Eng. Research Bull.
Faculty of Eng. & Tech.,
Menoufia University.
Vol. VII Part II, 1985.

PP. 1-16

PENETRATION DEPTH DISTRIBUTION FUNCTION OF BACKSCATTERED ELECTRONS IN THE ENERGY RANGE 0.02 TO 3 MeV

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ABSTRACT

The different aspects of electrons backscattered and transmitted from solid targets are calculated in terms of a simple analytical model, for normal incidence of the energy range 0.01 to 3 MeV.

A proposed analytical model for backscattered depth distribution function is given for various targets and incident energies.

An analytical formula to calculate the range of electron in solids is also given. The numerical results of these models provide a fairly good prediction in comparison with experimental results and Monte Carlo calculations.

1. INTRODUCTION

Information concerning the backscattering from solids is of great importance for its application in scaning electron microscopy, x-ray microanalysis, electron beam induced currents, electron beam lithography.

For this reason the backscattering from solid has been extensively studied but there is much interest in a theory which describes backscattering as it will be possible.

Electron backscattering can be described neither by theories based on single scattering only (1 to 4) nor on multiple scattering process only (5 to 9). These theories either describe certain limiting cases, or difficult to evalute.

The Monte Carlo simulation calculation reproduce well the experimental data AHMED (10 and 11), but they are usually computer-time-consuming.

The transport theory among the analytical treatments is well defined but it needs more approximations, for solving its equations (12). Therefore a strong interest exist in simplifying theoretical models which can be treated analytically. AHMED (13) combining the single scattering and multiple scattering give the backscattering coefficient for bulk and thin film targets as a function of energy which agrees well with experimental results.

In the following section we will give an analytical model for the transmission and backscattering coefficients. From these modeles a backscattered depth distrebution function deducied as a function of an electron incident energy and nature of elements.

All these treatments are carried out for an electron beam incident normally, of energy range 0.01 to 3 MeV and for targets of C, Al, Cu, Ag and Au.

2. ANALYTICAL STUDY OF TRANSMISSION AND BACKSCATTERING PHENOMENA

2.1 TRANSMISSION

The coeficient of transmission cannot be obtained theoretically over a wide energy range (14 to 15) and for large variation in atomic number Z.

Therefore a semi-empirical relations are preposed:

- Kanaya and Okayamma (17)

$$T = \exp - \left(\frac{\gamma' X}{1-X}\right) \text{ with } X = \frac{x}{r} \text{ and } \gamma' = 0.187\ Z^{2/3}$$

- Verdier and Arnal (18)

$$T = \exp - X^{P_t} \quad \text{with } X = \frac{x}{k.r} \qquad \dots \quad \dots \qquad (1)$$

where r is the electron range and k and P_t are parameters of target material. For a given atomic number Z for different energies, expression (1) give a "normalized" curve, which is not the case (19).

