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NUMERICAL MODELING OF AN L-SHAPED VERY STIFF CONCRETE RETAINING WALL

نموذج عددى لحائط خرسانى ساند جاسئ قوى على شكل حرف L.

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الخلاصه : تم دراسة نموذج عددى لتوضيح وبيان سلوك حائط ساند جاسئ على شكل حرف L من الخرسانة المسلحة وذلك باستخدام برنامج PLAXIS مع طريقة العناصر الدقيقة. ولقد تم تقييم ومعايرة النموذج العددى المقترح وذلك بمقارنة النتائج النظرية بالنتائج المعملية السابقة والتى قد تم إجرائها فى جامعة مانشستر. مع الأخذ فى الاعتبار الأبعاد والخصائص الهندسية للنموذج المعملي الذى أجريت عليه الاختبارات الطاردة عن المركز. وقد تم أيضا التوصل إلي وجود اتفاق كبير بين الضغوط الجانبية المحسوبة عدديا والمقاسة معمليا المؤثرة على الحائط، وذلك بالمقارنة مع نتائج الطريقة التقليدية القياسية والمبنية على طريقة K_o أو ضغط رانكن الفعال.

ABSTRACT

A numerical model incorporated in finite element PLAXIS program is presented in this paper to simulate the behavior of an "L-shaped" very stiff reinforced concrete prototype wall. The proposed numerical model has been calibrated and validated using centrifugal experimental results conducted previously at Manchester University. Taking into account the centrifuge model dimensions and geometry with respect to the centrifuge scaling law, the loading conditions and sequences during the test, and by considering for the modeling of the soil the hardening soil model, the numerical results obtained in terms of pattern and magnitude of the wall displacements were very close to the experimentally measured results at prototype scale. A good agreement was also obtained between the measured and the numerically computed lateral pressures acting on the wall, in comparison to the classical approach based on the K_o procedure or the Rankine active pressure.

Keywords: Retaining Wall, Centrifuge testing, numerical modeling, PLAXIS program.

INTRODUCTION

The continuous demands of accuracy in designing retaining structures projects in town areas clearly asks for bringing comprehensive analytical methods like the finite element method into practice. Many previous investigations put forward the requirement to bring together the large practical experience gained through field and laboratory testing and the modern numerical techniques. The fast developments in computer hardware and, more importantly, in geotechnical software enable the geotechnical engineer to perform very advanced numerical analyses at low cost and with relatively minimum computational effort. Commercial software, fully integrated into the PC-environment, have become so user-friendly that little training is required for operating the programme. However, before those numerical techniques could be used in common practice they have to be fully validated against reliable experimental results so that the designer develop a feeling of the potentials and pitfalls of the results obtained (Powrie & Li, 1991; Potts, 1992 and Carruba & Collonne, 2000).

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The first objective of the present investigation concerns the development of a numerical model to simulate the behavior of an "L-shaped" retaining structure, observed at centrifugal experiment testing conducted on a small scale model by Dr. Djerbib (1986) at Manchester University. In order to simulate correctly the behavior of the wall during the centrifuge test, and thus to predict the behavior of the prototype (full-scale) it is necessary to take into account during the development of the numerical model, all the parameters that governed the behavior observed during the experimental testing, and replicate numerically, as closely as possible the geometry of the structure, the boundary conditions, the loading conditions and testing sequences imposed during centrifuge model. The second objective would be to use the developed numerical model to undertake a parametric study to investigate further the factors that affect the complex behavior of this type of retaining structures; in an attempt to improve the overall understanding of their behavior and to produce a more realistic and economical design method for the particular "L-shaped" retaining wall. The paper presents the results of the first step of this study which concerns the development of the numerical model. The analysis and comparison for validation of the proposed numerical results in terms of wall displacements and lateral stresses acting on the wall stem are made using the experimental data obtained in the centrifugal testing model. The present approach could be of relevance, since relatively little attention has been paid in the literature on the validation and reliability of numerical models in general and on specific software in particular.

CENTRIFUGAL RESULTS USED FOR VALIDATION

Centrifuge model testing is useful in the investigation of geotechnical problems where idealized conditions may be created to allow the validation of analytical or numerical solutions. The primary purpose of centrifuge modeling is to create vertical stresses in the model which correspond to those at full scale and, thereby, to permit the soil in the model to display its correct deformation behavior. It is recognized that when a model geometrically similar to a field-scale structure is made at a scale 1:n and tested at a constant acceleration equivalent to n gravities, the self-weight stress distribution will be correctly modeled if the boundary conditions are also similar (Shofield, 1980).

