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ON SOLVING STIFF DIFFERENTIAL EQUATIONS OF ELECTRICAL POWER SYSTEMS

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ABSTRACT

The problem of solving the exact mathematical repressentation of electrical power systems is too complex even using digital computers. It becomes more difficult if the system performance analyses within a long period of time is needed (e.g. the limit cycle analysis).

This paper presents a new algorithm for solving stiff differential equations (SDE). This algorithm can also be used in solving the discontinuous and nonlinear differential equations with high degree of accuracy. Moreover, one can easily change the iteration steps as desired so as to obtain any detailed information within the period of interest.

To show the algorithm power and accuracy, an application for solving electrical power system model is introduced.

I. Introduction.

The study of systems may be divided into four parts: modeling, development of mathematical equation description, analysis and design. The development of models for physical systems, such as electrical power ones, along with the thorough understanding of the system is essential for the success of the design. The equations that describe the systems may assume many forms; differential equation description is of interest [1]. To solve such differential equations, the method of integration should be studied carefully.

There are many characteristics, upon which the integration method can be chosem, such as accuracy, simplicity, stability of the solution and ability of solving discontinuous and non-linear equations. Classical techniques can be used to solve simple differential equations [2]; however, such techniques fail to solve the so called "stiff differential equations(SDE)".

Recently, the systematic method [3] is successfully used to solve it.

This paper presents a new algorithm for solving SDE depending on both systematic and Rungy-Kutta methods of integration. For complex models, such as electrical power system.one in which the analysis should be considered within a long period of time. Rungy-Kutta will be difficult and a computer time consumer. Also, the systematic method with a large step of integration can not give any detailed information if needed. The proposed algorithm will use the norm of the coefficient matrix A to determine the step of integration and consequently determine the type of integration method.

Application of the proposed algorithm to the electrical power system model is introduced to clarify the power and accuracy of this algorithm.

II. Statement of the Problem

Determine the set of differential equations that represent the system model in state space form. Then, the stiffness test of this differential equations need be performed as follows, so as to assign the method of integration.

Let the system model be in the following state space form

$$\underline{\dot{X}} = A\,\underline{X} + b\,U$$

and λ_k be the λ_kth eigen value of the coefficient matrix A.

Then, the stiffness factor $K = \frac{\lambda\ max}{\lambda\ min}$

where

$$\lambda\ max = max_k\ /\ \lambda_k\ /\ ,\quad \text{and}$$

$$min = min_k\ /\ \lambda_k\ /\ ,$$

$$K = 1, 2, 3, \ldots \ldots ,n$$

if $K >> 1$, then the set of differential equation is stiff one. The infinite norm of A ($||A||$) need be calculated (the infinite norm of A is defined as

$$||A||_\infty = \max_i \left[\sum_{j=1}^{n} |a_{ij}| \right]$$

The problem now is: how to solve the SDE fast, and analyze the solution within a long period of time with the facility of getting any detailed information if needed?

III. Systematic Method of Integration

Assume that the system model is in the following state-space form:

$$\dot{\underline{X}} = A\underline{X} + bu \qquad \ldots \quad \ldots \quad (1)$$

Using newton-lebnetsa method (see Appendix A), the numerical solution of equation (1) will take the following form:

$$X_{n+1} = X_n + \phi(A,H/2)\ [\ 2f\ (t_n+H/2,\ X_n + \phi(A,H/2)\ .\ f(t_n,X_n)]$$
$$-\ A\ \phi(A,H.2)\ f\ (t_n,X_n)\ ] \qquad \ldots\ (2)$$

where H is the integration step,

$t_n = t_o + nH$ is the time interval

Assuming $A = 0$ in equation (2) will directly give the Rungy-kutta numerical solution form.

Assume that,

$$h = \frac{H}{2^N} \qquad \ldots \quad \ldots\ (3)$$

in other words the number of iteration steps $N = \log_2 \frac{H}{h}$ ... (4)

where

$$h = \frac{1}{||A||_\infty} \qquad \ldots \quad \ldots \quad \ldots \quad (5)$$

Now, to use equation (2) in soving the stiff D.E. without using pungy-kutta assumption, the transition matrix need be calculated numirically as follows:

$$\phi_N(A,H/2) = \int_0^{2^{n-1} h} e^{A\tau}\, d\tau \qquad \ldots \quad \ldots \quad (6)$$

Let N = 0 , then

$$\phi_o\,(A,H/2) = \int_0^{h/2} e^{A\tau}\, d\tau = A^{-1}\,[e^{Ah/2} - I] \quad \ldots \quad (7)$$

where I is the identity matrix

$$e^{Ah/2} = A\,\phi_o + I \qquad \ldots \quad \ldots \quad (8)$$

$$e^{2Ah/2} = (A\phi_o + I)^2 = A\phi_o(A\phi_o + 2I) + I = A\phi_1 + I \quad \ldots \quad (9)$$

comparing equation (8) and (9) gives

$$\phi_1 = \phi_o\,(A\,\phi_o + 2I) \qquad \ldots \quad \ldots \quad \ldots \; (10)$$

It is easy then to verify that

$$e^{2^{N-1}Ah} = A\,\phi_N + I \qquad \ldots \quad \ldots \quad \ldots \; (11)$$

i.e. $$\phi_N = \phi_{N-1}\,(2I + A\,\phi_{N-1}) \qquad \ldots \quad \ldots \quad \ldots \; (12)$$

Note that ϕ_o can easily be calculated approximately from the equation

$$\phi_o = \int_0^{h/2} e^{A\tau}\, d\tau = \sum_{\delta=0}^{2} \frac{A^{\delta+1}\,(h/2)^{\delta+1}}{(\delta+1)!} \qquad \ldots \quad \ldots \; (13)$$

Note that in the systematic method, the matrix A is taken into consideration.