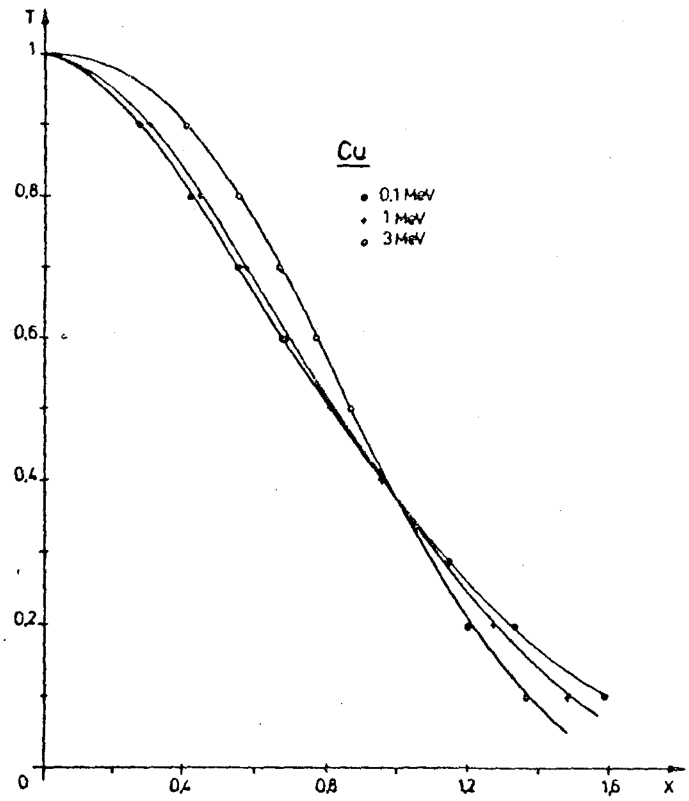

Fig.1. Normalized curve of copper at different energies.

As Fig. 1 shows, the normalization does not exist for energies greater than 1 MeV. It is necessary to have an expression for P_t which varies as a function of incident energy E_o.

The proposed form of the transmission coefficient is expressed as:

$$T = \exp - X^{P_t} \quad \text{with} \quad X = \frac{x}{x_H} \quad \dots \quad \dots \quad \dots \quad (2)$$

where

$$x_H = C\, V^n \left(\frac{V}{V^*} \right)^a$$

The parameters C, n, P_t and a are given by:

$$C = \frac{3.39 \times 10^{-2}}{\rho} \quad ; \quad n = 0.75 + 1.38\,\frac{Z}{A} + \frac{6.22}{Z^2 \rho^2}$$

$$P_t = \frac{-8.75}{\text{Log } Z}\ \frac{Z}{A} \left(\frac{V^*}{V}\right)^{0.284} \; ; \; a = 0.825\times10^{-2}\ Z^{0.38} \log(V\times10^{-2}) \dots (3)$$

where Z is the atomic number, A is the atomic weight and ρ is the density (gm.cm^3) of the studied element. The acceleration tension V of the incident electron and it's relativistic value is given in KV and the thickness x and x_H are given in microns.

The calculated results from Eq. (2) is plotted in Fig. (2) showing good agreement with the experimental values for energy range 0.05 to 3 MeV.

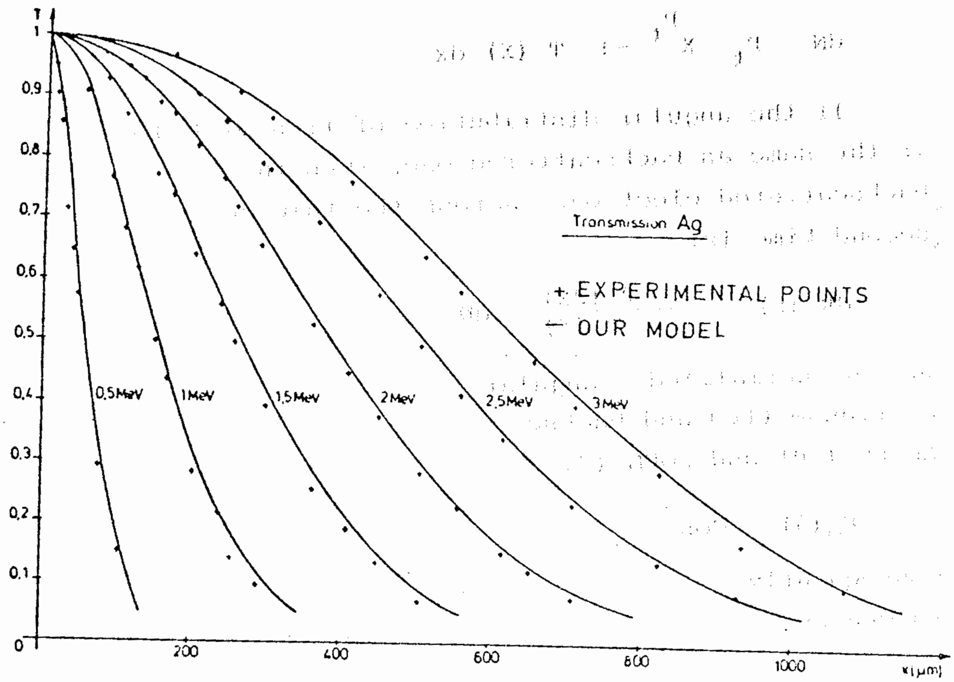

Fig.2. Transmission coefficient of silver as a function of target thickness and energy of electrons.

