ON THE VIBRATION SPECTRUM ANALYSIS OF PLANAR MECHANISMS

تحليل طيف الاهتزاز للآليات المستوية

by

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الخلاصة — يعرض البحث طريقة لايجاد أطياف الاهتزاز للآليات المستوية ذات العدد الاختيارى من الأعضاء التى تتزاوج مع بعضها بواسطة أنواع مختلفة من الوصلات بما فيها الوصلات الدورانية ذات الخلوص . أما هيكل الآلية فيمكن أن يكون مستندا على عدد اختيارى من الزنبركات والمخمدات فى مواضع اختيارية . وتم صياغة معادلات الحركة لكل من هيكل وأعضاء الآلية ، وتم حلها عدديا بواسطة طريقة رانج - كوتا - جيل . وبذلك - وبمساعدة خوارزم تحويل فورير السريع - فقد أمكن ايجاد طيف الاهتزاز للهيكل فى كل من الاتجاه الأفقى والرأسى والزاوى. والنموذج الرياضى المعروض يمكن أن يعتبر أساسا لانشاء برنامج حاسب يمكن بواسطته ايجاد أطياف الاهتزاز لأى آلية مستوية لها عدد اختيارى من الوصلات ذات المواضع الاختيارية وتحتوى على خلوصات وذلك بتغذية مصفوفة الآلية مع معطياتها الى برنامج الحاسب .

ABSTRACT- The paper presents a method for the determination of the vibration spectra of planar mechanisms with arbitrary number of links, which are paired by arbitrary kinds of joints including clearanced turning joints.

The mechanism frames may be supported with an arbitrary number of springs and dashpots having arbitrary locations. The equations of motion of the mechanism frame are formulated and solved numerically together with the equations of motion of the links using Runge-Kutta-Jill method. In doing so, and with the aid of Fast Fourier Transform Algorithm, the frequency spectra of the frame in horizontal, vertical, and angular directions are obtained.

The mathematical model presented may be considered as a basis to construct a computer program which can provide the vibration spectra of any planar mechanism with arbitrary number and locations of clearanced joints just by feeding its mechanism matrix with its data into the computer program.

NOMENCLATURE

C_i	: the viscous coefficient of supporting damper i
$C_{i,j}$	: the amount of clearance in joint $J_{i,j}$
$Cn_{i,j}$, $Ct_{i,j}$, $CT_{i,j}$	: the viscous coefficient in the normal, tangential and angular directions at joint $J_{i,j}$

$e_{i,j}$, $ex_{i,j}$, $ey_{i,j}$: the distance between the centers of the pairing element at a clearance joint $J_{i,j}$ and its component in x , y directions.

Fd_i, Fs_i, Fd_{xi}, Fd_{yi}, Fs_{xi}, Fs_{yi} : the force produced by the supporting damper i, the supporting spring i and their components in x , y directions.

Fe_{xi}, Fe_{yi} : the external force in x and y directions which act at the gravitational center of link i

$Fn_{i,j}$, $Ft_{i,j}$, $Fx_{i,j}$, $Fy_{i,j}$: the normal, tangential, x-and y-components of the reaction which acts on link i by link y through joint $J_{i,j}$

$H_{i,j}$: the distance between the centers of gravity of links i,j

I_i : the moments of inertia of link i about its gravitational center.

K_i, $K_{i,j}$: the stiffness of the supporting spring i and the stiffness of the pairing surfaces at clearance joint $J_{i,j}$

Ls_i, Lfs_i : the length of spring i and its free length

Ldf_i, Lsf_i : the distance between the center of gravity of the frame and the suspension point of the supporting damper and spring

$L_{i,j}$: the length between the center of gravity of link i and the center of pairing element of joint $J_{i,j}$

M_i : the mass of link i

$r_{i,j}$: the radius of the pairing element of link i at the turning joint $J_{i,j}$

Te_i, $T_{i,j}$: the external torque acting on link i and the reaction torque acting on link i by link j through joint $J_{i,j}$

$Vn_{i,j}$, $Vt_{i,j}$: the relative normal and tangential velocity components of the two pairing elements at $J_{i,j}$

x_i, y_i, x_{pi}, y_{pi}, x_{si}, y_{si} : the horizontal and vertical coordinates of the gravitational center of link i and the fixation points of damper i and spring i

$\delta_{(i-io)}$: delta function

δ_{st_i} : static deflection of supporting spring i

INTRODUCTION

A good design of a certain machine will produce low levels of inherent vibrations. As the machine parts wear, however, foundations settle, parts deform, and subtle changes in the dynamic characteristics of the machine begin to occur. Shafts become mis-aligned, rotors become unbalanced, and clearances between mating parts increase. All of these factors are reflected in an increase in the vibration energy which, when dissipated throughout the machine, excites resonances and exerts considerable dynamic loads on bearings. This may eventually lead to ultimate deterioration or breakdown of the machine.