The centrifugal experiment reported by Hird & Djerbib (1994) and Djerbib *et al.* (2001) was conducted using a wall-model fabricated from a stiff mild steel of 22 mm thick steel plate, with overall dimensions of 650 mm x 150 mm x 90 mm. The wall-model was instrumented with several displacement transducers to monitor the displacement of the wall during the course of loading. According to the centrifuge scaling law and by considering the acceleration of 60g produced during the testing, this wall represents at full scale (at 1g) a prototype of very stiff concrete structure with overall dimensions of height 9m x base width 5.5m and 3m thick. Figure 1 shows the overall configuration of the centrifuge model wall and the container. The soil used in the experiment was the Leighton buzzard sand.

For the stiff wall the bending deflection are negligible and the measured horizontal and vertical displacements reported concerns the rigid body movements. Table 1 presents the centrifuge measured displacements of the wall after an acceleration of 60g (which correspond to the state of backfilled prototype at 1g, without surcharge loading). As illustrated in Figure 2, δ_{ht} is the horizontal movement of the top of the wall (displacement of the point A); δ_{hb} is the horizontal movement of the bottom of the wall (horizontal displacements of the points B and C); δ_v is the vertical movement of the wall (vertical displacement of the points A and B) and θ is the angle of rotation. The corresponding displacement of the prototype according to the scaling law are also presented.

Table 1. Displacements of the wall (model and prototype) measured in the Centrifuge Test.

Scale	δ_{ht}	δ_{hb}	δ_v	0	$\frac{\delta_{ht}}{H}$
Centrifuge Model	-1.23	0.32	-0.63	0.53	-0.008
Prototype	-73.8	19.2	-37.8	31.8	-0.008

NUMERICAL ANALYSIS

Model Description:

The development of the numerical model for the analysis was carried out using the PLAXIS Program (version 7.2). Plaxis is a finite element package which has been developed specifically for the analysis of deformation and stability in geotechnical engineering projects. In order to carry out finite element analysis using Plaxis, the user has to create a finite element model and specify the material properties and boundary conditions. The simple graphical input procedures enable a quick generation of complex finite element models, and the output facilities provide a detailed presentation of computational results. The calculation itself is fully automated and based on robust numerical procedures.

The numerical analysis was carried out in plane strain two dimensional analysis. A cross section through the wall and backfill modeled is given in Figure 3. The finite element model extends 28m horizontally and 14m vertically to account for the centrifuge dimensions box converted to the prototype scale. The retaining wall is defined through an L-Shaped beam with a rigid slab footing representing the prototype dimensions of the centrifuge model.

The conditions of plane strain were assumed throughout. One of the first steps in any numerical simulation is to determine where to place the boundaries so that their influence on the results will be minimised. Figure 4 shows a typical finite element mesh and the displacement boundary conditions. The vertical boundaries of the mesh were pinned in the horizontal direction but free to move vertically, and the horizontal boundary at the base of the mesh was assumed to be pinned in both the vertical and the horizontal directions

Modeling mesh data:

Table 2 gives the modeling mesh data adopted in the finite element computation for the soil, the wall and the interface. The soil model was run with a fine coarse mesh, 15 nodes triangular elements leading to 344 elements, 2929 nodes and 4128 stress points. The wall was modeled as a linear concrete beam element.

Modeling experience showed that problems of soil structure interaction might involve points, which require special attention. Corners in stiff structures and an abrupt change in boundary condition (L-shaped geometry of the wall) may lead to peaks in the stresses and strains. Conventional finite elements analyses are not capable of reproducing these sharp peaks and will, as a result, produce non-physical stress oscillation. PLAXIS code prevents this phenomenon by entering additional interface elements inside the soil body which enhances the flexibility of the Unite element mesh and prevent non-physical result. The theoretical background on the special use of interface element in the modeling of soil-structure interaction was thoroughly investigated by Van Lange & Vermeer (1991).