2.2. BACKSCATTERING

2.2.1. Backscattering Model Deduced From Transmission Expression

The backscattered electron is a transmitted electron with its direction towards the entering surface. Then we can apply the same law of transmission i.e. exp $-X^{P_t}$, the reduced depth X is the crossed thickness which is different from the trajectory of electron in the target.

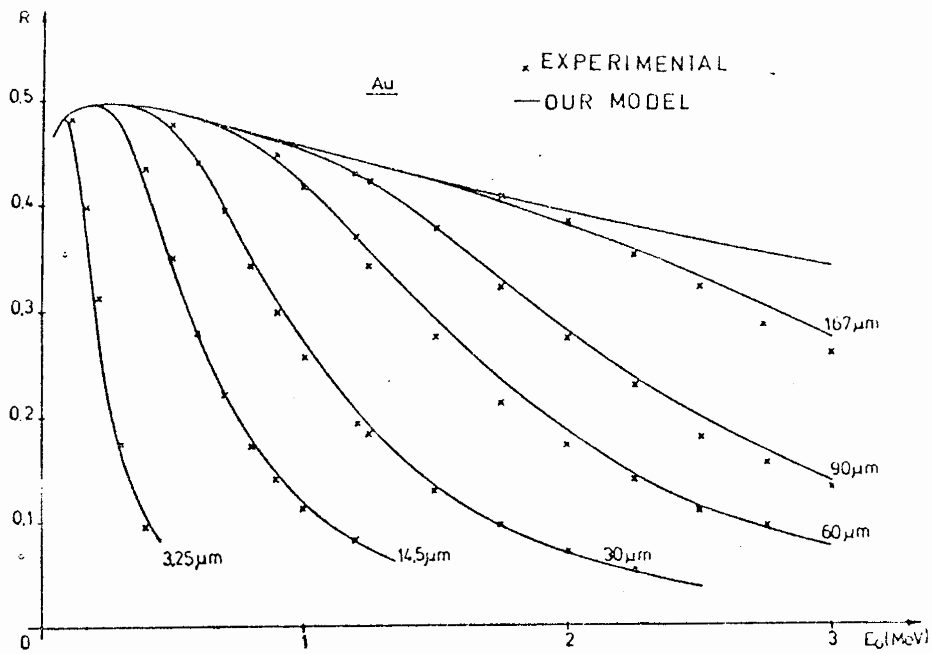

Fig. 4. Comparison of our model with our experimental results (19) for Au.

In Figs. (3) and (4) a good agreement between experimental results and the predection obtained from Equation (7) is observed in the energy range 0.01 to 3 MeV.

For bulk targets we compare the results obtained from Equation (8) for C, Al, Cu, Ag and Au with the exprimental results, (which are collected by TABATA (23)); in Figs. (5) and (6) which show a good agreement for a wide energy range up to 20 MeV.

The number of backscattered electrons at the depth X and thickness dx is given by:

$$dN = P_t \; X^{P_t - 1} \; T(X) \, dx$$

If the angular distribution of transmitted electrons is the same as backscattered one, then the fraction of backscattered electrons across the thickness dx for second time is:

$$dR(x) = \frac{T(2X)}{T(X)} \, dN$$

but the normalized angular distribution function of transmitted and backscattered electron are different, AHMED (20) and SOUM (21), which are given respectively by:

$$F_T(\theta) = \cos^2 \theta \text{ and } F_R(\theta') = \cos \theta'$$

Consequently, the backscattered electrons must cross a thickness $X_R > X$, which we proposed as:

$$\frac{X_P}{X} = \frac{\int_0^{\pi/2} \cos \theta' \, 2\pi \sin \theta' \, d\theta'}{\int_0^{\pi/2} \cos^2 \theta \, 2\pi \sin \theta \, d\theta} = 1.5$$