The majority of the puplished literature on the subject of clearance in mechanisms is mainly concerned with the effect of clearance on the reaction forces, driving torques and kinematics of mechanisms [1-9].

In 1979, Cempel [10] presented a method for detecting clearance in mechanisms by defining the'harmonic index' which depends on measurements of the root mean square of displacement, velocity, and acceleration. The presence of clearance was anticipated for certain ranges of this index, which was considered a fair estimate of the overall clearance in all kinematic pairs. There was no way for identifying the location or magnitude of individual clearances. Schraut [11] studied the vibration of elastically mounted and damped frames supporting mechanisms with rigid links and the effect of different parameters on the width and location of unstable zone. A criterion for the speed range causing instability for the frame was obtained. Tomaszewski [12] investigated a method of identifying the vibroacoustic effects in mechanisms. Crandall and Kuluets [13] investigated random vibrations of mechanisms supported on plates. To the auther's knowledge there is no puplished work on the effect of clearance on the vibration of machines and mechanism frames.

The aim of this work is to construct a mathematical model which is capable of accurately deducing the vibration spectra of wide range of planar mechanisms having arbitrary number and location of clearanced turning joints through systematical procedures. The mechanism frames are supported by arbitrary number and location of springs and dampers. The mathematical model may be helpful in the detection of either magnitude or location of clearanced joints [14].

MATHEMATICAL MODEL:

The mathematical model, in general, represents a planar mechanism having a frame supported by an arbitrary number of linear springs and dampers in arbitrary locations. The parts moving on the frame, the links constituting the machine, may be paired by either a sliding pair, a turning pair without clearance, or a turning pair with clearance.

The analysis is carried out assuming rigid links, perfectly straight sliding pairs, and the mechanism is running in a vertical plane. It is further assumed that hydrodynamic effects and Hertizian nonlinearity are negligible and that coulomb and viscous frictions only are present. The clearanced joints are simulated using the 'Impact Pair Model'presented by Dubowsky and Freudenstein [15] and Dubowsky [16,17]. The "Impact Pair Model" is advantageous over other models for its high accuracy in the simulation of clearanced connections and it has been, therefore, used by other investigators [18,19].

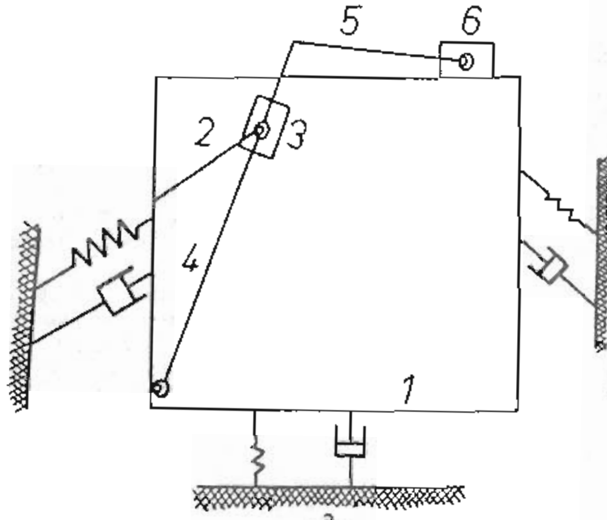
Successful application of the mathematical model to a certain mechanism requires the knowledge of some quantities and arrays. To facilitate the determination of these quantities and arrays is necessary to define the mechanism matrix, mech. (NXN) whose element mech. (i,j) represents the type of connection between links i and j and where N is the number of links in the mechanism. The element mech. (i,j) is set as O when no connection exists between links i and j. It is set as + A or - A for a sliding joint depending on whether i is larger or smaller than j. It is set as B for a turning non-clearanced joint, and as + C or -C for a turning clearanced joint depending on whether the bearing element of link i is a pin or a socket, respectively. The number of spring, NSP, and the number of dampers, NDP, supporting the frame represent elements (1,1) and (N,N), respectively.

In order to demonstrate the mechanism matrix, the quick return mechanism shown in Fig. (1-a) is considered with its matrix shown in Fig. (1-b).

The quantities and arrays needed for the application of the mathematical model are listed below.

(1) The number of links, N, the number of supporting springs, NSP, and the number of supporting dampers, NDP.

(2) The factor $\xi_{i,j}$ which is set as O for each B and as 1 for each C in the mechanism matrix.