Moreover, the integration step H can be decreased to get detailed information as long as it satisfies the condition

$$H > \frac{2}{||A||_\infty} \qquad \ldots \quad \ldots \quad (14)$$

From the first glance, it seems that the increasing of H will consequently cause increasing of N and this makes the computation of transition matrix longer. This is not true, since H is not directly proportional to N (eqn 4) and $\phi_1, \phi_2, \ldots, \phi_N$ need be calculated once.

Solving the SDE by using the systematic method of integration is not only easier and faster than Rungy-kutta method, but also will guarantee the solution convergence.

The algorithm for solving the SDE will easily continue according to the flow chart shown in Fig. (1).

IV. APPLICATION OF THE PROPOSED ALGORITHM TO THE ELECTRIC POWER SYSTEM MODEL

Consider the following power system model which is consists of synchronous machine transmission line and infinite busbar as shown in the schematic diagram of Fig. (2).

Synchronous machine representation [4]

$$\dot{\psi}_d = [V \sin\delta - (1+S)\psi_q - r\, i_d]\, w_o$$
$$\dot{\psi}_q = [-V \cos\delta + (1+S)\psi_d - r i_q]\, w_o$$
$$\dot{\psi}_f = [V_f - r_f\, i_f]\, w_o$$
$$\dot{\psi}_{1d} = [-r_{1d}\, i_{1d}]\, w_o$$
$$\dot{\psi}_{1q} = [-r_{1q}\, i_{1q}]\, w_o$$
$$\dot{S} = [P_T - \psi_d\, i_q + \psi_q\, i_d]/T$$
$$\dot{\delta} = S\, w_o$$

$$\begin{bmatrix} X_d & 0 & X_{ad} & X_{ad} & 0 \\ 0 & X_q & 0 & 0 & X_{aq} \\ X_{ad} & 0 & X_f & X_{ad} & 0 \\ X_{ad} & 0 & X_{ad} & X_{id} & 0 \\ 0 & X_{aq} & 0 & 0 & X_{1q} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_f \\ i_{1d} \\ i_{1q} \end{bmatrix} = \begin{bmatrix} \psi_d \\ \psi_q \\ \psi_f \\ \psi_{1d} \\ \psi_{1q} \end{bmatrix}$$

AVR representation according to the block diagram in [5]:

$$\dot{X}_1 = (\Delta V_g - X_2 - 1.3 \times 10^{-2} X_1)/0.828 \times 10^{-4},$$

$$\dot{X}_2 = X_1$$

$$\dot{X}_3 = (X_1 - X_2) / 0.26 \times 10^{-2}$$

$$\dot{X}_4 = (X_2 - X_4) / 0.26 \times 10^{-2}$$

$$\dot{X}_5 = (\Delta S - X_5) / 1.27 \times 10^{-2}$$

$$\dot{X}_6 = (X_5 - X_6) / 3.39 \times 10^{-2}$$

$$\dot{X}_7 = (X_6 - X_7) / 0.94 \times 10^{-2}$$

$$\dot{X}_8 = X_9$$

$$\dot{X}_9 = (X_7 - X_8) / 0.93 \times 10^{-2}$$

$$\dot{X}_{10} = (29.5 X_5 - 135.88 X_6 + 106.38 X_7)/0.94 \times 10^{-2}$$

$$\dot{X}_{11} = (X_{10} - X_{11}) / 0.026$$

$$\dot{X}_{12} = [0.93 K_{of} (9.3 \times 10^{-2} X_9 + X_7 - X_8)/9.3 \times 10^{-2} + K_{1f} (X_{10} - 0.74 - 0.74 X_{11})/0.026 + K_\phi K_{OV} (X_2 - 0.914 X_4)/0.026 + K_\phi K_{1V} (X_1 - 0.974 X_3) / 0.026 - K_{FB} K_F V_F - X_{12}/K_T) / 0.66 / K_T$$

$$\dot{X}_{13} = (X_{12} - X_{13}) / 0.005$$

$$\dot{X}_{14} = (K_{FB} K_{15} - X_{14}) / 0.1$$

$$\dot{V}_f = (K_f X_{13} - V_F) / 0.05$$

Where

$K_{OF} = 13$, $K_{1F} = 4.8$, $K_{OV} = 50$, $K_{1V} = 7$,

$K_\phi = 0.87$, $K_F = 0.087$, $K_T = 75$, $K_{FB} = 0.27$.

Results

Three phase short circuit was applied at the busbar for o.4 seconds and then it was cleared. The purpose of that was to study the behavior of the synchronous machine and moreover, to investigate the system transient and steady state stability.