Then, the fraction dR at the depth X and thickness dx obey the following relation:

$$dR(x) = \frac{T(2.5X)}{T(X)} = P_t \, X^{P_t - 1} \; T(2.5X) \, dx \quad \ldots (4)$$

By a simple integration, we have the backscattering coefficient as:

$$R(X) = \frac{1}{2.5^{P_t}} \left[1 - T(2.5X)\right] \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

For bulk targets we have the following expression:

$$R = \frac{1}{2.5^{P_t}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots (6)$$

2.2.2. Comparison With Experimental Results:

Equations (5) and (6) are simple ones like that of Everhart (1) and Archard (9). As Everhart died , we shall adept our model to the experimental results. The expression of P_t is modified such that Equations (5) and (6) agree well with the experimental results. P_r (modify P_t) is given by:

$$P_r = a_1 \cdot a_2 \cdot a_3$$

with

$$a_1 = 8.7\ (Z-1)^{-\frac{1}{2}} - 0.34$$

$$a_2 = 1 + \tfrac{1}{2}\ \mathrm{Exp} - \left(\frac{V}{0.19\ Z^3}\right)^{\frac{1}{2}}$$

$$a_3 = \left[\ \frac{V^*}{V} + 0.4 \times 10^{-16}\ Z^{1.2}\ V^2\ \right]^{0.35}$$

in this expression the tension V is given in volt. Then the backscattered coefficient is given by:

$$R\ (X) = \frac{1}{2.5^{P_r}}\ [\ 1 - T\ (2.5X)\ ] \quad \ldots \quad \ldots \qquad (7)$$

$$R = \frac{1}{2.5^{P_r}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (8)$$

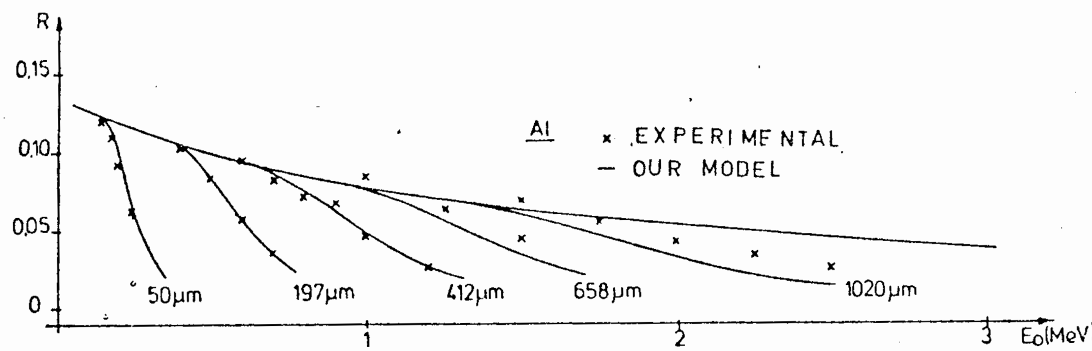

Fig. 3. Comparison of our model with our experimental results (19) for Al.

