

A PROJECTION TECHNIQUE FOR THE DESIGN OF
FIR DIGITAL FILTERS

BY

Ibrahim A. Mandour^{*}, Rashid M.El-Awady^{**},
Hamdi A. Elmikati^{**} & Ahmed A. Abou-Taleb^{**}

ABSTRACT:

A method for synthesis of Chebyshev FIR digital filters is presented. The best approximation in the chebyshev (L_{∞}) sense is obtained making use of the method of successive projections, which reduces the problem to one of finding a point in the intersection of a system of convex sets. The method is fast converging and does not require solving a set of non-linear equations as in other minimax techniques. An example is presented to illustrate the procedure and the results are compared with a recently published minimax technique.

I. INTRODUCTION:

Several techniques have been developed for the design of FIR digital filters with prescribed frequency response. Among these techniques are: Windowing, frequency sampling and minimax approximation [1], [2]. Each of these techniques has its own strength and weakness. For example; the windowing technique is tedious unless a closed form expression for the window coefficients is found. The frequency sampling technique is amenable only for filters having frequency responses that are reasonably smooth. Present minimax techniques [1], [2] such as the Remez algorithm and its modifications require the solution of a set of nonlinear equations to derive the filter frequency response. In addition, the band edges in these techniques are not specified by the designer, thus leaving the transition bands unconstrained. This leads to relatively large transition bands and sometimes undesirable spikes in these bands. The recently developed CONNIP technique [3], [4], though very efficient, is confined to filters having relatively large transition bands.

This paper presents a relatively simple digital filter design method, in which the approximation in L_{∞} norm is obtained making use of the method of successive projections.

* Department of Electrical eng., Alexandria University.

** Department of Electronic eng., el-Mansoura University.

The method takes in to account all of the frequency band, thus preventing any undesired spikes in the transition bands. In addition, the computational effort involved is reduced greatly due to simplicity of each step of iterations (projections). In the following sections, the method is described briefly and an example is solved for illustrating the technique.

II. The Method of Successive Projection:

A projection of a point x onto a set R in a normed space E is defined as a point $P \in R$ such that

$$\|x - p\| = \inf_{y \in R} \|x - y\| \quad \dots\dots(1)$$

$\|\cdot\|$ denotes the length, or "norm", of the vector joining the two points x and p . In words, the projection of a point x onto a set R is the point in R nearest to x .

Consider a family of sets $Q_\alpha \in E$, where $\alpha \in A$ and A is a set of subscripts-not necessarily countable. Let it be required to find a point x in the common intersection of these sets, i.e. a point x such that

$$x \in Q = \bigcap_{\alpha \in A} Q_\alpha \quad \dots\dots(2)$$

According to the method of successive projections [5], [6], to find x we proceed as follows:

- 1) An arbitrary starting point x^0 is chosen and a set-also arbitrary- Q_1 is selected from the given family.
- 2) The projection x^1 of x^0 onto Q_1 is determined according to the definition in (1).
- 3) Another set Q_2 is selected and the projection x^2 of x^1 onto Q_2 is determined.
- 4) The preceding scheme is continued until the process converges to a point x , satisfying (2).

It is shown in [5], [6] that the sequence x^n is strongly convergent to a point x in a finite number of steps if any one of the following conditions is satisfied:

- a) E is finite dimensional.
- b) $A = \{\alpha_1, \alpha_2, \dots, \alpha_m\}$ is finite and all Q_α are half spaces i.e.

$$Q_\alpha = \{x : (C_i, x) \leq \beta_i\}$$

Where C_i 's are given numbers and β_i 's are defined scalars.

In the next section, it is shown that the problem of digital filter design can be reduced to one of finding some common point of a system of convex sets in a finite dimensional space. Each of these sets is such that it is not difficult to project onto it a given point of the underlying space.

III. FIR digital filter synthesis:

Consider the synthesis of an FIR digital filter having a prescribed frequency response, which is defined and sectionally continuous over the interval $[0, \pi]$. It is known from the theory of digital filters that the frequency response of FIR filters can be represented as a linear combination of trigonometric functions, the exact form of which depends on the filter type [1]. The expansion coefficients are to be chosen such that the synthesized response approximates the desired one in the chebyshev sense. In other words, we seek a set of coefficients a_1, a_2, \dots, a_n which minimizes the error function E defined by:

$$E(\omega) = \text{Max}_{0 \leq \omega \leq \pi} \left| H_d(\omega) - \sum_{i=1}^N a_n \phi_n(\omega) \right| \dots\dots(3)$$

where H_d is the desired response and $\phi_1, \phi_2, \dots, \phi_n$ are the basis functions representing the response of the synthesized filter. We specify some positive number δ and seek the solution of the inequalities

$$-\delta \leq H_d(\omega) - \sum_{i=1}^N a_n \phi_n(\omega) \leq \delta, \quad 0 \leq \omega \leq \pi \dots\dots(4)$$

