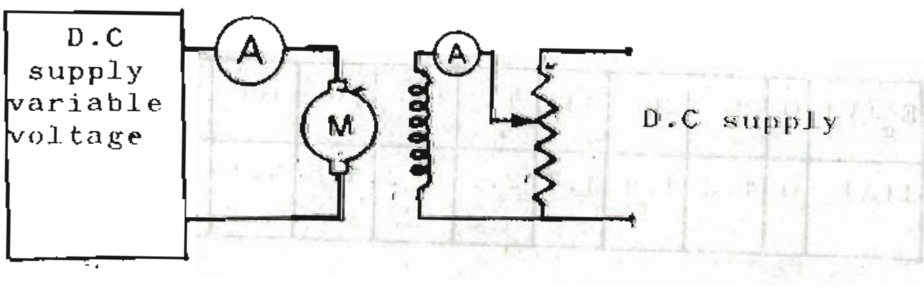


Fig.(2)

I_f (A)	0.05	0.06	0.13	0.17	0.21	0.25	0.3	0.36
K_5	0.90	0.80	0.65	0.55	0.45	0.35	0.25	0.1
$K_6 \times 10^{-4}$	1.58	1/68	1.65	1.56	1.66	1.58	1.56	1.56

Experimentally, it is found that the values of K_6 is almost constant, we shall consider it as ,

$$K_6 = 1.6 \times 10^{-4}$$

b) The Dynamic torque constant K_7

Experimentally, it is difficult to determine the dynamic torque constant K_7. But at the moment of transient, if we consider that all input power to the motor armature is consumed in accelerating the motor then,