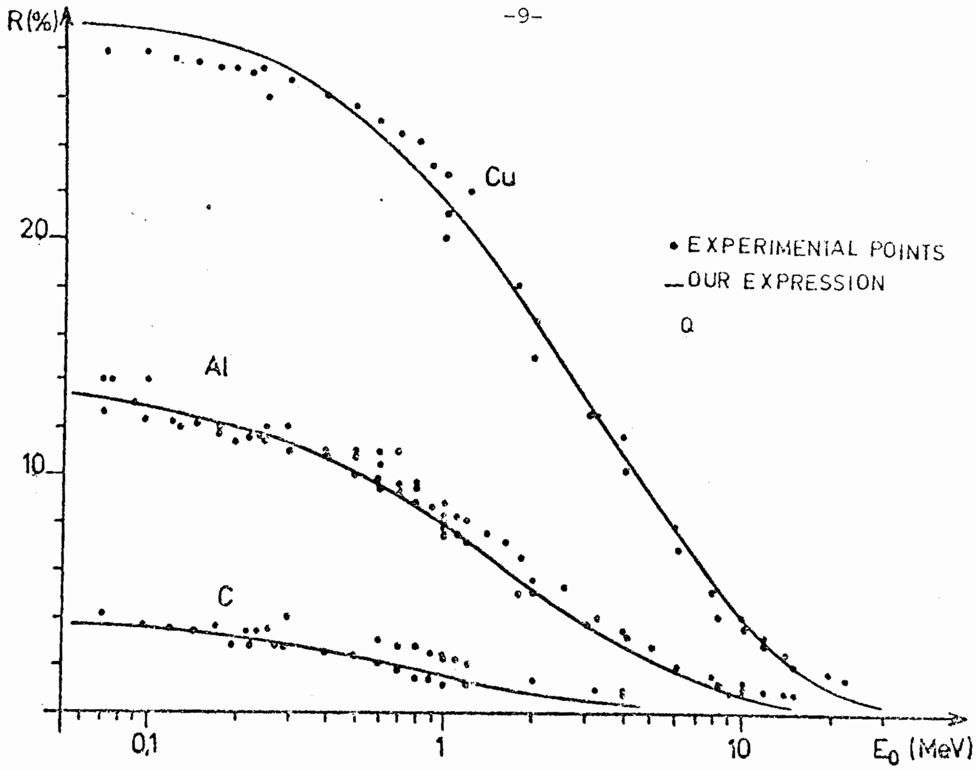

Fig. 5. Comparison of our model with experimental results (23) for C, Al and Cu.

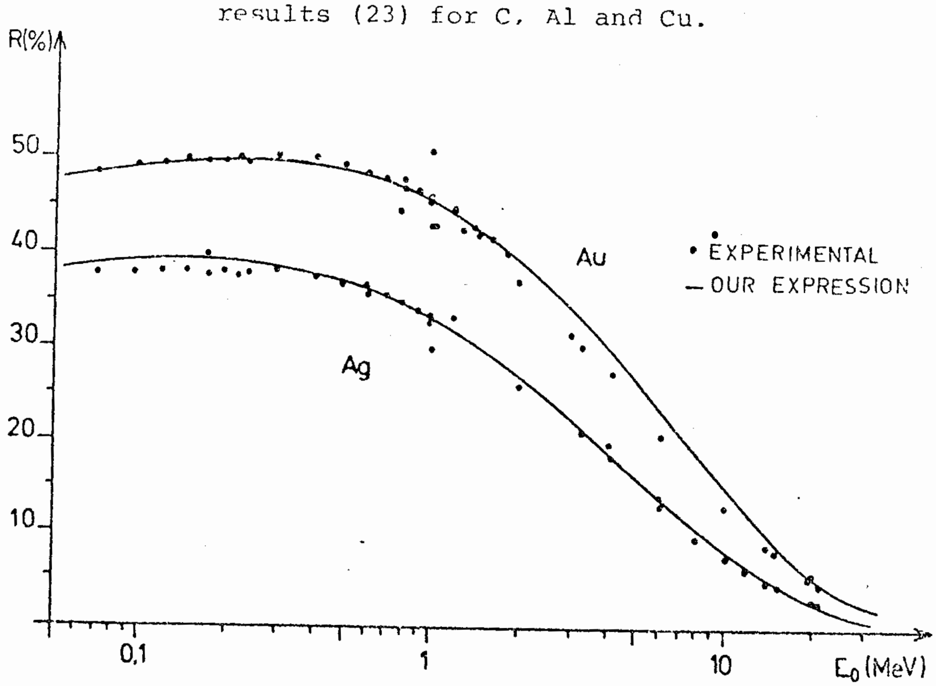

Fig. 6. Comparison of our model with experimental results (23) for Ag and Au.

3. DEPTH DISTRIBUTION OF BACKSCATTERED ELECTRONS

For each backscattered electron, it is of interest to know the depth at which it begins to return to the entry surface of the sample, Electrons thus provide information from different depths in the target.