If the solution of (4) exists, then $\delta \geq f^* = \inf f(a)$

where $f(a) = \sum_{i=1}^N a_n \phi_n(\omega)$. If, on the other hand, a

solution to (4) does not exist, then $\delta < f^*$. In the first case δ can be reduced, and in the second it can be increased. In this way we can obtain the solution of the original problem (3) within certain specified tolerance. Thus the problem has been reduced to the solution of the auxiliary problem of solving the set of linear inequalities (4). But this problem is equivalent to finding a point of the set $Q = \bigcap_{\alpha \in A} Q_\alpha$,

where: A is the interval $[0, \pi]$,

$$Q_\alpha = \left\{ a : \pm H_d(\omega_\alpha) - \delta \leq \pm \sum_{n=1}^N a_n \phi_n(\omega_\alpha) = \pm(a, \Phi(\omega_\alpha)) \right\} \dots(5)$$

a is the vector whose components a_1, a_2, \dots, a_N ,

Φ is the vector whose components $\phi_1(\omega_\alpha), \phi_2(\omega_\alpha), \dots, \phi_N(\omega_\alpha)$,
 (a, Φ) is the scalar product.

Each of Q_α is a half space and the space of all vectors a is evidently of finite-dimension. Hence condition (a) of section II is valid. Also, if the frequency interval $[0, \pi]$ is divided into a grid of frequencies with finite but large number of frequency points, then condition (b) is valid—the family of sets Q_α becomes finite. Conditions (a) and (b) ensure the convergence of the process.

IV. The Projection Formulae:

Consider a projection step n and let the current value of the vector $a = (a_1, a_2, \dots, a_N)$ be a^n . To find a^{n+1} we apply the definition of projection given in equation (1), i.e. find the point a^{n+1} in the space b , of all vectors a , which is nearest to a^n and lies inside Q_{n+1} . This requires the minimization of the functional,

$$M = \| a^{n+1} - a^n \| + \beta (|H_d(\omega_\alpha) - (a^{n+1}, \Phi)| - \delta) \dots (6)$$

where β is a lagrangian multiplier. Setting the derivatives $\partial M / \partial a^{n+1}$ equal to zero and solving the resulting equations it follows that

$$a_i^{n+1} = a_i^n - (\delta - |E|) \left(\sum_{i=1}^N \phi_i^2(\omega_\alpha) \right)^{-1} \phi_i(\omega_\alpha) \text{sign}(E) \dots (7)$$

where $i = 1, 2, \dots, N$.

$$E = H_d(\omega_\alpha) - (a^n, \Phi) \dots \dots \dots (8)$$

and ω_α is the frequency corresponding to the set onto which projection is made. In principle, this set can be chosen at random. However, to accelerate convergence, projection is made on to the set corresponding to the frequency ω_α at which filter specifications (desired response) deviates largely from the current design at iteration step n [4]. Thus is determined from a condition of the type:

$$\left| H_d(\omega_{\alpha p}) - \sum_{i=1}^N x_i^n \phi_i(\omega_{\alpha p}) \right| = \text{Max}_{0 \leq \omega_\alpha \leq \pi} \left| H_d(\omega_\alpha) - \sum_{i=1}^N x_i^n \phi_i(\omega_\alpha) \right| \dots \dots (9)$$

The algorithm starts by assuming arbitrary values for Q^0 and Eqn.(7) is applied repeatedly until specifications are

met. eqns.(7-9) show that each step of successive projections can be realised very simply. The amount of calculations associated with each step is usually small. Within each step, the method only requires that the maximum of a numerical function over an interval be found.

V. Example and Discussion:

Fig. 1 and Fig. 2 give an example of an FIR linear phase digital filter designed through the method of successive projections. The corresponding CONRIP design [4] of the filter is shown in Fig. 3. The designed filter is chosen to have the following specifications:

Filter length	43
1st stop-band edge	0.17
1st transition band edge	0.22
2nd transition band edge	0.29
2nd stop-band edge	0.34
Maximum deviation Delta	0.028

The successive projections were found to converge in 43 iterations to a specified tolerance $\Delta = 0.028$. It is seen from curves that the CONRIP design does not yield specific band edges as those produced by the method of successive projections. Those band edges in the case of CONRIP design are:

1st Stop-band edge	0.0728
1st transition band edge	0.227
2nd transition band edge	0.29
2nd stop-band edge	0.3345

The CONRIP design does, in fact, specify regions rather than edges, within which the frequency response should lie. This explains why the pass band in the CONRIP design is wider than it is in the technique of successive projections. Fig. 4 shows how the rate of convergence speeds up when the specified tolerance Δ is increased. The increase of Δ results in a rapid convergence towards the specified tolerance with no oscillations.