Table 2. Modeling Mesh Data

Type	Type of element	Type of integration	Total number
Soil Wall	15-noded	12-point Gauss	344
Interface	5-node line	4-point Gauss	10
	5-node line	4-point Newton Cotes	14

In the model presented in Figure 4 the interface element are shown to have finite thickness, but in the PLAXIS finite element formulation the coordinates of each node pair are identical, which means that the element has zero thickness. Each interface has assigned to it a "virtual thickness" which is an imaginary dimension used to obtain the material properties of the interface. The thickness is defined as the Virtual thickness factor times the average element size. The average element size is determined by the global coarseness setting for the mesh generation. The default value of the virtual thickness factor is 0.1.

The Wall modeling:

The retaining wall structure was simulated with one-dimensional linear beam element that can resist axial load and bending moments. The stiffness for the wall element is represented by means of the flexural rigidity EI and the normal stiffness EA, where A and E are the cross section area and the Young modulus of the concrete structure wall. The wall modeling parameters are presented in Table 3.

The Soil modeling :

The modeling of the soil backfill behavior and foundation is very a important task in the analysis of the retaining structure. It is clear that a simple elasto-plastic soil models may be rather misleading, thus more sophisticated soil models must be used in common practice so that the designer might have a closer approach to the real behavior of the retaining walls.

In the present numerical analysis the soil was modeled using the hardening soil model incorporated into the Plaxis program. This constitutive model is based on the well known formulation of Duncan & Chang (1970), but formulated within the theory of plasticity. It incorporates shear hardening and volumetric hardening, a stress dependent stiffness for primary loading and unloading/reloading and the stress dilatancy theory by Rowe (1962).

The hardening-soil model is an advanced model for simulating the behaviour of different type of soil, both soft and stiff soils. When subjected to primary deviatoric loading, soil shows a decreasing stiffness and simultaneously irreversible plastic strains develop. In the special case of a drained test, the observed relationship between the axial strain and the deviatoric stress can be well approximated by hyperbola. Such a relationship was first formulated by Kondner (1963) and later used in the hyperbolic model of Duncun & Chang (1970). The hardening-Soil model, however, superseded the hyperbolic model by far. Firstly by using the theory of plasticity: rather than the theory of elasticity. Secondly by including soil dilatancy and thirdly by introducing a yield cap. Some basic characteristics of the model are:

- Stress dependent stiffness according to a power law: parameter m
- Plastic straining due to primary deviatoric loading: parameter E_{50}^{ref}
- Plastic straining due to primary compression: E_{oed}^{ref}
- Elastic unloading/reloading: E_{ur}^{ref}, $\nu_{ur}^{(nu)}$
- Failure according to the Mohr-Coulomb model: c, φ and ψ

The full modeling parameters of the soil used in this analysis are presented in Table4.

Table 3. Wall Modeling Parameters

Type	Type of Material	Young's modulus E [kPa]	Poisson's ration ν	Normal stiffness EA [kN/m]	Flexural rigidity EI [kNm^2/m]	Equivalent thickness D [m]
Wall	Reinforced Concrete	2.3E7	0.30	6.9E7	5.175E7	3.00

Table 4. Soil modelling Data Sets Parameters

Soil model: Hardening Soil	Units	Soil Material: Leighton Buzzard Sand
Type		Drained
γ_{unsat}	[kN/m^3]	17.00
γ_{sat}	[kN/m^3]	20.00
K_x	[m/day]	1.000
K_y	[m/day]	1.000
E_{50}^{ref}	[kN/m^2]	30000.00
E_{oed}^{ref}	[kN/m^2]	30000.00
power (m)	[-]	0.50
C_{ref}	[kN/m^2]	1.00
φ	[°]	32.00
ψ	[°]	2.00
E_{ur}^{ref}	[kN/m^2]	90000.00
$\nu_{ur}^{(nu)}$	[-]	0.200

CALIBRATION OF THE NUMERICAL MODEL

Before the modeling mentioned above was adopted, the PLAXIS program was used in many runs to calibrate the Finite element model. The calibration process consisted of finding the set of modeling parameters and numerical calculation parameters that led to the best match between the experimental data and the computed results. Many attempts were investigated in this stage, including the mesh coarseness and the calculation process in addition to the geometry, the boundary, the loading conditions, the centrifuge testing sequences, and considering different constitutive soil models incorporated in the Plaxis code. To account for the testing procedure and to replicate numerically as close as possible the experimental testing process, the calculation process was run in one phase, taking into account the initial conditions according to the numerical sequences adopted in the chart shown in Figure 5.