- a -

i \ j	1	2	3	4	5	6
1	3	B	0	-C	0	-A
2	B	0	+C	0	0	0
3	0	-C	0	-A	0	0
4	+C	0	+A	0	B	0
5	0	0	0	B	0	+C
6	+A	0	0	0	-C	3

- b -

Fig.1-Quick-return motion mechanism with clearance and vibrating frame (a), and the corresponding mechanism matrix (b)

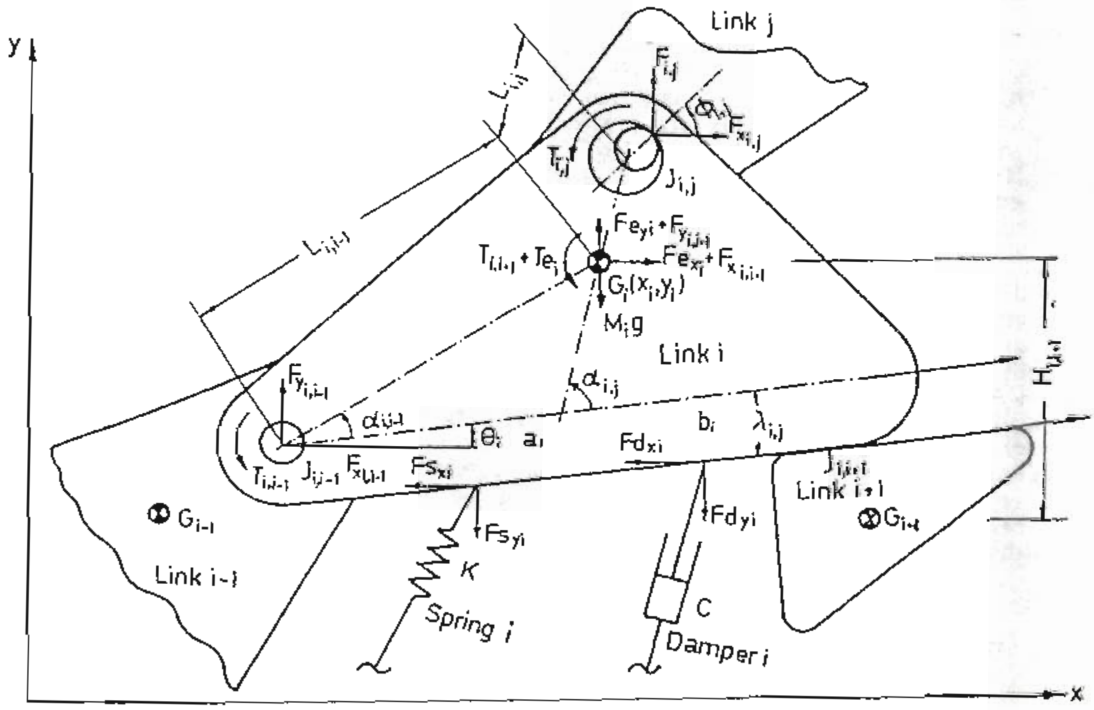


Fig.2-The free body diagram of the general case

(3) The factor $n_{i,j}$ which is set as 1 for each + C and as 2 for each - C in the mechanism matrix.

(4) The factor $s_{i,j}$ which is set as 1 for each + A and as O for each - A in the mechanism matrix.

(5) The array NT (N) which is obtained by counting the B and C elements associated with each row in the mechanism matrix.

(6) The arrays NS (N) and NL (N) which are obtained in a similar manner as the array NT (N) except working with A elements and A, B, C elements instead of B and C elements, respectively.

(7) The array NTR (N) which is obtained by deleting all elements of the mechanism matrix except those above the principal diagonal then counting the B elements associated with each row in the mechanism matrix.

(8) The arrays NTC (N), NS' (N), and NL' (N) which are obtained in a similar manner as the NTR (N) array except working, respectively, with C elements, A elements, and A, B and C elements instead of B elements.

(9) The array JT (NxN) which is obtained by:

a- deleting all elements of the mechanism matrix except the B and C elements,
b- replacing each B and C by its jth, the number of its columns,
c- shifting the elements to the left, so there is no empty elements between them, and
d- setting the remaining elements equal O.

(10) The arrays JS (NxN) and JL (NxN) which are obtained in a similar manner as the JT (NxN) array except working, respectively, with A elements and A, B, and C elements instead of B and C elements.

(11) The array JTR (NxN) which is obtained by:

a- deleting all elements of the mechanism matrix except those above the principal diagonal,
b- deleting the remaining elements except the B elements,
c- replacing each B by its jth, the number of its columns,
d- shifting the elements to the left, so there is no empty elements between them, and
e- setting the remaining elements equal O.