VI. CONCLUSION:

Use of successive projection technique reduces the problem of minimax approximation to one of finding a common point of a system of convex sets. This makes it possible to avoid solving a set of non-linear equations in filter equation in filter coefficients as in other minimax approximation algorithms. In addition, the method takes into consideration the transition bands and thus prevents any undesirable behaviour in these bands.

L.60. Mandour, El-Awady, Elmikati, Abou-Taleb.

REFERENCES:

1. Rabiner L.R. and Gold B., "Theory and Applications of digital signal processing", Prentice-Hall, N.J., 1975.
2. R.W. Hamming, "Digital Filters", Prentice - Hall, N.J. 1977.
3. M.I. McCallig and B.J. Leon", "Constrained ripple design of FIR digital filters". IEEE, Trans. on circuits and systems, Vol. CAS - 25, No. 11, Nov. 1978.
4. M.T. McCallig and B.J. Leon, "Design of nonrecursive digital filters to meet maximum and minimum frequency response constraints" Tr-EE 75-73 Dec. 1975. School of electrical Engineering, Purdue University.
5. Bregman L.M., "Finding the common point of convex sets by the method of successive projection", Dokl. Akad. Nauk. SSSR, 162, 3, pp. 487, 1965. (In Russian).
6. L.G. Gubin, B.T. Polak, E.V. Raik: "The Method of projections for finding the common point of convex sets" Zn. vychisl. Mat. Mat. Fiz. 7, 5, 1211 - 1228, 1967. (Journal of computational math. and math. Physics. In Russian).

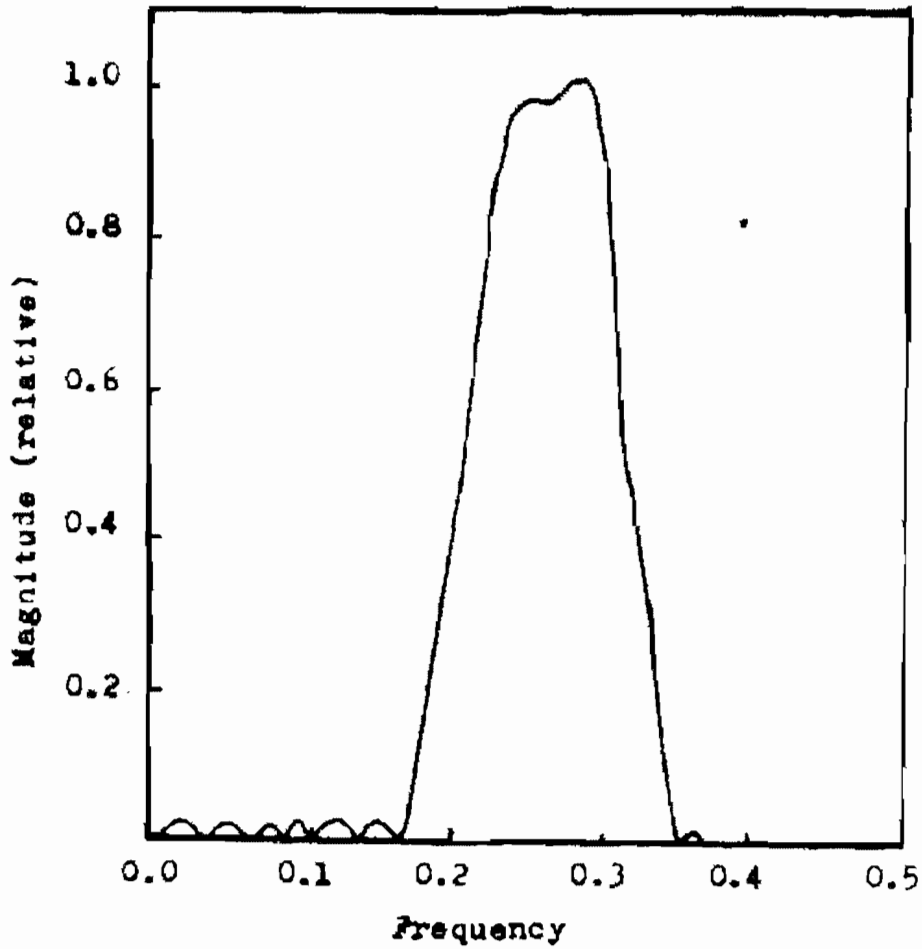


Fig.(1): Magnitude Response of a Band-pass FIR digital filter designed by successive projection Technique).

