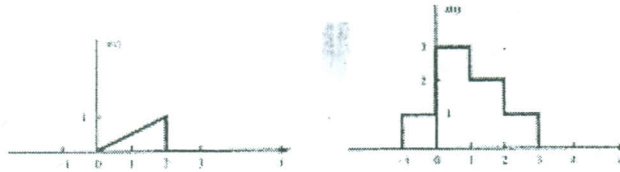




**Attempt all questions. Assume any missed data. Full mark is 100**

**Q.1.a) Express the signals shown in terms of unit step functions [5 Marks]**



**Q.1.b) Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental period. [5 Marks]**

- i.  $x(t) = e^{j\pi t} + \cos(2t)$
- ii.  $x(n) = \cos(\frac{n}{2}) \cos(\frac{\pi n}{4})$

**Q.1.c) Evaluate each of the following integrals [5 Marks]**

- i.  $\int_0^{2\pi} t \sin \frac{t}{2} \delta(\pi - t) dt$
- ii.  $\int_{-\infty}^t \cos(\tau) \delta(\tau) d\tau$

**Q.1.d) A discrete-time system has an impulse response  $h(n)$  given by:**

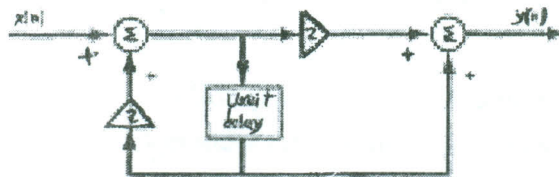
$$h(n) = 0.5\delta(n) + \delta(n - 1) + 0.5\delta(n - 2)$$

- i. Plot  $h(n)$  versus  $n$ . Is the filter causal? Why?
- ii. Is the difference equation recursive or non-recursive?
- iii. Is the filter specified by  $h(n)$  BIBO stable? Why?
- iv. Is the filter FIR? Why?

**[5 Marks]**

**Q.2.a) An LTI system has an impulse response given by  $e^{-3t}u(t)$ . Determine the output of the system for an input,  $x(t)$ , given by  $u(t - 1)$ . [5 Marks]**

**Q.2.b) Write the input-output equation for the system shown in figure [5 Marks]**



**Q.2.c) Find the inverse Laplace transform of  $X(s)$  given by**

$$X(s) = \frac{2 + 2se^{-2s} + 4e^{-4s}}{s^2 + 4s + 3} \quad -3 < \text{Re}(s) < -1 \quad \text{[5 Marks]}$$

**Q.2.d) The step response of a continuous LTI system is give by  $S(t) = 2e^{-3t}u(t)$ . Find the output of the system when the input is given by  $x(t) = e^{-t}u(t)$ . [5 Marks]**

**Q.3.a)** Consider an LTI system described by the differential equation

$$y''(t) + 5y'(t) + 6y(t) = x(t), \quad y(0) = 2, \quad y'(0) = 1$$

- Find the system function. Locate poles and zeros in the s-plane.
- Find the impulse response of the system.
- Find the output of the system if  $x(t) = u(t)$ .
- What are the zero-input response and the zero-state response? **[10 Marks]**

**Q.3.b)** Find the inverse Z-transform of each of the following functions using power series expansion: **[10 Marks]**

i.  $X(z) = \frac{z}{(z-1)(z-2)} \quad 1 < |z| < 2$

ii.  $X(z) = \log\left(\frac{1}{1-2z^{-1}}\right) \quad |z| > 2$

**Q.3.c)** Consider a system described by

$$y(n) - 5y(n-1) + 6y(n-2) = x(n), \quad y(-1) = 3, \quad y(-2) = 2, \quad x(n) = u(n)$$

- Find the system function and locate its poles and zeros in the complex plane.
- Determine the output of the system.
- Express the output  $y(n)$  as a sum of two components; the zero-state response and the zero-input response. **[10 Marks]**

**Q.4.a)** Consider a periodic square wave  $x(t)$  given by:

$$x(t) = \begin{cases} 10 & 0 \leq t \leq 1 \\ 0 & 1 \leq t \leq 2 \end{cases}, \quad x(t) = x(t+2)$$

- Sketch  $x(t)$ . State the conditions required for the convergence of Fourier series.
- Find the complex exponential Fourier series of  $x(t)$ .
- Use the result of (ii) to get the trigonometric Fourier series of  $x(t)$ .
- If  $x(t)$  is applied as an input to a high-pass filter with frequency response

$$H(\omega) = \begin{cases} 1 & |\omega| \geq 4\pi \\ 0 & |\omega| < 4\pi \end{cases}$$

Find the output of the filter.

**[15 Marks]**

**Q.4.b)** An ideal phase shifter is represented by the following equation

$$H(\omega) = \begin{cases} e^{-j\frac{\pi}{2}} & \omega > 0 \\ e^{j\frac{\pi}{2}} & \omega < 0 \end{cases}$$

- Find the impulse response  $h(t)$  of this phase shifter.
- Find the output  $y(t)$  when the input is  $x(t) = \cos \omega_0 t$  **[5 Marks]**

**Q.4.c)** Sketch the Bode plot for the following frequency response

$$H(\omega) = \frac{100(1 + j\omega)}{(10 + j\omega/10)(100 - j\omega/10)} \quad \text{[10 Marks]}$$

*My best wishes to all of you!*

*Assis. Prof. Hossam El-Din Moustafa*