(12) The arrays JTC (NxN), JS' (NxN), and JL' (NxN) which are obtained in a similar manner as the JTR (NxN) array except working, respectively, with C elements, A elements and A, B and C elements instead of B elements.

EQUATIONS OF MOTION:

Following the determination of all the quantities and arrays previously mentioned, for any planar mechanism, the mathematical model may be applied, as follows, to obtain the equations of motion of the system.

$$\begin{bmatrix} M_i\ddot{x}_i \\ M_i\ddot{Y}_i \\ I_i\ddot{\theta}_i \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{nl(i)} Fx_{i,jl(i,k)} + Fex_i - \delta(i-1)\left[\sum_{k=1}^{NSP} Fex_k + \sum_{k=1}^{NDP} Fdx_k\right] \\ \sum_{k=1}^{nl(i)} Fy_{i,jl(i,k)} + Fey_i - M_i\theta - \delta(i-1)\left[\sum_{k=1}^{NSP} Fey_k + \sum_{k=1}^{NOP} Fdy_k\right] \\ \sum_{k=1}^{nt(i)} Fx_{i,jt(i,k)}\left[L_{i,jt(i,k)}\sin(\theta_i+\alpha_{i,jt(i,k)}) - (-1)^{n_{i,jt(i,k)}}\xi_{i,jt(i,k)}\, r_{i,jt(i,k)} \sin\phi_{i,jt(i,k)}\right] - Fy_{i,jt(i,k)}\left[L_{i,jt(i,k)}\cos(\theta_i+\alpha_{i,jt(i,k)}) - (-1)^{n_{i,jt(i,k)}}\xi_{i,jt(i,k)}\, r_{i,jt(i,k)}\cos\phi_{i,jt(i,k)}\right] + (1-\xi_{i,jt(i,k)})T_{i,jt(i,k)} + T_{s,i} + \sum_{k=1}^{ns(i)} \delta_{i,js(i,k)} \\ \left\{Fx_{i,js(i,k)}(y_i - y_{js(i,k)}) - Fy_{i,js(i,k)}(x_i - x_{js(i,k)})\right\} + T_{i,js(i,k)} + \delta(i-1)\left\{\sum_{k=1}^{NSP} \left[-Fsx_k Lsf_k \sin(\eta_k+\theta_1) + Fsy_k Lsf_k \cos(\eta_k+\theta_1)\right] + \sum_{k=1}^{NDP}\left[-Fdx_k Ldf_k \sin(\psi_k+\theta) + Fdy_k Ldf_k \cos(\psi_k+\theta_1)\right]\right\} \end{bmatrix} \quad ...I$$

$i = 1, \ldots, N$

$$\begin{bmatrix} Fx_{i,jl'(i,k)} \\ Fy_{i,jl'(i,k)} \\ T_{i,jl'(i,k)} \end{bmatrix} = - \begin{bmatrix} Fx_{jl'(i,k),i} \\ Fy_{jl'(i,k),i} \\ T_{jl'(i,k),i} \end{bmatrix} \quad ...II$$

$i = 1, \ldots, N$

For each i $k = 1, \ldots, nl'(i)$

$$\begin{bmatrix} Fx_{i,js'(i,k)} \\ Fy_{i,js'(i,k)} \\ Ft_{i,js'(i,k)} \end{bmatrix} = \begin{bmatrix} -Fn_{i,js'(i,k)}\sin(\theta_i+\lambda_{i,js'(i,k)}) - Ft_{i,js'(i,k)}\cos(\theta_i+\lambda_{i,js'(i,k)}) \\ Fn_{i,js'(i,k)}\cos(\theta_i+\lambda_{i,js'(i,k)}) - Ft_{i,js'(i,k)}\sin(\theta_i+\lambda_{i,js'(i,k)}) \\ -\mu_{i,js'(i,k)}\,\sigma_{i,js'(i,k)}\,Fn_{i,js'(i,k)} - Ct_{i,js'(i,k)}\,yt_{i,js'(i,k)} \end{bmatrix}$$