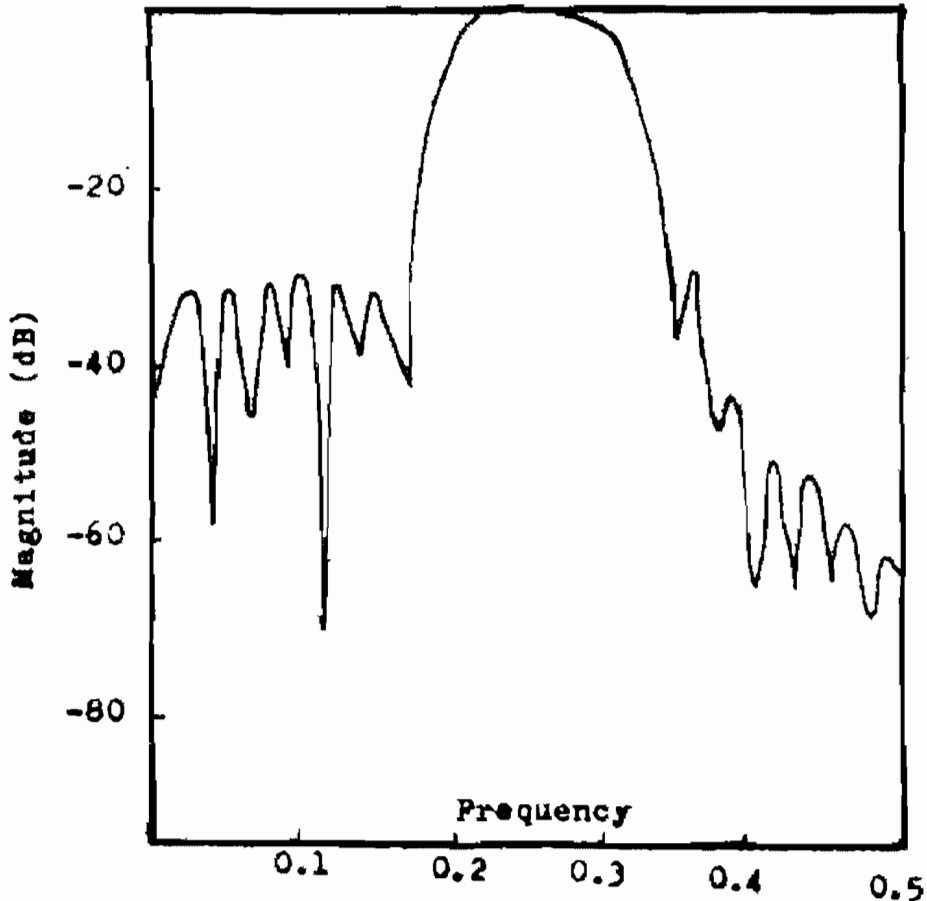


Fig.(2): Magnitude Response in dB.

