Menofia University Faculty of Engineering Shebien El-kom Basic Engineering Sci. Department. Academic Year : 2017-2018 Date : 28 / 05 / 2018	Minoufia University	Subject : Introduction to Complex Analysis Code: BES 502 Time Allowed: 3 hours Year : Master Total Marks: 100 Marks

Answer all the following questions: [100 Marks]

Q.1 [20]

(A) Prove the identities:

i) $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$

ii) $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

iii) $\sin^3\theta = \dfrac{3}{4}\sin\theta - \dfrac{1}{4}\sin 3\theta$

iv) $\cos^4\theta = \dfrac{1}{8}\cos 4\theta + \dfrac{1}{2}\cos 2\theta + \dfrac{3}{8}$

(B) Find the roots 0f $\sqrt[4]{-i}$. And find the value of $(1+i)^{\frac{2}{3}}$.

Q.2 [20]

(A) Define the following:

i) Analytic function.

ii) Cauchy-Riemann equations.

iii) Harmonic functions

(B) Prove that a necessary condition that $w = f(z) = u(x,y) + iv(x,y)$ is analytic in a region R is that the Cauchy-Riemann equations are satisfied in R.

Q.3 [20]

(A) Prove that the function $u = e^{-x}(x\sin y - y\cos y)$ is harmonic, find v such that $f(z) = u + iv$ is analytic and find $f(z)$.

(B) Define the singular points? And state the various types of singularities?

Q.4 [20]

(A) Prove that $\dfrac{d}{dz}\left(z^2\bar{z}\right)$ do not exist anywhere. (i.e. the function $f(z) = z^2\bar{z}$ is non-analytic)

(B) Find the orthogonal trajectories of the following families of curves

i) $x^3y - xy^3 = \alpha$

ii) $e^{-x}\cos y + xy = \alpha$

Q.5	**(A)** Verify Green theorem in the plane for $\oint_c (2xy - x^2)dx + (x + y^2)dy$ where C is the closed curve of the region bounded by $y = x^2$ and $y^2 = x$ **(B)** Evaluate each of the following integrals $I = \frac{1}{2\pi i}\oint_c \frac{e^z}{z-2}dz$ if **i)** The circle $\lvert z \rvert = 3$ **ii)** The circle $\lvert z \rvert = 1$

Good Luck

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