$i = 1, \ldots, N$

For each i $k = 1, \ldots, ns'(i)$

$$\begin{bmatrix} \ddot{X}_1 + L_{i,jtr(i,k)}\ddot{\theta}_i \sin(\theta_i+\alpha_{i,jtr(i,k)}) + L_{i,jtr(i,k)}\dot{\theta}_i^2\cos(\theta_i+\alpha_{i,jtr(i,k)}) \\ \ddot{Y}_1 - L_{i,jtr(i,k)}\ddot{\theta}_i\cos(\theta_i+\alpha_{i,jtr(i,k)}) + L_{i,jtr(i,k)}\dot{\theta}_1^2\sin(\theta_i+\alpha_{i,jtr(i,k)}) \end{bmatrix}$$

$$= \begin{bmatrix} \ddot{X}_{jtr(i,k)} + L_{jtr(i,k),i}\, \ddot{\theta}_{jtr(i,k)} \sin(\theta_{jtr(i,k)} + \alpha_{jtr(i,k),i}) + L_{jtr(i,k),i}\, \dot{\theta}^2_{jtr(i,k)} \cos(\theta_{jtr(i,k)} + \alpha_{jtr(i,k),i}) \\ \ddot{Y}_{jtr(i,k)} - L_{jtr(i,k),i}\, \ddot{\theta}_{jtr(i,k)} \cos(\theta_{jtr(i,k)} + \alpha_{jtr(i,k),i}) + L_{jtr(i,k),i}\, \theta^2_{jtr(i,k)} \sin(\theta_{jtr(i,k)} + \alpha_{jtr(i,k),i}) \end{bmatrix} \quad \ldots III$$

For each i $\quad i = 1, \ldots, N$; $\; k = 1, \ldots, ntr(i)$

$$\begin{bmatrix} \ddot{\theta}_i \\ \ddot{Y}_{js'(i,k)} - \ddot{y}_i + \ddot{\theta}_i H_{js'(i,k),i} \tan(\theta_i + \lambda_{i,js'(i,k)}) \sec(\theta_i + \lambda_{i,js'(i,k)}) + H_{js'(i,k),i}\, \dot{\theta}^2_i \\ \sec^3(\theta_i + \lambda_{i,js'(i,k)}) + H_{js'(l,k),i}\, \dot{\theta}^2_i \tan^2(\theta_i + \lambda_{i,js'(i,k)}) \sec(\theta_i + \lambda_{i,js'(i,k)}) \end{bmatrix} \quad \ldots IV$$

$$= \begin{bmatrix} \ddot{\theta}_{js'(i,k)} \\ (\ddot{X}_{js'(i,k)} - x_i) \tan(\theta_{js'(i,k)} + \lambda_{js'(i,k),i}) + 2(x_{js'(i,k)} - x_i)\theta_{js'(i,k)} \sec^2(\theta_{js'(i,k)} \\ + \lambda_{js(i,k),j}) + \theta_{js'(i,k)} (x_{js'(i,k)} - x_i) \sec^2(\theta_{js'(i,k)} + \lambda_{js'(i,k),i}) + 2(x_{js'(i,k)} - x_i) \\ \dot{\theta}^2_{js}(i,k) \sec^2(\theta_{js(i,k)} + \lambda_{js'(i,k),i}) \tan(\theta_{js'(i,k)} + \lambda_{js'(i,k),i}) \end{bmatrix}$$

For each i $\quad i = 1, \ldots, N$; $\; k = 1, \ldots, ns'(i)$

$$\begin{bmatrix} Fx_{i,jtc(i,k)} \\ Fy_{i,jtc(i,k)} \end{bmatrix} = c \begin{bmatrix} Fn_{i,jtc(i,k)} \cos\phi_{i,jtc(i,k)} - Ft_{i,jtc(i,k)} \sin\phi_{i,jtc(i,k)} \\ Fn_{i,jtc(i,k)} \sin\phi_{i,jtc(i,k)} + Ft_{i,jtc(i,k)} \cos\phi_{i,jtc(i,k)} \end{bmatrix} \quad \ldots V$$

where:

$c = 1$ For $e_{i,jtc(i,k)} > c_{i,jtc(i,k)}$ & $c = 0$ for $e_{i,jtc(i,k)} < c_{i,jtc(i,k)}$

For each i $\quad i = 1, \ldots, N$; $\; k = 1, \ldots, ntc(i)$

$$\begin{bmatrix} X_i - L_{i,jtr(i,k)} \cos(\theta_i + \alpha_{i,jtr(i,k)}) \\ Y_i - L_{i,jtr(i,k)} \sin(\theta_i + \alpha_{i,jtr(i,k)}) \\ \dot{X}_i + L_{i,jtr(i,k)}\, \dot{\theta}_i \sin(\theta_i + \alpha_{i,jtr(i,k)}) \\ \dot{Y}_i - L_{i,jtr(i,k)}\, \dot{\theta}_i \cos(\theta_i + \alpha_{i,jtr(i,k)}) \\ T_{i,jtr(i,k)} \end{bmatrix}$$