Figure (3) illustrates the transient characteristics of the currents i_d , i_q , i_F , i_{1d} and i_{1q} against the time. Also to study the slips and the electrical power angle δ after the short circuit.

Because of oscillation found in the response, a long period of time was needed for investigating the system behavior after short circuit.

Moreover, the detailed information is needed so as to analyze precisely what had happened during the time period of interest.

Therefore, the integration step H has been chosen to satisfy this condition $H < 2/ ||A||_\infty$ in order to solve the SDE very easy and fast (in our application H=0.2). However in the period between $t_I = 3.5$ to $t_f = 3.55$ (i.e. the period in which the detailed information was needed) H did not satisfy that condition. Therefore Rungy-Kutta method has been used with integration step H = 0.001 within that period only.

Figure (4) illustrates all of the results along with the detailed information within the period of interest (3.5-3.55sec.).

CONCLUSIONS

The physical system mathematical models has been studied. The SDE solution that takes a very long computer time using the classical techniques has been investigated.

A new algorithm to solve such SDE easy and fast has been introduced in this paper.

The infinite norm of the coefficient matrix A has been used so as to assign the integration method and moreover, the integration step.

Using the systematic method guarantees the solution convergence and in addition saves a computer time. The nice property of using the systematic method of integration in solving SDE is that, one can get a detailed information about any period of interest just by assigning the initial and final time of such a period.

The mentioned algorithm has been applied to the electrical power system.

APPENDIX A

Systematic Method Numerical Solution .

Consider the following first order differential equations.

$$\underline{X} = F(\underline{X}, t)$$

with the initial condition

$$X(t_o) = X_o$$

Newton-Lebnetsa Solution

$$X_{n+1} = X_n + \int_o^H \frac{d}{d\tau} X(t_n + \tau)\, d\tau$$

where

$$t_n = t_o + n H$$

Multiplying $\frac{dX}{d\tau}$ by $\phi^{-1} \phi = I$ gives

$$X_{n+1} = X_n + \int_o^H \phi^{-1}(t_n+\tau)\, \phi(t_n+\tau) \frac{d}{d\tau} X(t_n+\tau)\, d\tau$$

By integrating the preceding equation one gets

$$X_{n+1} = X_n + [\int_o^H \phi^{-1}(t_n+\tau)\, d\tau + C]\, \phi(t) \frac{d}{dt} X(t) \Big|_{t=t_n} -$$

$$C\, \phi(t) \frac{d}{dt} X(t) \Big|_{t=t_{n+1}} - \int_o^H [\int_o^\tau \phi^{-1}(t_n+\tau)\, d\tau +$$

$$C][\phi(\tau) \frac{d^2}{dt^2} X(t) + \frac{d}{d\tau} \phi(\tau) \frac{d}{dt} X(t)] \Big|_{t=t_{n+1}-\tau} d\tau$$

where C is an independant matrix.

Putting $\phi(t_n+\tau) = e^{-A(t_n+\tau)}$ and neglecting the last term

$$X_{n+1} = X_n + [\int_o^H e^{A(t_n+\tau)}\, d\tau + C]$$

$$e^{-At_n} \frac{d}{dt} X(t) \Big|_{t=t_n} - C\, e^{-At} \frac{d}{dt} X(t) \Big|_{t=t_{n+1}}$$

Let $C = 0$

$$X_{n+1} = X_n + \int_o^H e^{A\tau}\, d\tau \,.\, f(X_n, t_n)$$

put

$$\phi(A, H/2) = \int_0^{H/2} e^{A\tau} \, d\tau$$

Then, the systematic method numerical solution;