NUMERICAL RESULTS

Prediction of displacements and forces are amongst the key objectives of performing soil-structure interaction analysis. For validation purpose the results of the present numerical analysis are presented mainly in terms of wall displacements predictions and computation of earth pressures acting on the wall stem. A brief indication of the predicted soil displacements is also given.

Soil displacements:

Figure 6 shows the deformed mesh of the numerical prediction corresponding to the state of 'end of backfilling'. In this figure the displacements are scaled up 10 times to highlight the deformation pattern of the wall (rigid body movements) and the soil. Figure 7 gives an indication of the total displacement arrows predicted at a considered stage of wall loading. In this Figure it could be noticed that there is clear concentration of the displacements beneath the wall base and within the part of the backfill inside what is commonly called the virtual wall. Common design practice of "L-Shaped" retaining wall assume the soil mass resting on the wall base as par of the wall and its deformations are not taken into account.

Wall displacements:

The resulting predicted total displacements of the retaining wall are shown in Figure 8 at an exaggerated scale. It is clear that the settlement of the wall base is greater at the stem bottom. In addition to the forward tilt of the wall stem away from the original backfill, the base of the wall has also translated forward. In addition to the accordance observed in the centrifuge experiment, these results are consistent with some previous gravity wall analyses. The position of the instantaneous centre of rotation for the unpropped wall indicates that rotational movements are dominant throughout the test.

Figures 9 and 10 show the computed displacements of the nodes corresponding to the points A, B and C representing the geometry of the retaining wall plotted against the multiplier. The predicted displacement of the point A is representative of the displacement of the top of the wall stem. and could thus be compared to the experimental value of δ_{ht}. The horizontal movement of the bottom of the wall δ_{hb}, is compared to the horizontal displacement (translation) of the point (B or C). The vertical movement of the wall δ_v is compared to the average displacement of the points B and C to account for the rotation of the wall.

The prediction of the wall movement by the proposed numerical model indicates that rotational movements are generally dominant. This is in agreement with the reported behavior observed throughout the centrifuge test, and confirms the movements pattern of the wall which is a combination of the rotation and translation.

Figure 11 illustrates the correlation obtained between the centrifuge measured (converted to the prototype scale) and the numerically predicted displacements of the wall. As can be seen in this figure, the proposed numerical model has the ability to produce a very close prediction of both: the displacement pattern and the magnitude of the displacements. The small discrepancy observed between the numerical and the experimental approaches can be attributed to the possible limitation of the proposed numerical model.

Earth pressure:

Figure 12 shows the associated distribution of the horizontal earth pressures at the vertical plane: immediately adjacent to the wall stem (computed from the stress-points corresponding to the soil elements adjacent to the interface). In this plot a good agreement between the output of the proposed numerical model and the experimental results obtained on the centrifuge by Hird & Djerbib (1993) is clearly apparent. Also for the appreciation of the obtained results, the at-rest and the classical Rankine active earth pressures are also shown in the Figure 12.

The lateral pressures acting on the wall stem are generally of interest for structural design of reinforced concrete retaining walls. The common practice in design consist of multiplying the active pressures by a load factor that usually has a value greater than unity. The lateral movement of the wall resulted in the reduction of the horizontal earth pressures acting on the wall stem from the at- rest condition for almost the entire height of the wall. The numerically predicted as well as the experimentally measured lateral pressures at the wall stem correspond closely to the classical Rankine active pressures for the top two-thirds of the wall. In the lower third of the stem the numerically predicted lateral pressures are significantly in excess of the active pressures. This could be attributed to the effect of the wall base. These few observations are closely in agreement with the findings of Goh (1993) in his finite element approach to investigate the behavior of concrete cantilever retaining walls.

Limitations of the numerical modeling procedures:

Although all efforts have been made to reproduce numerically the centrifugal tests referred to in the calibration and validation processes, it will remains obvious that perfect numerical simulation of geotechnical experimental testing is a hard goal to achieve. Since some details of the experiment, reported by

Djerbib *et al.* (2001) and related to handling of the equipment, the appliance of the loading charges through the increase of the gravitation, the limitations related to the system of wall deformations measurement used (standards LVDTs), the disturbances induced to the soil backfill and foundation during the mounting process of the centrifuge testing box, in addition to the tolerated finite element computational errors, remains factors that might have introduced uncertainties in the correlation made between the experimental and numerically computed results.