$$IE_m = JW \frac{dw}{dt} \qquad w = \frac{2\pi N}{60}$$

having $E_m = K_3 N$

then

$$I = \frac{0.011\ J}{K_3} \frac{dN}{dt}$$

then

$$K_7 = \frac{0.011\ J}{K_3} = \frac{0.00188}{K_3}$$

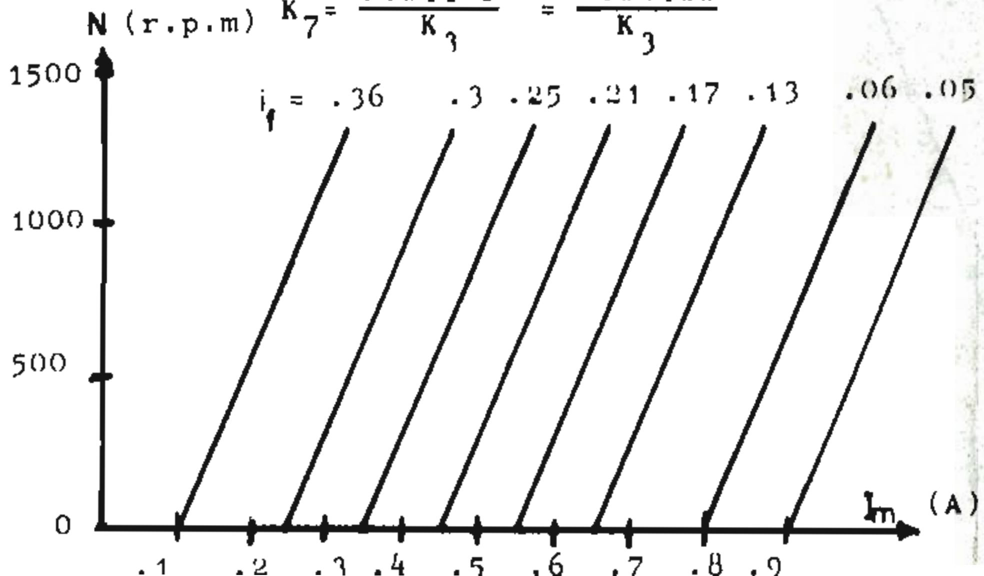
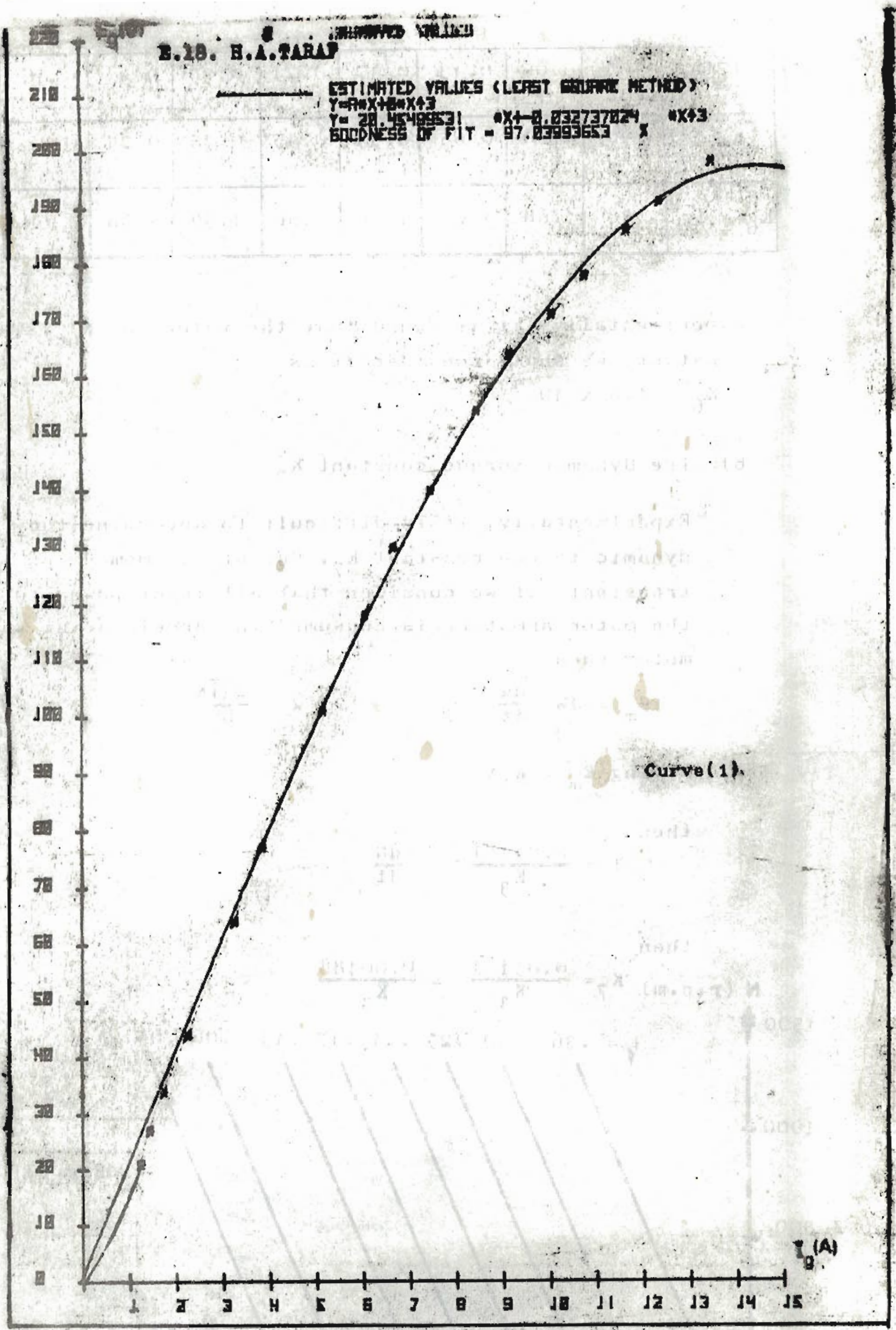


Fig.(3)


E.18. H.A.TARAF
ESTIMATED VALUES (LEAST SQUARE METHOD)
Y=A*X+B*X↑3
Y= 20.45489531 *X+-0.032737024 *X↑3
GOODNESS OF FIT = 97.03993653 %
210
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
I_g(A)