$$X_{n+1} = X_n + \phi(A , H/2) \; [\; 2f \; (t_n+H/2, \; X_n+ \phi(A,H/2).$$

$$f \; (t_n,X_n)] - A \; \phi \; (A,H/2) \; f \; (t_n,X_n)]$$

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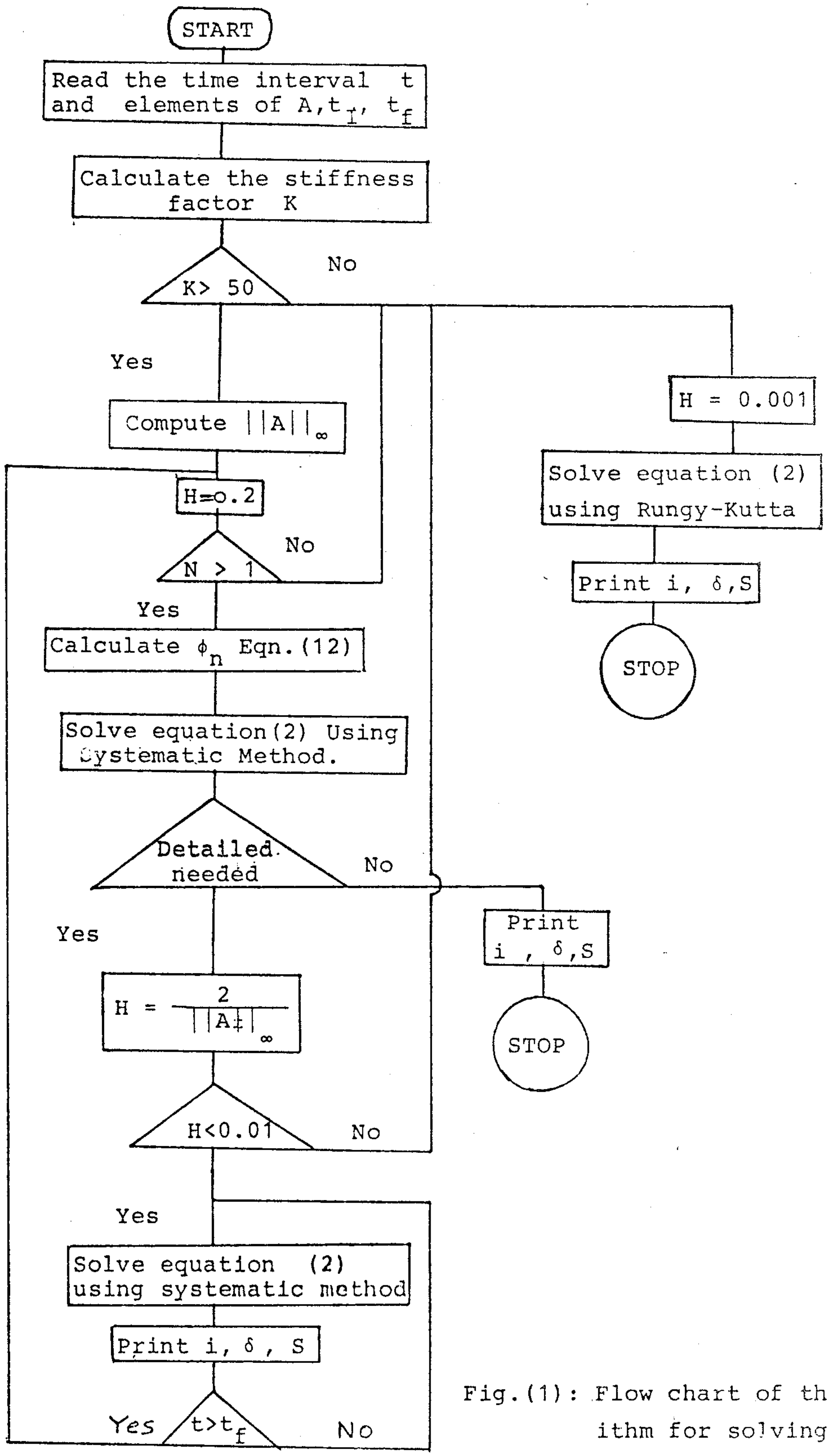


Fig.(1): Flow chart of the algorithm for solving SDE.

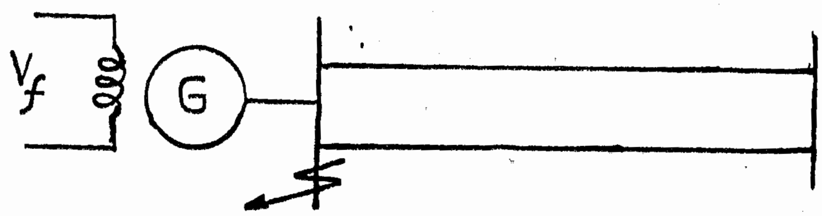
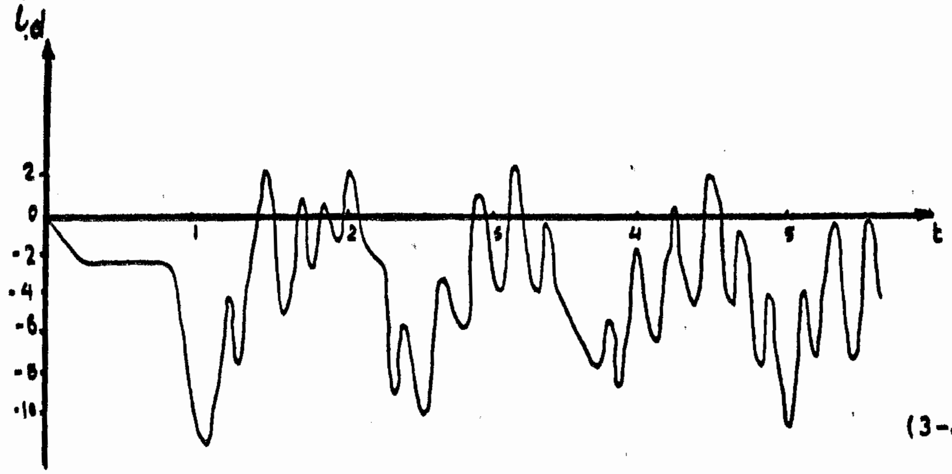
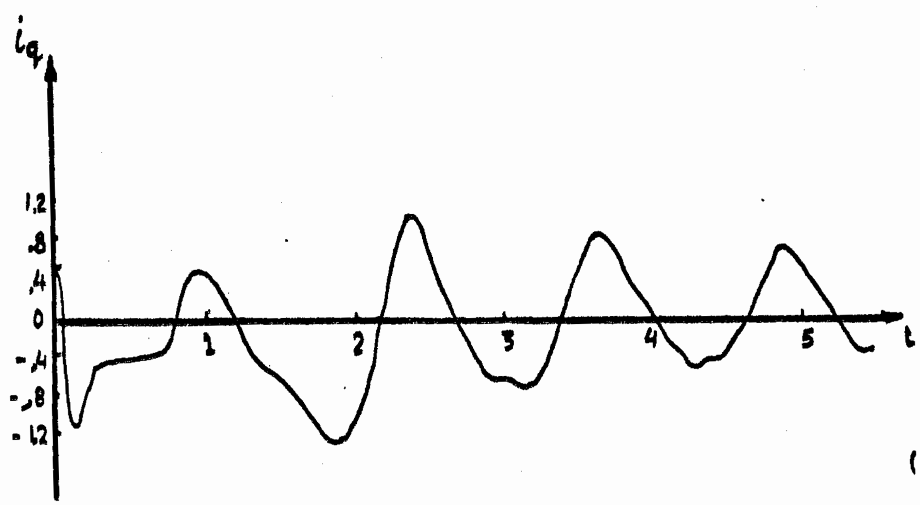