AHMED (13) combined the single and multiple scattering in a combined diffustion model. In this model the differential backscattered coefficient is given by:

$$dR\ (y) = f_r\ (y)\ \tfrac{1}{2}\ \frac{1-2y}{1-y}\ dy \qquad \dots \quad \dots \qquad (9)$$

where

$\tfrac{1}{2}\ \frac{1-2y}{1-y} = \frac{2\ (1-\text{Cos}\ \quad)}{4}$ is the ratio of solid angle and f_r (y) is the probability function of backscattered electrons at the reduced depth y = x/r. From Equation (9) we can obtain:

$$f\ \ (y) = \frac{2\ (1-y)}{(1-2\ y)}\ \frac{dR(y)}{dy} \qquad \dots \quad \dots \quad \dots\ (10)$$

Differentiating Equation (7) we can have:

$$dR\ (x) = \frac{-\ dT\ (2.5X)}{2.5^{\ P_r}} \qquad \dots \quad \dots \quad \dots \quad \dots\ (11)$$

From Equation (1) we can obtain:

$$dT\ (2.5X) = P_t\ \ (2.5)^{P_t}\ x^{P_t-1}\ T(2.5X)\ dx\ \dots(12)$$

Subestituting Equation (12) in Equation (11), and taking the reduced depth y = x/r, then we can obtain:

$$dR\ (y) = P_t \frac{(2.5)^{P_t}}{(2.5)^{P_r}}\ \left(\frac{r}{x_H}\right)^{P_t}\ y^{P_t-1}\ T\ \left(\frac{2.5\ yr}{x_H}\right)..(13)$$

The depth backscattered distribution function is given from Equations (10) and (13) by:

$$f_r(y) = 2\ \frac{(2.5)^{P_t}}{(2.5)^{P_r}}\ \left(\frac{r}{x_H}\right)^{P_t}\ y^{P_t-1}\ \frac{(1-y)}{(1-2y)}\ T\left(\frac{2.5\ yr}{x_H}\right)\ \ldots(14)$$

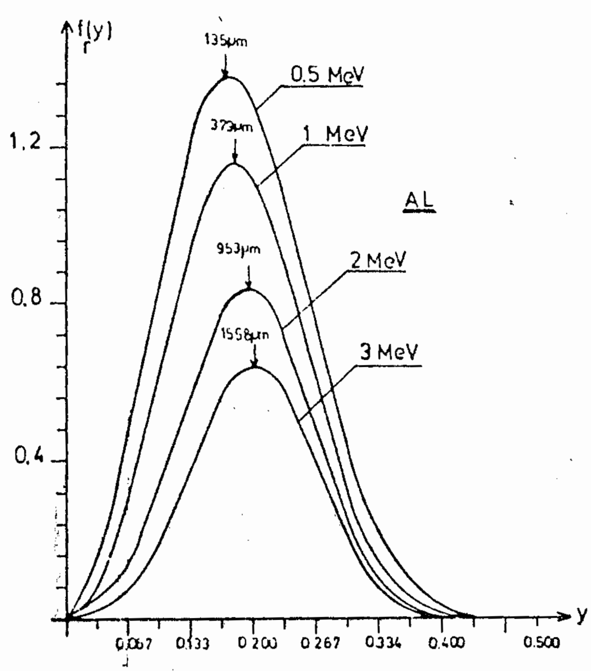

Fig.7. The depth distribution function for Al as a function of normalized depth y for different energies.

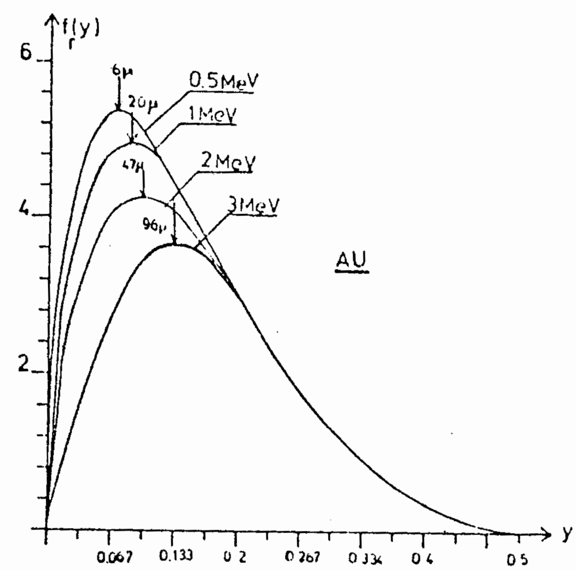

Fig.8. The depth distribution function of Au as a function of the normalized depth y for different energies.