CONCLUSION

This paper described the development of a simple numerical model using the finite element code Plaxis program (version 7.2) to simulate the behavior of an "L-shaped" retaining wall supporting a sand backfill. The proposed numerical model was calibrated and validated using centrifugal experimental results conducted on a small scale model stiff wall. Attempts has been made to simulate the behavior of the retaining wall by taking into account the wall geometry, the boundary conditions and considering for the constitutive modeling of the soil, the advanced hardening soil model. The proposed numerical model was found to give good prediction of the displacement pattern and magnitude of the prototype wall. The lateral pressures computed using this model was also found to be significantly in agreement with the experimental results.

The possible implication of the developed numerical model is to undertake a parametric study to further investigate the factors which affect the complexes behavior of this type of retaining structures and to improve its overall understanding, in an attempt to produce amore realistic and economical design method for this particular type of retaining wall.

The present approach could be of relevance, since relatively little attention has been paid in the literature on validation and reliability of numerical models in general and on specific software in particular. The comparison of the experimental results to the numerical analyses approach used in this study could constitute a powerful mean in investigating the behavior of retaining structures. However, it remains relevant for a complete geotechnical investigation to monitor field observations which remains the best way to fully validate numerical simulations.

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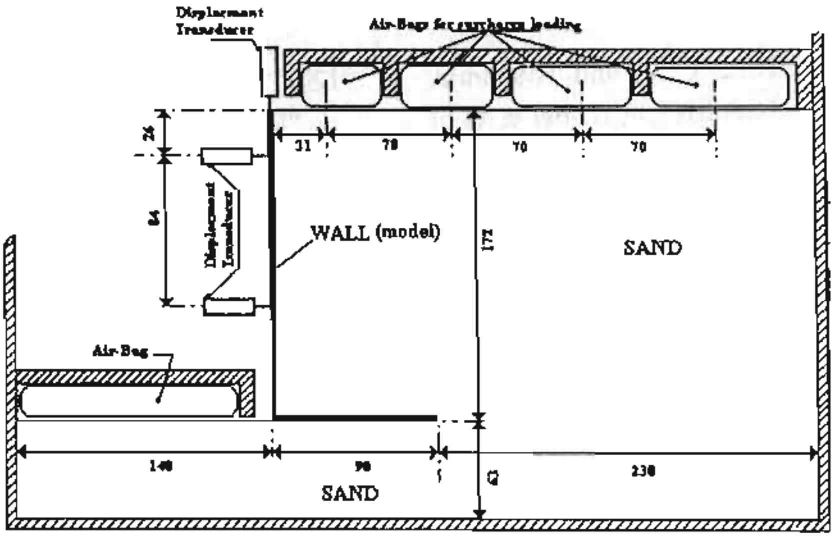


Fig. 1. Transverse section of the centrifuge Testing model (after Hird & Djerbib 1993)

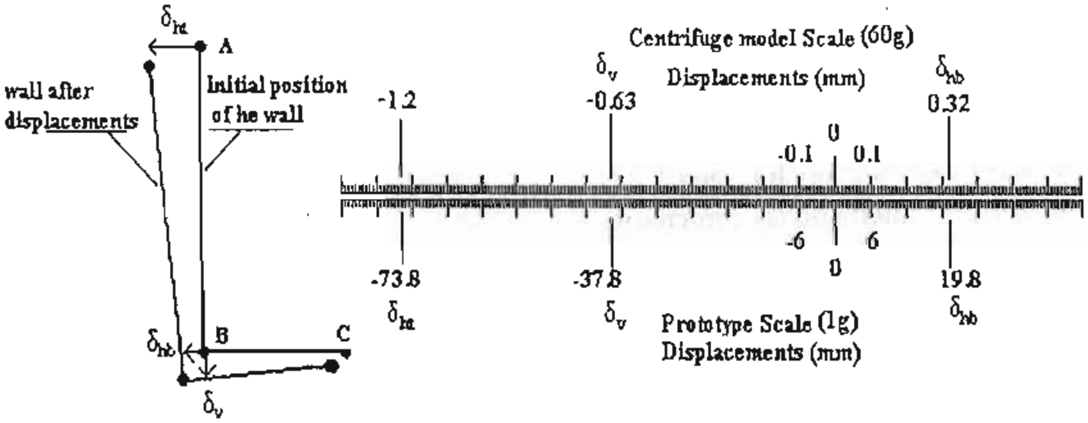
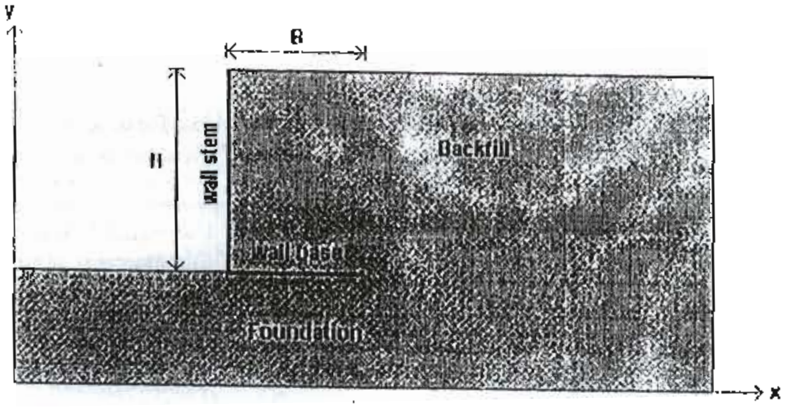