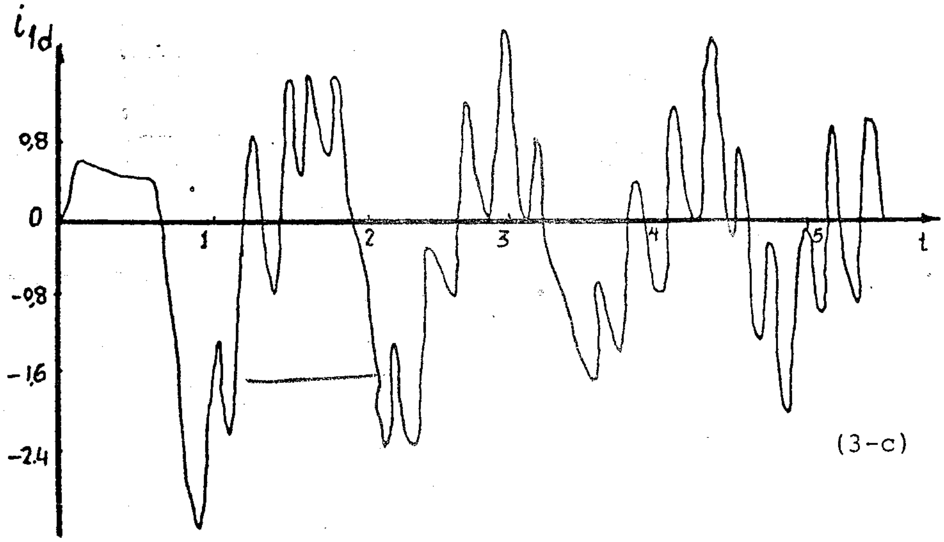
Fig. (2): Schematic diagram of the power system.



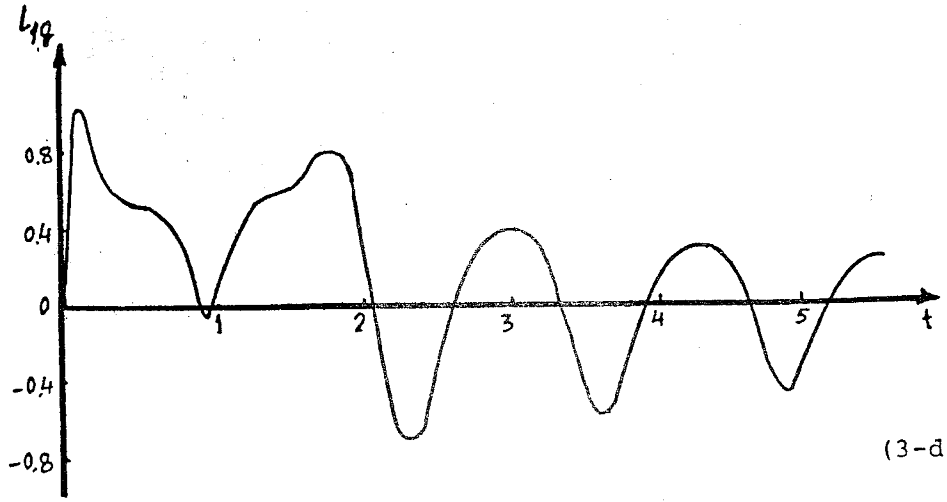
(3-a)



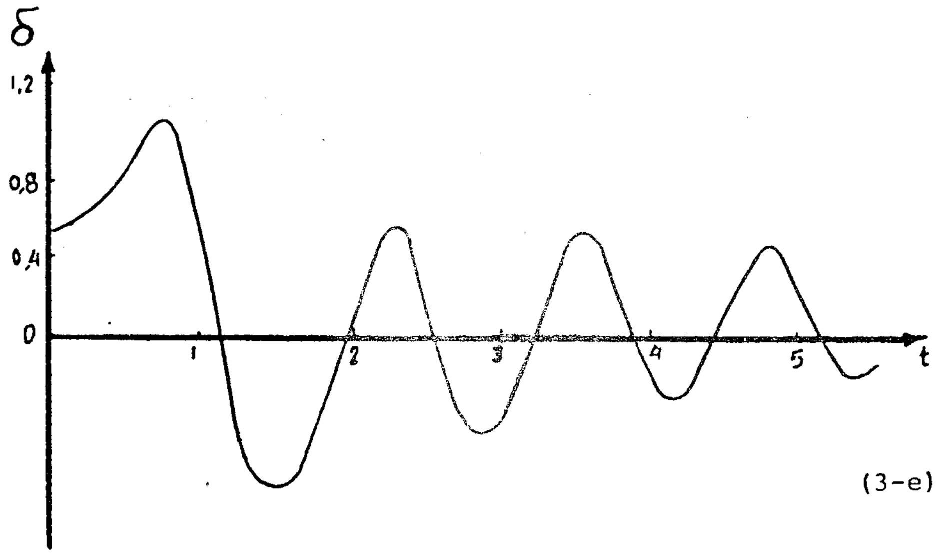
(3-b)


