



Solve the Following Questions

(Question Number-1) :(20 Marks)

(A) Let f be a scalar field and \vec{F} be a vector field. Check the appropriate box (Vector, Scalar, or Nonsense) for each quantity.

	Quantity	Vector	Scalar	Nonsense		Quantity	Vector	Scalar	Nonsense
1	$\nabla \cdot (\nabla f)$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	5	$\text{curl}(\nabla f)$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
2	$\text{grad}(\text{div} f)$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	6	$\nabla \cdot (\nabla \times \vec{F})$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
3	$\text{div}(\text{grad} \vec{F})$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	7	$\text{div}(\text{curl} f)$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
4	$\text{div}(\text{div} \vec{F})$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	8	$\text{curl}(\text{curl} \vec{F})$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

(B) Prove that $\iint_S \vec{r} \cdot \vec{n} ds = 3$; S is the surface of the cube bounded by $x=0, x=1, y=0, y=1, z=0, z=1$.

(C) Show that the area bounded by a simple closed curve C is given by $\frac{1}{2} \oint_C x dy - y dx$.

(Question Number-2) :(20 Marks)

(A) If $\vec{U}(x, y, z) = (2x^2z)\vec{i} - (xy^2z)\vec{j} + 3yz^2\vec{k}$. State whether: $\boxed{1}$ \vec{U} is irrotational or not?

$\boxed{2}$ \vec{U} is solenoidal or not?

$\boxed{3}$ $\text{div} \vec{U}$ is harmonic function or not?

(B) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$. $\boxed{1}$ Find $\text{grad} \phi$ if $\phi = \ln r$, $\boxed{2}$ Find $\nabla \phi$ if $\phi = \frac{1}{r}$

(Question Number-3) :(20 Marks)

(A) Verify Green's theorem in the plane for $\oint_C (x^2y + y)dx + y^2dy$ where C is the closed curve between the two curves $y = x, y = x^2$.

(B) Verify Stokes' theorem for $\vec{F} = (yz)\vec{i} + (xz)\vec{j} + xy\vec{k}$; S is the surface of the cube $x=0, y=0, z=0, x=1, y=1, z=1$ above $y-z$ plane.

(Question Number-4) :(20 Marks)

(A) Prove that the area of a parallelogram with sides \vec{A} and \vec{B} is $|\vec{A} \times \vec{B}|$.

$$\begin{aligned} \text{Max } z &= 4x_1 + 5x_2 \\ \text{s.t. } 5x_1 + 4x_2 &\leq 200 \\ 3x_1 + 6x_2 &= 180 \\ 8x_1 + 5x_2 &\geq 160 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(B) Solve the following problem by the simplex method:

(Question Number-5) :(20 Marks)

(A) Evaluate the following integrals $\boxed{1} \int_0^{\infty} y^{\frac{1}{2}} e^{-y^3} dy$ $\boxed{2} \int_0^2 \frac{x^2}{\sqrt{2-x}} dy$

(B) Prove that $\beta(m, n) = 2 \int_0^{\frac{\pi}{2}} (\sin \theta)^{2m-1} (\cos \theta)^{2n-1} d\theta$, and evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\tan \theta} d\theta$

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This exam contributes " by measuring in achieving Programme Academic Standards according to NARS

Question Number	Q1-A	Q1-B,C	Q2, Q4-A	Q3	Q4-B	Q5
	a-1-1, a-1-2, a-1-3	a-8-1	a-1-3	b-3-1	b-7-1	c-1-1
Skills	Knowledge & Understanding Skills			Intellectual Skills		Professional Skills