Fig. 2. Displacement measured at Centrifuge experiment (after Djerbib et al., 2001)



	Min.	Max.
X	0.000	28.000
Y	0.000	14.000

Property	Symbol	Value
Wall-stem height (m)	H	9.0
Wall-base width (m)	B	5.4

Fig. 3. Numerical model Geometry

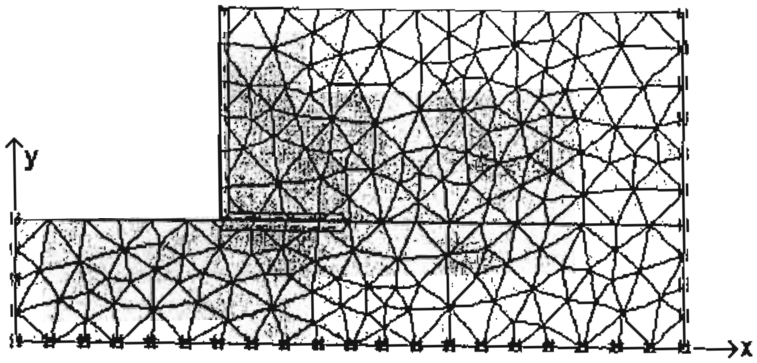


Fig. 4. Typical Finite element mesh and displacement boundary conditions.