Figs. (7) and (8) show the distribution function $f_r(y)$ of backscattered electrons for a semi-infinite Al and Au targets and for incident electron energies o.5 MeV, 2 MeV and 3 MeV calculated from relation (14). In this calculation we used the range of electrons in matter given by (22) as:

$$r\ (E_o) = qC\ E_o^{nn}\ (1 + \gamma E_o)^{-0.825} \quad \dots \quad \dots \quad \dots \quad (15)$$

with

$$\gamma = 0.9785 \times 10^{-3}$$

The constants C and n are characteristic of the target - element in question and the coefficient q results from the "normalization" of transmission curves. The energy is expressed in KeV and the thickness in μm. This relation faithfully represents the experimental results of electrons in the energy range from 0.01 to 3 MeV and for many elements from Aluminium to Platinum.

3.1. Examination of $f_r(x)$ Results:

Our calculated results obtained from Equation (14) are compared with, AHMED (23) Monte Carlo calculation for Al and Au at 1 MeV in Figs. (9) (a) and (b), which are in a good agreement.

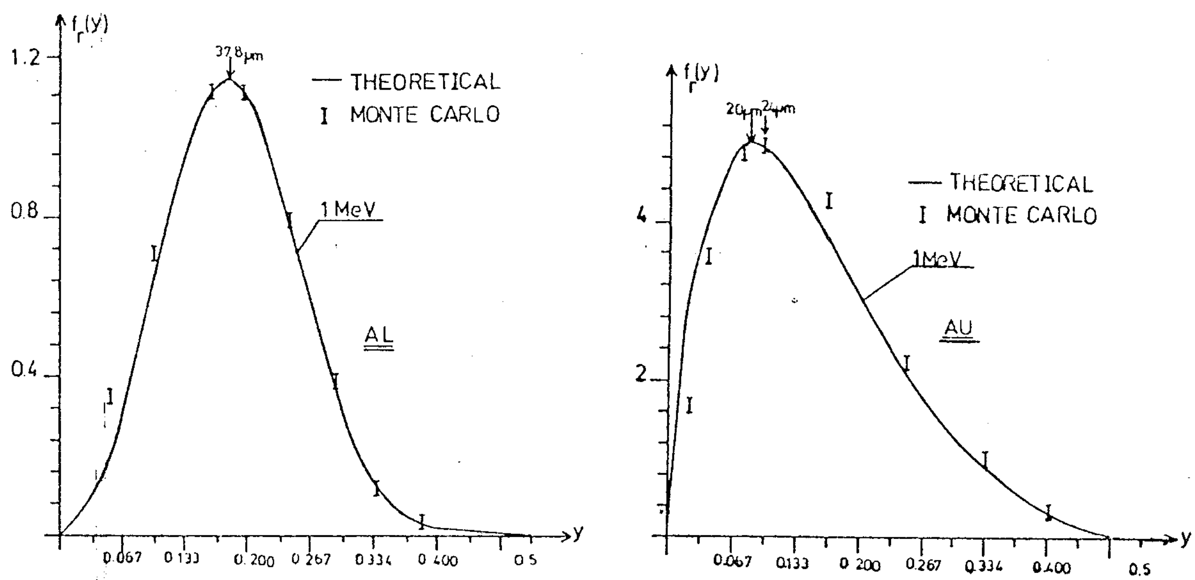

Fig.9. Comparison of our model and our Monte Carlo calculation results obtained for normalized depth distribution functions.

Also we compare the calculated results of Equation (14) with that of Murata (24) at 20 KeV for Al and Au, in Fig. (10), we find a small difference at the decaying part of the curves.