i_{1d}
0,8
0
-0,8
-1,6
-2,4
1
2
3
4
5
t

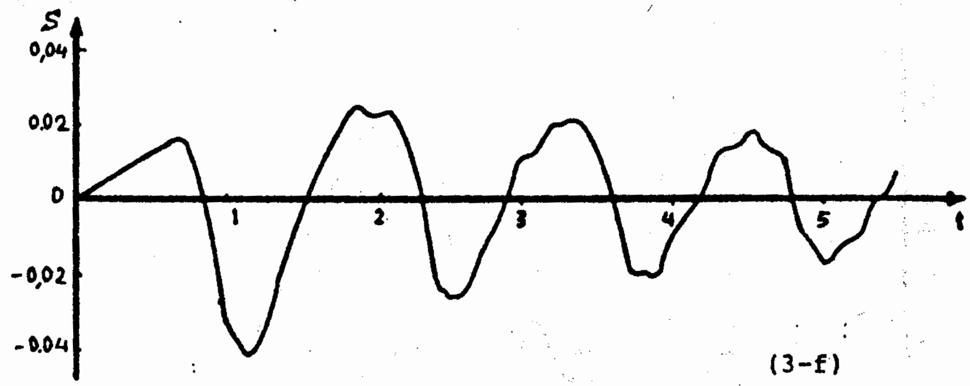
(3-c)


i_{1q}
0,8
0,4
0
-0,4
-0,8
1
2
3
4
5
t

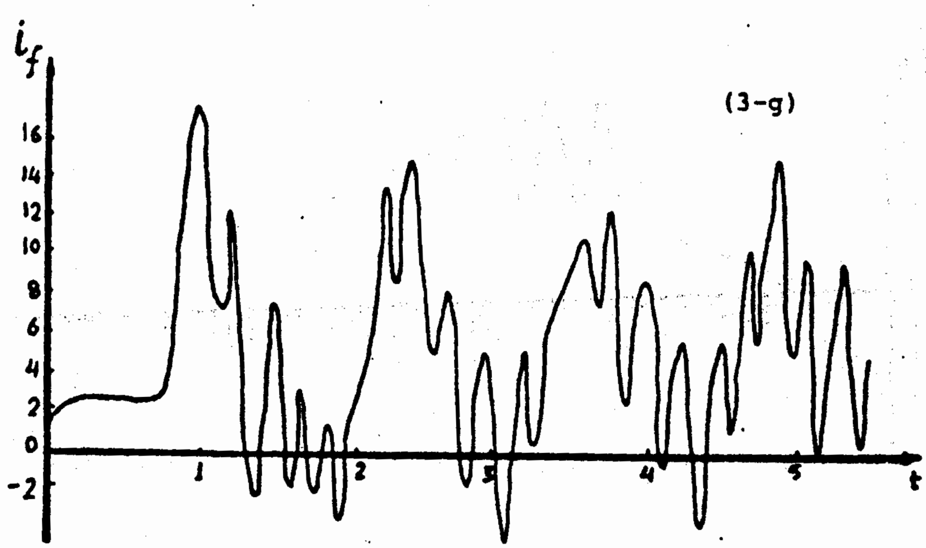
(3-d)


δ
1,2
0,8
0,4
0
1
2
3
4
5
t

(3-e)

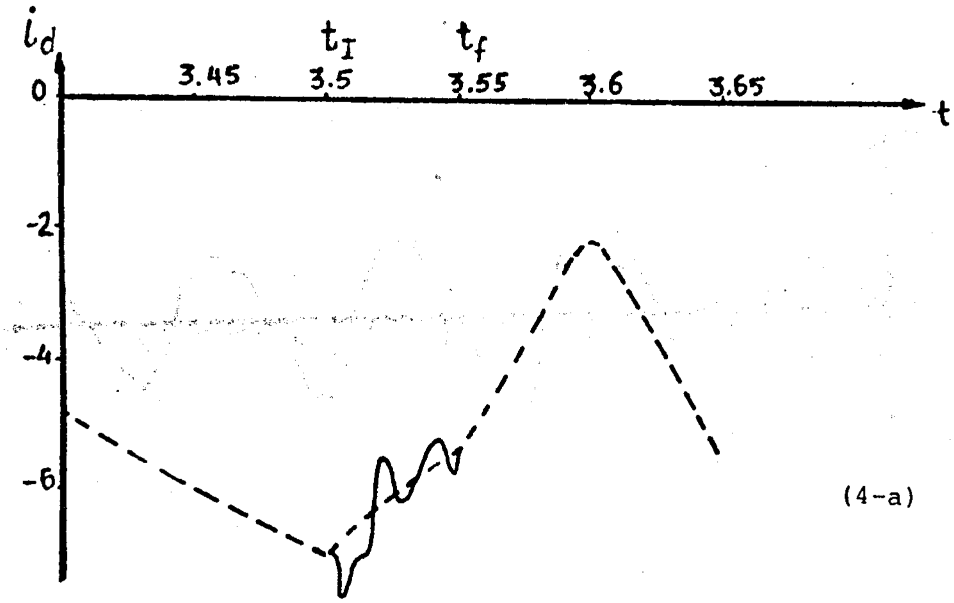


(3-f)



