

Menoufia University  
 Faculty of Engineering, Shebin El-Kom,  
 Basic Engineering science Department  
 First semester Examination, 2016-2017  
 Date of Exam: 4/1/2017



Subject: Mathematical Modeling  
 Code : BES 510  
 Year : Postgraduate students  
 Time Allowed : 3 hours  
 Total Marks: 100 marks

**Answer the following questions**

1) Define each of the following expressions:

Autonomous dynamical system - Difference equation- Lipschitz concept.

2) Derive the equation that minimizes the sum of the squared deviations between a set of data points as in the following table and the quadratic model.

$x$	0.1	0.2	0.3	0.4	0.5
$y$	0.06	0.12	0.36	0.65	0.95

3) Solve the difference equation ,  $x_{n+2} - 2x_{n+1} + 2x_n = 0$  , with initial conditions  $x_1 = 0$  and  $x_2 = 1$ .

4) Classify the fixed point for the following systems and then draw the phase-portrait for each system.

$$(i) \begin{cases} \dot{x}_1 = x_1 + x_2 \\ \dot{x}_2 = 4x_1 - 2x_2 \end{cases} \quad (ii) \begin{cases} \dot{x}_1 = x_1 - 2x_2 \\ \dot{x}_2 = 2x_1 - x_2 \end{cases}$$

5) Find the equilibrium point for the following systems then determine the stability each system.

$$(i) \begin{cases} \dot{x} = \frac{a}{b}(x)(b - x) \\ x(0) = x_0 \end{cases} \quad \text{with } a > 0 \text{ and } b > 0 \text{ (logistic model)}$$

$$(ii) \begin{cases} \dot{x} = ax \\ x(0) = x_0 \end{cases} \quad \text{with } a > 0 \text{ (Exponential growth model)}$$

6) Show that the function  $f(x) = 2x + 3, \forall x \in \mathbf{R}$  is Lipschitz continuous in  $\mathbf{R}$  and the function  $f(x) = x^{\frac{2}{3}}, \forall x \in (0, +\infty)$  is not Lipschitz continuous on  $(0, +\infty)$ .

7) Write the following equation,  $\ddot{y} + (1 + t^2)\dot{y} + 2ty = 0$ , as autonomous system and then find the Jacobian matrix for the following autonomous systems:

$$(i) \begin{cases} \dot{x}_1 = (x_1)^2(x_1 - x_2)^3 \\ \dot{x}_2 = (x_2)^3(x_1 - x_2)^2 \end{cases} \quad (ii) \begin{cases} \dot{x}_1 = \cos x_1 \sin x_2 \\ \dot{x}_2 = \cos x_2 \sin x_1 \end{cases}$$

8) A sewage treatment plant processes raw sewage to produce usable fertilizer and clean water by removing all other contaminants. The process is such that each hour 12% of remaining contaminants in a processing tank are removed. What percentage of the sewage would remain after 1 day? How long would it take to lower the amount of sewage by half? How long until the level of sewage is down to 10% of the original level?