$$
= \begin{bmatrix}
X_{jtr(i,k)} - L_{jtr(i,k),i} \cos(\theta_{jtr(i,k)} + \alpha_{jtr(i,k),i}) \\
Y_{jtr(i,k)} - L_{jtr(i,k),i} \sin(\theta_{jtr(i,k)} + \alpha_{jtr(i,k),i}) \\
\dot{X}_{jtr(i,k)} + L_{jtr(i,k),i}\, \dot{\theta}_{jtr(i,k)} \sin(\theta_{jtr(i,k)} + \alpha_{jtr(i,k),i}) \\
\dot{Y}_{jtr(i,k)} - L_{jtr(i,k),i}\, \dot{\theta}_{jtr(i,k)} \cos(\theta_{jtr(i,k)} + \alpha_{jtr(i,k),i}) \\
CT_{i,jtr(i,k)} (\dot{\theta}_{jtr(i,k)} - \dot{\theta}_{i})
\end{bmatrix} \quad \text{...VI}
$$

For each i $\quad i = , \ldots ,N$; $\quad k = 1, \ldots , ntr(i)$

$$
\begin{bmatrix}
\theta_i + \lambda_{i,js'(i,k)} \\
y_{js'(i,k)} - y_i + H_{js'(i,k),i} \sec(\theta_i + \lambda_{i,js'(i,k)}) \\
\dot{\theta}_i \\
\dot{y}_{js'(i,k)} - \dot{y}_i + H_{js'(i,k),i}\, \dot{\theta}_i \tan(\theta_i + \lambda_{i,js'(i,k)}) \sec(\theta_i + \lambda_{i,js'(i,k)}) \\
v_{i,js'(i,k)}
\end{bmatrix}
$$

$$
= \begin{bmatrix}
\theta_{js'(i,k)} + \lambda_{js'(i,k),i} \\
(x_{js'(i,k)} - x_i) \tan(\theta_{js'(i,k)} + \lambda_{js'(i,k),i}) \\
\theta_{js'(i,k)} \\
(\dot{x}_{js'(i,k)} - \dot{x}_i) \tan(\theta_{js'(i,k)} + \lambda_{js'(i,k),i}) + (x_{js'(i,k)} - x_i) \sec^2(\theta_{js'(i,k)} + \lambda_{js'(i,k),i})\, \dot{\theta}_{js'(i,k)} \\
\dot{x}_i \cos(\theta_i + \lambda_{i,js'(i,k)}) + \dot{y}_i \sin(\theta_i + \lambda_{i,js'(i,k)}) - \dot{x}_{js'(i,k)} \cos(\theta_{js'(i,k)} + \lambda_{js'(i,k),i}) \\
- \dot{y}_{js'(i,k)} \sin(\theta_{js'(i,k)} + \lambda_{js'(i,k),i})
\end{bmatrix} \quad \text{...VII}
$$