Curve(1).

```
10 DIM X[20],Y[20],Z[20]
20 N=10
30 FOR I=1 TO N
40 INPUT X[I],Y[I]
50 NEXT I
60 S=T=R=S1=S2=S3=S4=S5=E=S7=0
70 FOR I=1 TO N
80 S=S+Y[I]
90 S1=S1+Y[I]*X[I]
100 S2=S2+X[I]↑2
110 S3=S3+X[I]↑4
120 S4=S4+Y[I]*X[I]↑3
130 S5=S5+X[I]↑6
140 NEXT I
150 Y1=S/N
160 D=S2*S5-S3↑2
170 A=(S1*S5-S3*S4)/D
180 B=(S2*S4-S3*S1)/D
190 PRINT
200 PRINT "A = "A
210 PRINT "B = "B
220 PRINT
230 PRINT
240 PRINT "X(I)","Y(I)","A*X(I)+B*X(I)↑3"
250 PRINT
260 FOR I=1 TO N
270 Z[I]=Y=A*X[I]+B*X[I]↑3
280 PRINT X[I],Y[I],Y
290 S7=S7+Y
300 T=T+(Y[I]-Y1)↑2
305 E=E+(Y[I]-Y)↑2
310 NEXT I
312 Y2=S7/N
314 FOR I=1 TO N
316 R=R+(Z[I]-Y2)↑2
318 NEXT I
320 G=100*R/T
325 G1=100*(1-E/T)
330 PRINT
340 PRINT
350 PRINT "GOODNESS OF FIT = "G"%  OR  ="G1"%"
355 PRINT
356 PRINT "T="T,"E="E,"R="R
360 STOP
370 SCALE 0,260,0,21
380 LABEL (*,1,1.7,PI/2,2/3)
390 OFFSET 240,3
400 XAXIS 0,-10,0,-220
410 YAXIS 0,1,0,15
420 FOR I=0 TO 220 STEP 10
430 PLOT -I,-1
440 CPLOT -1,0
450 LABEL (*)I
460 NEXT I
470 FOR I=1 TO 15
480 PLOT 5,I
490 CPLOT -1,0
500 LABEL (*)I
510 NEXT I
520 STOP
530 FOR I=1 TO N
540 PLOT -Y[I],X[I]
550 LABEL (*)"*"
560 PLOT -Y[I],X[I]
570 NEXT I
580 PEN
590 PLOT -220,3
600 PLOT -220,4
610 LABEL (*)"*"
620 PLOT -220,4
630 PLOT -220,5
640 CPLOT 2,0
650 LABEL (*)"OBSERVED VALUES"
660 STOP
670 FOR X=0 TO 15 STEP 0.1
680 PLOT -A*X-B*X↑3,X
690 NEXT X
700 PEN
710 PLOT -210,3
```

Appendix II

Numerical solution of the differential equation

$$Y'' = \mu y'(1-y^2) - y - py^3 + Q$$

Programme No.2 is used to solve this equation for different values of the field current and hence the constant K_3(K [8]) and K_5 (J [8]) for the eight conditions. Fig (4) shows the flow chart of this programme