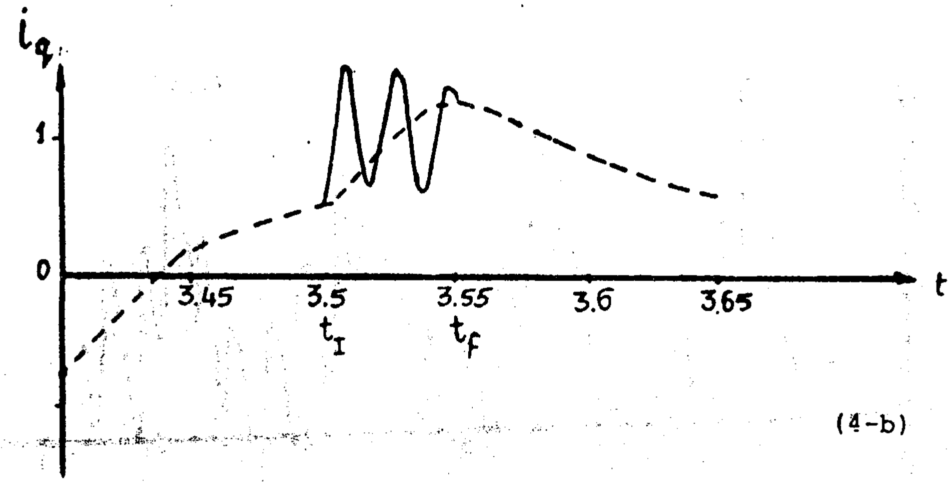
(3-g)

Fig. (3): Transient characteristics of the generator parameters.



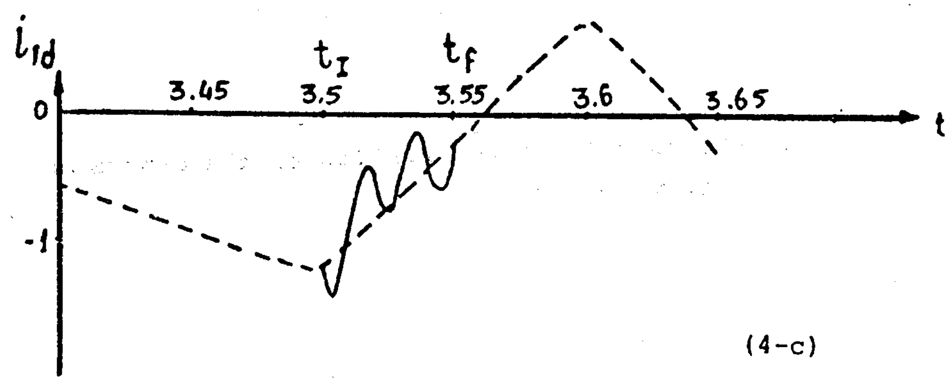
i_d
t_I
t_f
0
3.45
3.5
3.55
3.6
3.65
t
-2
-4
-6

(4-a)



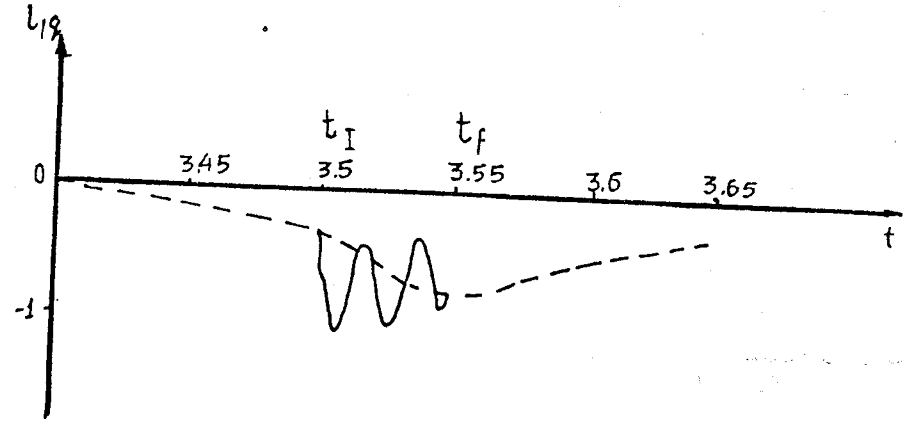
i_q
1
0
3.45
3.5
3.55
3.6
3.65
t
t_I
t_f

(4-b)