Examination of $f_r(x)$ results leads to the following coments. First, these curves show that, for a given energy, the incident electron cannot be backscattered from beyond a certain depth, hence the notation of thin and bulk targets. The thickness x_r can thus be identified with the half-maximum range r/2 of the electrons, this confirms the suggestion put forward by certain authors that backscattering occurs up to the half range.

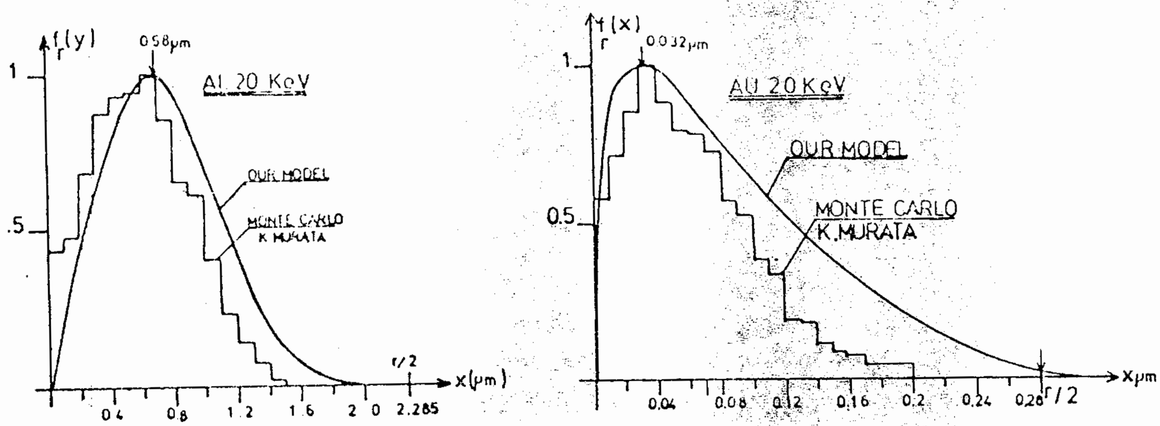

Fig.10. Comparison of our normalized depth distribution function with Monte Carlo calculation of MURATA(24).

From Fig. (10) the normalized depth distribution backscattered function $f_r(x)$ is equal to zero for Al at x_r = 2.28 μm and for Au = 0.32 μm which is in accordance with r/2 values.

Secondly, the characteristics of this distribution function depend on the nature of the target and the energy of incident electrons.

The most probable depth of backscattered electron x_p and the average depth x_{av} in μm for different atomic numbers Z and different energies is given in Table (1).

From this table (1) for the same energy x_p and x_{av} become smaller as Z increase. Thus the portation of single scattering in case of Al is stronger than multiple scattering at the same energy, thus backscattered appear near the surface for heavy elements and more deeply for light elements, (approximately x/8 for Au and x/5 for Al).

Table (1) x_{av} and x_p for different elements and different energies.

Energy / Element	0.5 MeV		1 MeV		2 MeV		3 MeV	
	x_p	x_{av}	x_p	x_{av}	x_p	x_{av}	x_p	x_{av}
Al	135	145.4	378.5	385	945.5	953	1553	1558
Cu	30	37.2	80.5	97.1	202.3	236.5	394	458
Ag	18	28.2	52.5	73	142.8	176.5	250	289.5
Au	6	12.5	20	31.6	47	75.5	96	123

The integral $\int_0^x f_r(x)\,dx$ give the area under the curve of $f_r(x)$, which decreases with increasing the incident energy, this area is proportional to the bac backscattering coefficient R, which decreases also with increasing the incident energy in practice.

This area also increases with the increasing of Z, which is also confirmed by the experimental results.

4. CONCLUSION

In this paper, analytical diffusion models for transmission coefficient and backscattered coefficient as a function of atomic number Z, thickness of targets and the energy of incident electrons are given, which are in agreement with the experimental results. From these modeles, we derived an expression for the backscattered depth distribution function, which is examiend for different elements and different energies, all these calculated results are in agreement with Monte Carlo calculation for different energies and elements. The most probable depth and average depth are given.

The experimental and analytical data presented provide evidence that the proposed simple analytical models are useful in the description of scattering processes in targets.

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