For each i $\quad i = 1, \ldots ,N$; $\quad k = 1, \ldots , ns(i)$

$$
\begin{bmatrix}
c_{i,jtc(i,k)} \\
ex_{i,jtc(i,k)} \\
ey_{i,jtc(i,k)} \\
e_{i,jtc(i,k)} \\
\phi_{i,jtc(i,k)} \\
\phi_{jtc(i,k),i} \\
\dot{ex}_{i,jtc(i,k)} \\
\\
\dot{ey}_{i,jtc(i,k)} \\
\\
\dot{e}_{i,jtc(i,k)} \\
\dot{\phi}_{i,jtc(i,k)} \\
Fn_{i,jtc(i,k)} \\
Ft_{i,jtc(i,k)} \\
Vn_{i,jtc(i,k)} \\
Vt_{i,jtc(i,k)}
\end{bmatrix}
=
\begin{bmatrix}
\left| r_{i,jtc(i,k)} - r_{jtc(i,k),i} \right| \\
x_{jtc(i,k)} - L_{jtc(i,k),i} \cos(\theta_{jtc(i,k)} + \alpha_{jtc(i,k),i}) - x_i + L_{i,jtc(i,k)} \cos(\theta_i + \alpha_{i,jtc(i,k)}) \\
y_{jtc(i,k)} - L_{jtc(i,k),i} \sin(\theta_{jtc(i,k)} + \alpha_{jtc(i,k),i}) - y_i + L_{i,jtc(i,k)} \sin(\theta_i + \alpha_{i,jtc(i,k)}) \\
\sqrt{ex^2_{i,jtc(i,k)} + ey^2_{i,jtc(i,k)}} \\
\tan^{-1}\left[\left(\dfrac{ey_{i,jtc(i,k)}}{ex_{i,jtc(i,k)}}\right)\right] \\
\phi_{i,jtc(i,k)} + \pi \\
\dot{x}_{jtc(i,k)} + L_{jtc(i,k),i}\, \dot{\theta}_{jtc(i,k)} \sin(\theta_{jtc(i,k)} + \alpha_{jtc(i,k),i}) - \dot{x}_i - L_{i,jtc(i.k)} \\
\dot{\theta}_i \sin)\theta_i + \alpha_{i,jtc(i,k)}) \\
y_{jtc(i,k)} - L_{jtc(i,k),i} \dot{\theta}_{jtc(i,k)} \cos(\theta_{jtc(i,k)} + \alpha_{jtc(i,k)}) - \dot{y}_i + L_{i,jtc(i,k)} \\
\dot{\theta}_i \cos(\theta_i + \alpha_{i,jtc(i,k)}) \\
\dot{ex}_{i,jtc(i,k)} \cos\phi_{i,jtc(i,k)} + \dot{ey}_{i,jtc(i,k)} \sin\phi_{i,jtc(i,k)} \\
\dot{ex}_{i,jtc(i,k)} \cos\phi_{i,jtc(i,k)} - \dot{ex}_{i,jtc(i,k)} \sin\phi_{i,jtc(i,k)}) / e_{i,jtc(i,k)} \\
k_{i,jtc(i,k)} (e_{i.jtc(i,k)} - C_{i,jtc(i,k)}) + Cn_{i,jtc(i,k)} Vn_{i,jtc(i,k)} \\
\mu_{i,jtc(i,k)} \sigma_{i,jtc(i,k)} Fn_{i,jtc(i,k)} + Ct_{i,jtc(i,k)} Vt_{i,jtc(i.k)} \\
\dot{e}_{i,jtc(i,k)} \\
-r_{jtc(i,k),i} \dot{\theta}_{jtc(i,k)} + e_{i,jtc(i,k)} \dot{\phi}_{i,jtc(i,k)} + r_{i,jtc(i,k)} \dot{\theta}_i
\end{bmatrix}
\quad .VIII
$$

For each i $i = 1, \ldots, N$

$k = 1, \ldots, ntc(i)$

$$\begin{bmatrix} L_{si} \\ \mu_i \\ Fs_i \\ Fsx_i \\ Fsy_i \end{bmatrix} = \begin{bmatrix} \sqrt{[(y_l - y_{si} - Lsf_i \sin(\eta_i + \theta_l))^2 + (x_l - x_{si} - Lsf_i \cos(\eta_i + \theta_l))^2} \\ \tan^{-1}\left(\dfrac{y_l - ys_i - Lsf_i \sin(\eta_i + \theta_l)}{x_l - xs_i - Lsf_i \cos(\eta_i + \theta_l)}\right) \\ k_i (Ls_i - Lfs_i) \\ Fs_i \cos \mu_i \\ Fsi \sin \mu_i \end{bmatrix} \quad \text{... IX}$$

$i = 1, \ldots, NSP$

$$\begin{bmatrix} \dot{x}Q_i \\ \dot{y}Q_i \\ \nu_i \\ VQ_i \\ Fd_i \\ Fd_{ix} \\ Fd_{iy} \end{bmatrix} = \begin{bmatrix} \dot{x}_l + \dot{\theta}_l Ldf_i \sin(\psi_i + \theta_l) \\ \dot{y}_l - \dot{\theta}_l Ldf_i \cos(\psi_i + \theta_l) \\ \tan^{-1}\left(\dfrac{y_l - yp_i - Ldf_i \sin(\psi_i + \theta_l)}{x_l - xp_i - Ldf_i \cos(\psi_i + \theta_l)}\right) \\ \dot{x}Q_i \cos\nu_i + \dot{y}Q_i \sin\nu_i \\ c_i VQ_i \\ Fd_i \cos\nu_i \\ Fd_i \sin\nu_i \end{bmatrix} \quad \text{... X}$$

$i = 1, \ldots, NDP$

Groups I through V represent the main structure of the mathematical model. Group I introduces the equations of motion of any link i attached to an arbitrary number of links through arbitrary type of joints subjected to forces having arbitrary magnitudes and directions. Moreover, it introduces the equations of motion of a frame supported by an arbitrary number of springs and dampers at arbitrary locations. Groups I and II combined represent the dynamic equations of plane multilink mechanism having a number of arbitrarily located clearanced joints, and supporting springs and dampers. On the other hand, groups III, IV, and V represent the constraint equations for a non-clearanced turing pair, a sliding pair, and a clearanced turing pair in a general plane multilink mechanism. The free body diagram of the general case is shown in Fig. (2). Fig. (3) shows the impact model representation of a clearanced joint. The surfaces of the pairing elements are represented by a spring and a damper. The elastic effect of the surfaces, represented by the spring, is assumed linear, according to the Hertizian theory of contact. The viscous damper represent the material damping while the friction in the joint is represented by coulomb and viscous frictions. Fig. (4) indicates the forces acting on a clearanced turning pair.