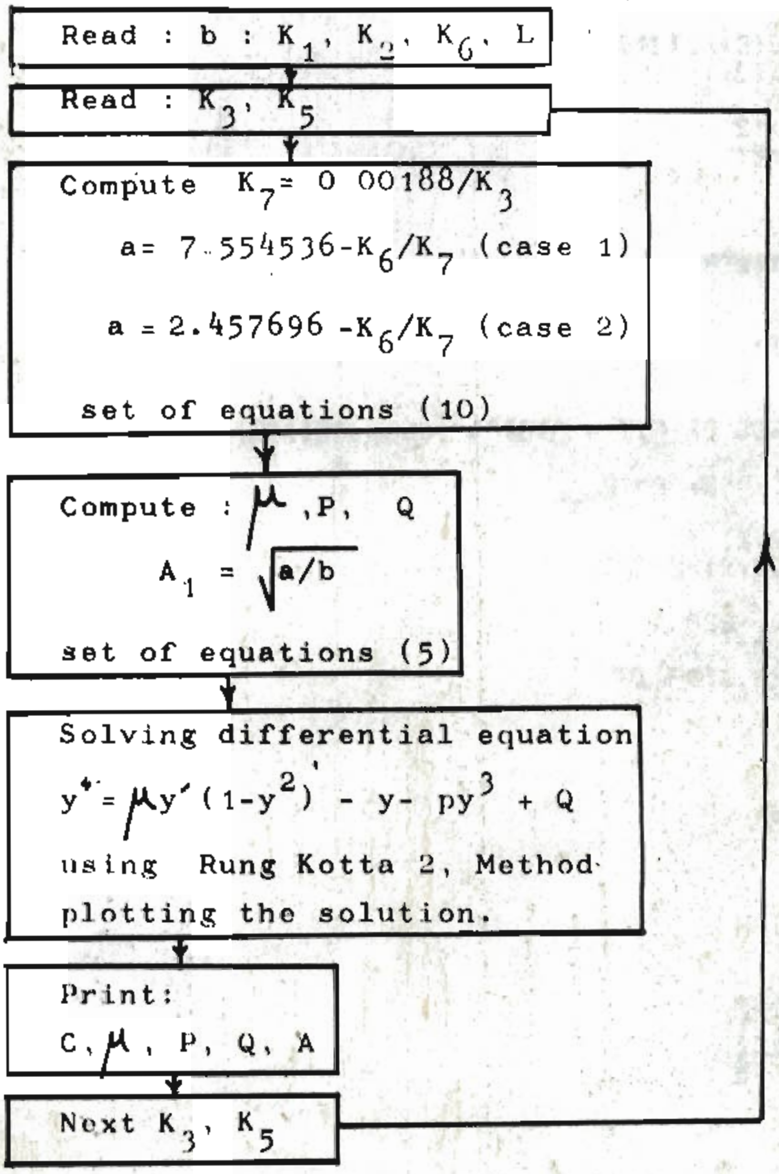


Fig.(4)

Programme 2

K7 K5

```
10 DIM K[8],J[8]
20 H=0.01
30 READ B,K1,K2,K6,L
40 DATA 0.1,20.457,0.0327,0.00[illegible],0.[illegible]
50 READ K[1],K[2],K[3],K[4],K[5],K[6],K[7],K[8]
60 DATA 0.02,0.03,0.075,0.1,0.12,0.14,0.1[illegible],0.2[illegible]
70 READ J[1],J[2],J[3],J[4],J[5],J[6],J[7],J[8]
80 DATA 0.9,0.8,0.65,0.55,0.45,0.35,0.25,0.1
90 FOR N=1 TO 8
100 X=Y=Y1=0
110 K7=0.00189/K[N]
120 A=7.554536-K6/K7
130 C2=(K[N]-K6*7.411)/(L*K7)
140 C=C2↑0.5
150 A1=(A/B)↑0.5
160 D=K2*K6/K7*L
170 E=K[N]*J[N]/K7
180 M=A/C
190 P=A*D/(B*C2)
200 Q=(E/C2)/A1
210 Y2=M*Y1*(1-Y↑2)-Y-P*Y↑3+Q
220 SCALE -5,35,-5,5
230 LABEL (*,1.2,1.7,0,2/3)
240 XAXIS 0,1,0,30
250 YAXIS 0,1,-3,3
260 FOR I=1 TO 30
270 PLOT I,0
280 CPLOT -2,-2
290 LABEL (*)I
300 NEXT I
310 FOR I=-3 TO 3
320 PLOT 0,I
330 CPLOT -3,0
340 LABEL (*)I
350 NEXT I
360 PLOT X,Y
370 FOR X=H TO 30 STEP H
380 U=Y+H*Y1
390 U1=Y1+H*Y2
400 U2=M*U1*(1-U↑2)-U-P*U↑3+Q
410 Y=Y+0.5*H*(Y1+U1)
420 Y1=Y1+0.5*H*(Y2+U2)
430 Y2=M*Y1*(1-Y↑2)-Y-P*Y↑3+Q
440 PLOT X,Y
450 NEXT X
460 PEN
470 PLOT 1,4
480 LABEL (*,2.5,2.2,0,0.6)" C="[illegible]
490 LABEL (*)" P="P;" Q="Q;" (A/B)↑.5 ="A[illegible]
500 PLOT -5,-5
510 IPLOT 40,0
520 IPLOT 0,10
530 IPLOT -40,0
540 IPLOT 0,-10
550 DISP "CHANGE PAPER"
560 STOP
570 NEXT N
580 END
```

Note:

Initial condition at x=0 y=0 y=0

For low values of field current, the interval H will be 0.001

From the numerical solution we get

Case 1

i_f	0.05	0.06	0.13	0.17	0.21	0.25	0.3	0.36
C	0.45169	0 684	1.7325	2.3147	2.7804	3.24614	3.9447	4.9433
X_o	28	20	10.6	9	8.4	7.8	7.5	7.1
A_1	8 6907	8.6902	8.688	8.6867	8.6858	8.6848	8.6833	8.6818
f	0.00518	0.0342	0.1634	0.257	0.331	0 4161	0.526	0.6962

Case 2

i_f	0.05	0.06	0.13	0.17	0.21	0.25	0.3	0.36
C	0.4611	0.6940	1.74190	2.3240	2.7897	3.25548	3.95405	4.652617
X_o	11.8	10	7.1	7	6.6	6.6	6.55	6.5
f	0.0390	0.06940	0.2453	0.332	0.4226	0.4932	0.6036	0.715
A_1	4.9573	4.95725	4.95688	4.95665	4.95648	4.95630	4.95605	4.95579