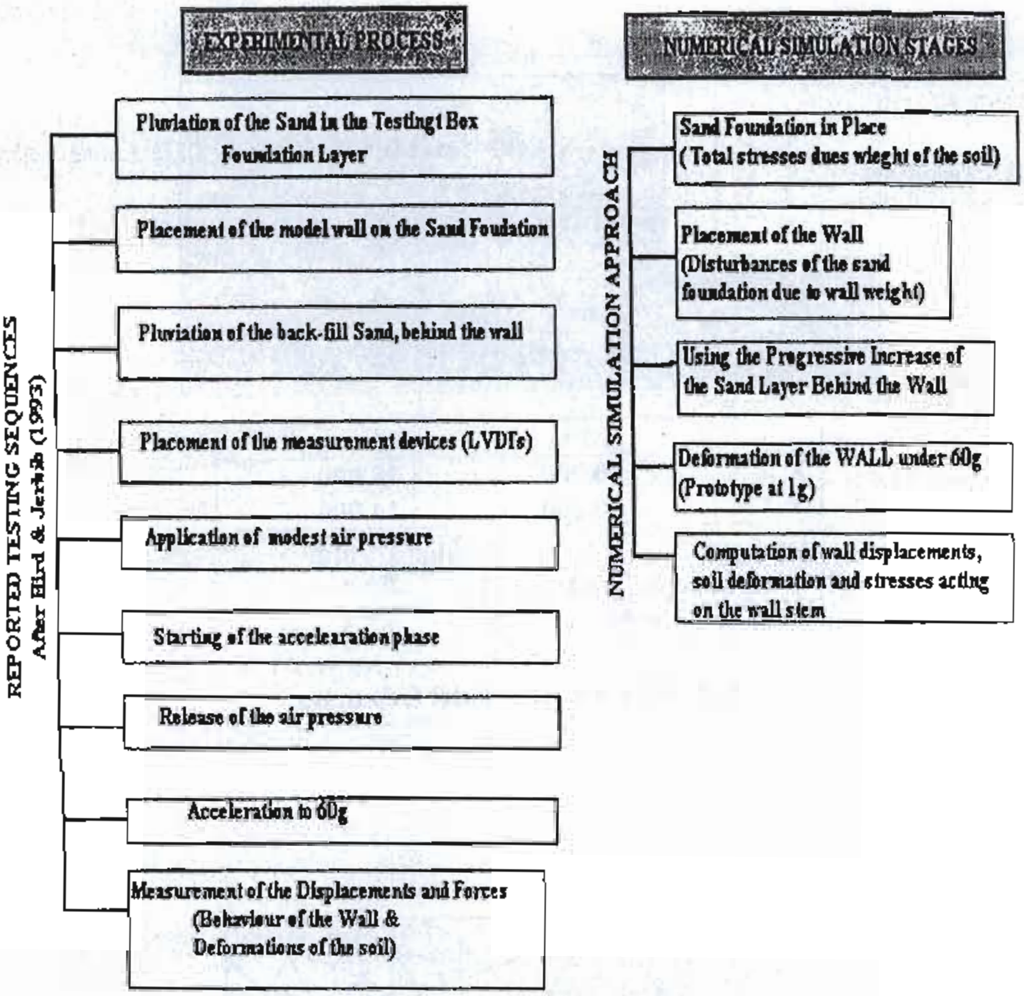


Fig.5. Numerical Simulation stages

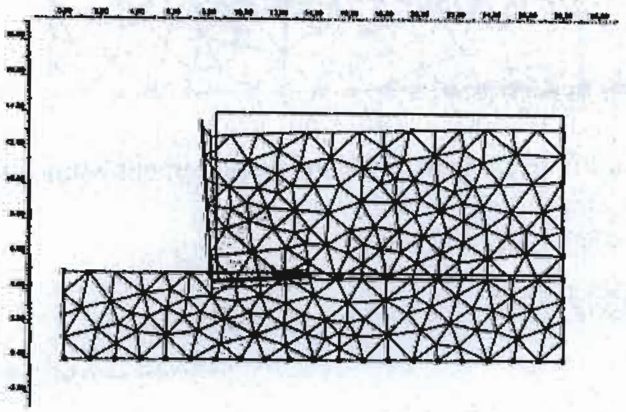


Fig.6. Deformed mesh, displacements scaled up 10 times.

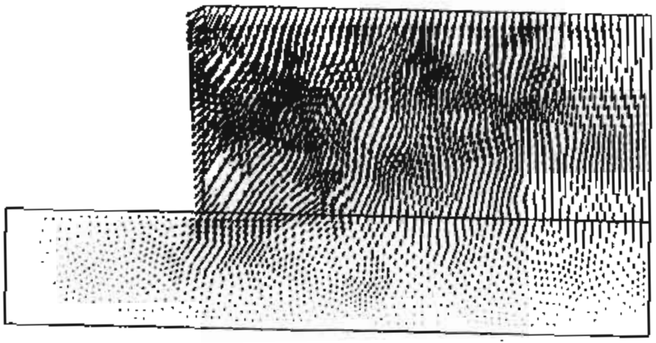


Fig.7. Total displacement arrows of the soil.

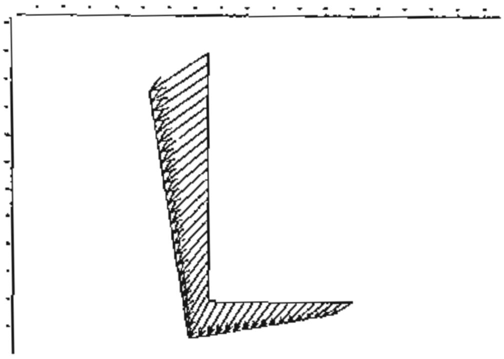


Fig.8. Wall total displacement.

