Faculty of Engineering, | 2nd year Eng. Students | Second Semester 2012/2013
Math. & Eng. Physics Dept. | Mechanical Power | Date: Saturday 25/5/2013
| Engineering Dept. | Time allowed: 3 hours
| Numerical Methods & | Code number: BAS 4221
| Statistics |

Answer the following questions (Full mark: 110)

1. (a) **[11 marks]** By the least square technique, derive the normal equations to fit the curve $y = ax + b$ to the data $(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$. Use this technique to fit the curve $y = \ln(ax + b)$ to the readings:

x	1	3	6	7	8	10
y	2.154	2.661	3.129	3.246	3.351	3.533

(b) **[11 marks]** Derive the Newton forward divided-difference formula and then use it to find $f(2)$ from the above table of readings. Use a suitable method to find $f(9)$.

(c) **[11 marks]** Use a suitable iterative method to solve the linear system of equations

$$
\begin{aligned}
10x_1 &- 2x_2 &+ 2x_3 &+ x_4 &= -11 \\
4x_1 &+ 10x_2 &- 2x_3 &+ 2x_4 &= -24 \\
2x_1 &- 2x_2 &+ 10x_3 &- 3x_4 &= 21 \\
2x_1 &- x_2 &+ x_3 &- 10x_4 &= -40
\end{aligned}
$$

correct to one decimal place, starting from $(x_1, x_2, x_3, x_4) = (-2, 2, 4, 3)$.

2. (a) **[11 marks]** Derive the convergence condition of the simple iteration method to solve the equation $f(x) = 0$. Hence use this method to approximate the smallest positive root of the equation

$$x^3 + 4x^2 - 25 = 0,$$

correct to two decimal places.

(b) **[11 marks]** Given the initial value problem

$$4y' = \sqrt{x + y}, \quad x_0 = 0.4, \quad y_0 = 0.41.$$

(i) Evaluate $y(0.8)$ by Runge-Kutta approximation of order 4,

(ii) Find $y(0.2)$ by the use of Euler method.

3. (a) [10 marks] We are given three boxes with the same items Box I : has 10 items و
which 4 are defective, **Box II** : has 6 items of which one is defective and **Box III** : has ٤
items of which **3** defective, three Boxes have been emptied (الصناديق تم تفريغها) and
then draw an item at random . If an item is defective what is the probability that the
item from the first box.

(b) [12 marks] A pair of fajr dice is tossed . Let **X** and **Y** be the random variables on **S**
where **X** assigns the absolute difference between two number and **Y** the sum of two
numbers appears. Find the correlation coefficient between **X** and **Y**.

(c) [8 marks] Given the following probability function :

(i) $f(x) = \frac{x^2}{18}$, $-3 < x < 3$, zero elsewhere,

(ii) $f(x) = \frac{x+2}{18}$, $-2 < x < 4$, zero elsewhere,

Find $P(|X| < 1)$ and $P(X^2 < 9)$ of each probability function.

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4. (a) [10 marks] The marks of **200** student are normally distributed with mean **60** (out of
100) and standard deviation **5** marks. Find the number of student with marks **(i)** Less
than **61** mark. **(ii)** Less than **65** mark. **(iii)** Between **61** and **65** mark **(v)** Greater than **70**
mark **(iv)** Less than **57** mark.

(b) [12 marks] If X is a random variable having poisson distribution

(i) Find the value of $E(X)$ and $V(X)$.

(ii) If $P(x = 2) = P(x = 3)$, find $P(x = 5)$.

(c) [8 marks] We want to draw samples of **2** items from a box containing **10** items, **3** of
which are defective .Find $f(x)$ of the random variable **X** which assigns the number of
defective items by two methods (drawing with replacement) .

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