جامعة المنوفيـــــــــــــة
كلية الهندسة بشبين الكوم
قسم العلوم الاساسية الهندسية
امتحان دبلومة] مستوى -500]

اسم المادة : مقدمة فى التحليل المركب
النهاية العظمى : مائة درجة

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الزمن : 3 ساعات

Q1 A) prove that a necessary conditions that $W = f(z) = u(x,y) + i\,v(x,y)$ be analytic in region R is that the Cauchy- Rieman equations are satisfied in R.

B) - Show that $\frac{d}{dZ}\left(Z^2\overline{Z}\right)$ does not exist any where , i.e. $f(z) = Z^2\overline{Z}$ is non-analytic any where.

C) Prove that the function $U = 2x(1-y)$ is harmonic . then find a function **v** such that $f(z) = u + i\,v$ is analytic, then express $f(z)$ in terms of Z.

D) Find the orthogonal trajectories of the following family of curve

$$x^3 y - x y^3 = \alpha$$

Q2 A) Verify Green's theorem in the plane for

$$\oint_c \left(2xy - x^2\right)dx + (x+y^2)dy$$

where c is the closed curve of the region bounded by $y = x$ and $y^2 = x$.

B) Evaluate each of the following integrals $I = \frac{1}{2\pi i}\oint_C \frac{e^Z}{Z-2}dZ$ if C is: -

i) the circle $|Z| = 3$ ii) the circle $|Z| = 1$.

Q3 A) Compute the following integral

$$\oint_c \frac{2Z}{(Z-1)^2(Z+3)}dZ$$ where

i] C: $|Z| = 2$

ii] C: $|Z| = 4$

B) Prove that

$$\int_{-\infty}^{\infty} \frac{dx}{\left(x^2 + a^2\right)^3} = \frac{3\pi}{8a^5}$$

With my best wishes

Dr. osama N. saleh