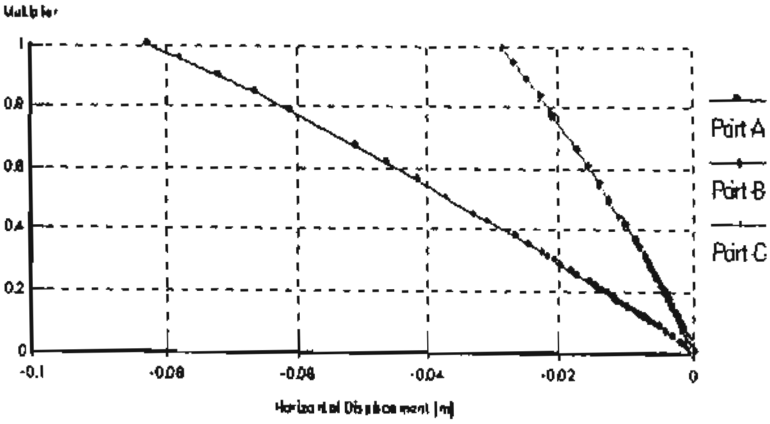


Fig.9. Wall horizontal displacement

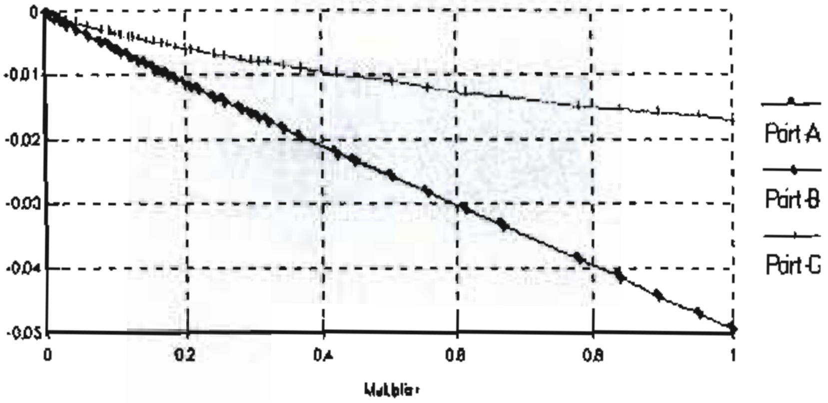


Fig.10. Wall vertical displacement.

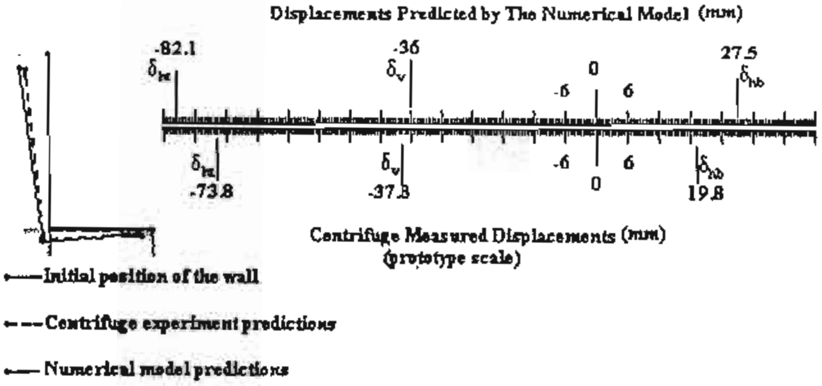


Fig ll. Comparison of computed and measured displacements.

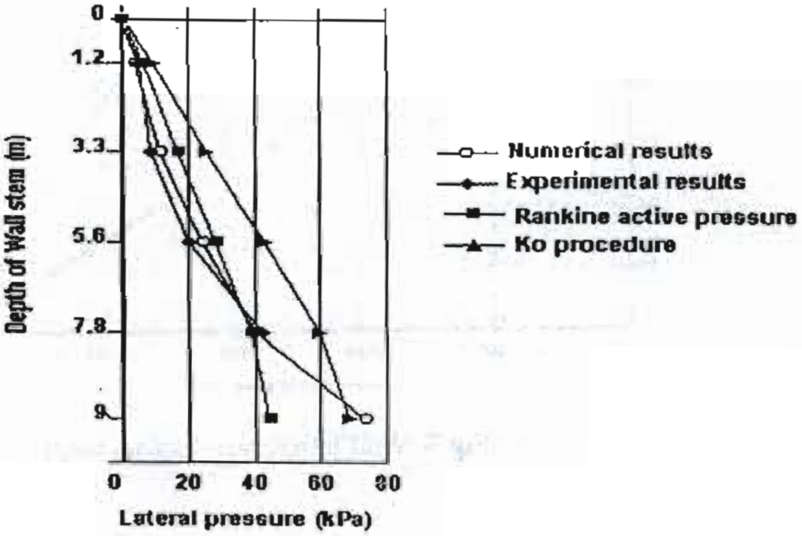


Fig12. Lateral Pressure acting on the Wall stem.