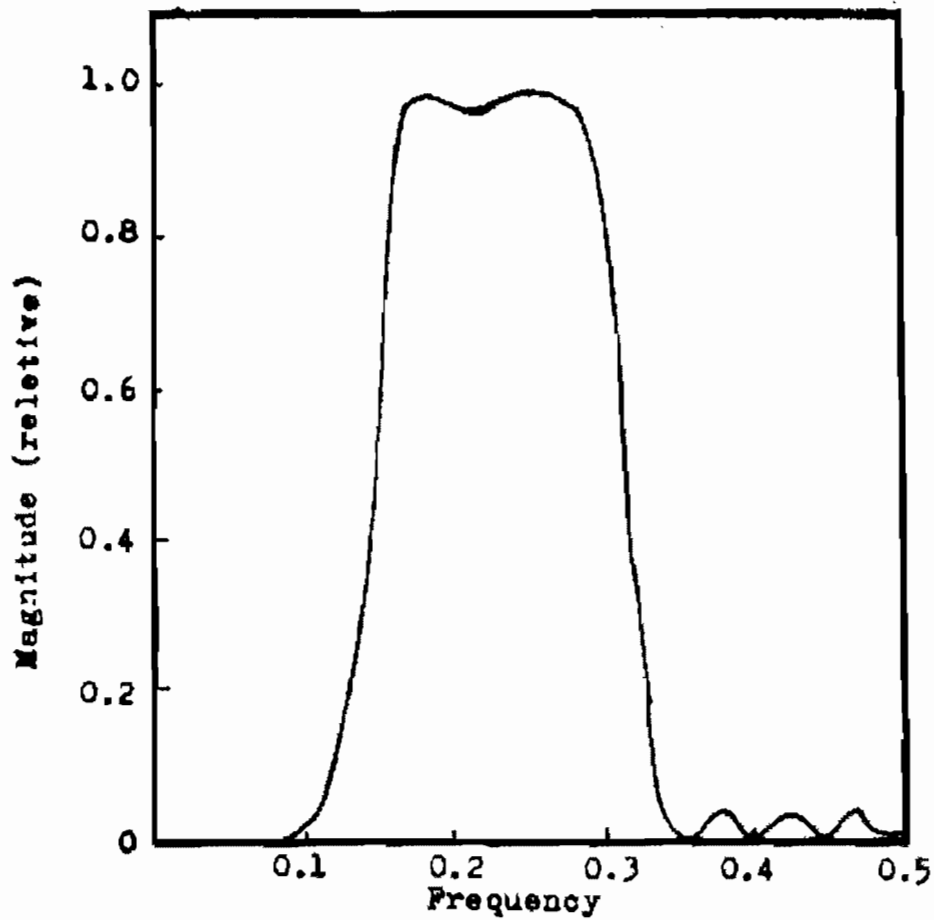


Fig.(3): Magnitude Response of Band-pass filter
Designed by CONRIP Technique.

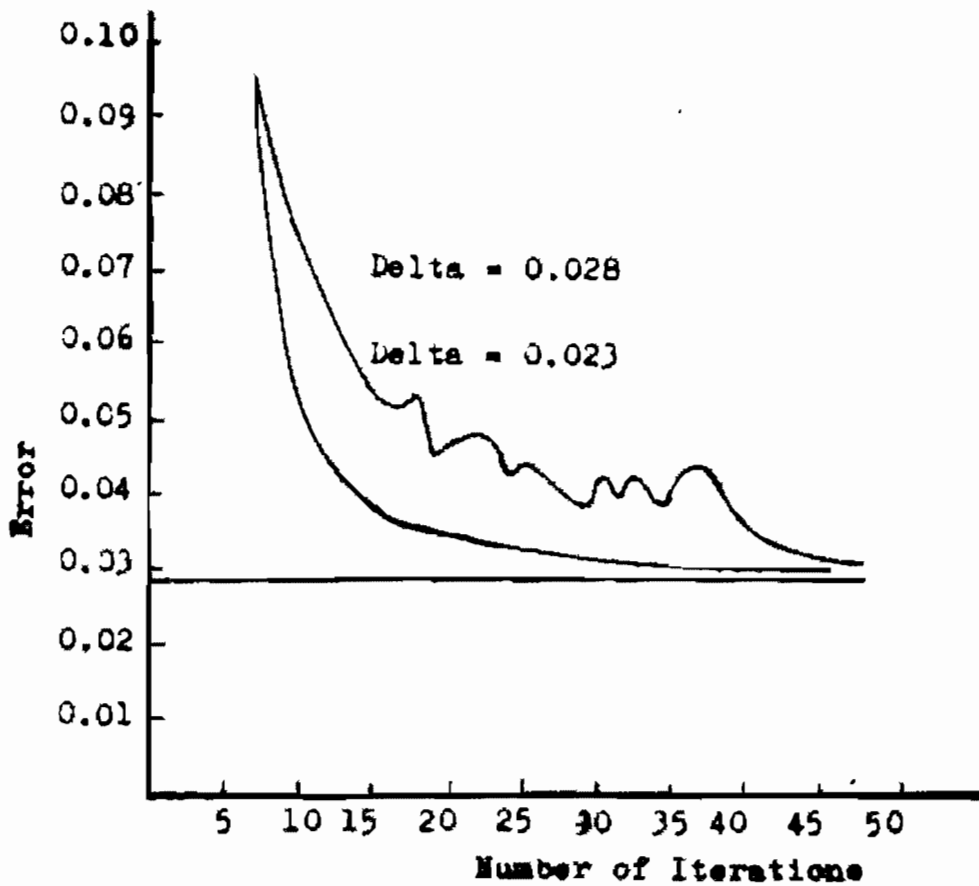


Fig.(4): Change in the maximum absolute error in the
designed response with the number of iterations.