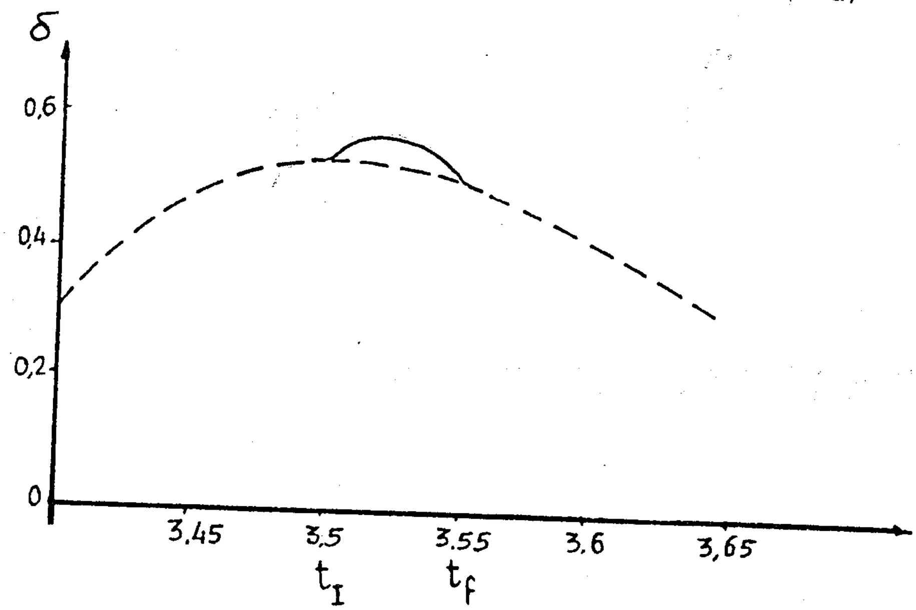


i_{1d}
t_I
t_f
0
3.45
3.5
3.55
3.6
3.65
t
-1

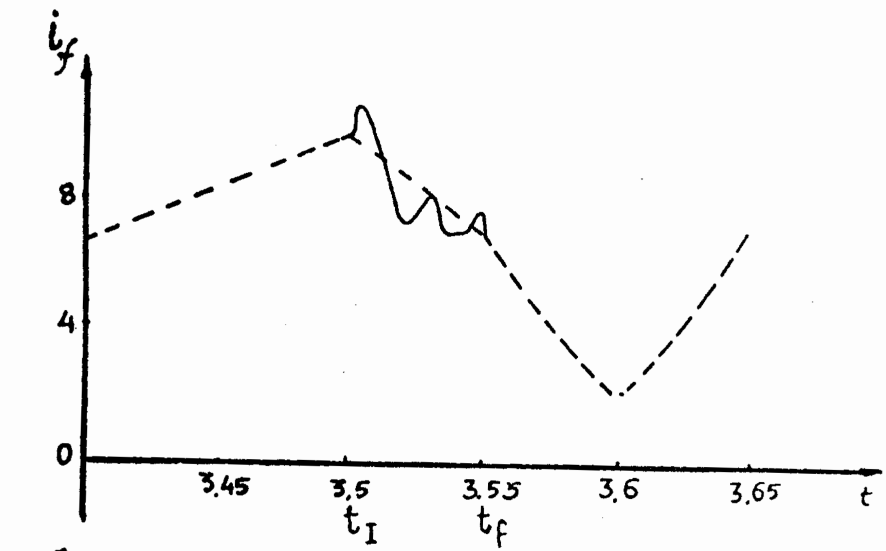
(4-c)



(4-d)



(4-e)



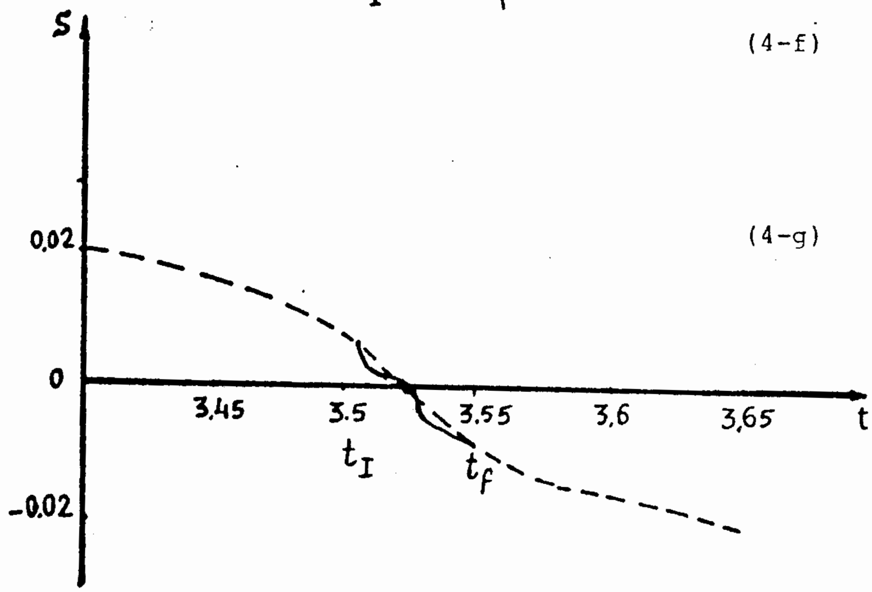


Fig. (4): Transient characteristics of the generator parameters along with the detailed information.