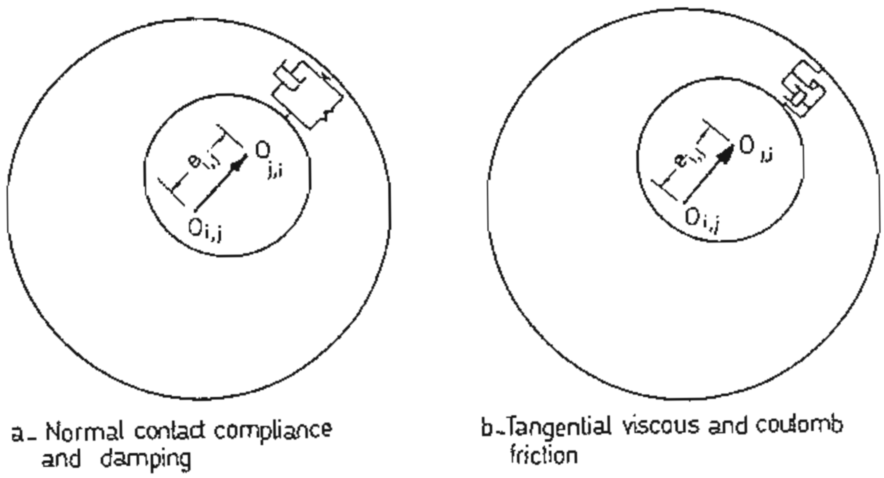


Fig.3_Model of clearance connection

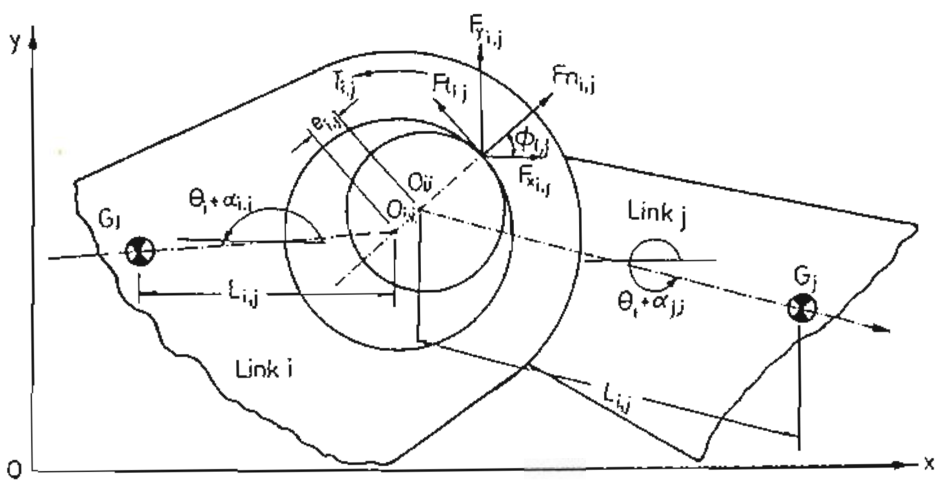


Fig.4_Forces acting on a turning pair with a clearance

The remaining groups VI through X represent the sub-structure of the mathematical model. It provides the auxiliary equations necessary to obtain the relations between displacements, velocities, and forces of the mechanism. One can use the sub-structure in addition to the main structure systematically to obtain all the necessary equations required to describe the dynamics and kinematics of any planar mechanism with clearanced joints and vibrating frame.

The number of equations represented by the main structure of the model may be found, upon substitution of group II into group I, to be equal 3N + 2J. There are three unknown quantities associated with each link i. Those three quantities are generally the acceleration $\ddot{x}_i$, $\ddot{y}_i$ and $\ddot{\Theta}_i$. For driver link 2, the above-mentioned quantities may become either (Te_2, x_2 and y_2),(Θ_2, Fex_2 and y_2) or (Θ_2, x_2 and Fey_2) if the input motion is known and the input excitation is unknown. Moreover, there are two unknown quantities associated with each joint. They are $F_{x_{i,j}}$ and $F_{y_{i,j}}$ for the turning pair $J_{i,j}$ and $T_{i,j}$ and $F_{x_{i,j}}$ (or $F_{y_{i,j}}$) for the sliding pair $J_{i,j}$. Therefore, the total number of unknown quantities is 3N + 2J which is equal to the number of equations introduced by the main structure of the mathematical model.

It must be noted that, if a mechanism has no clearance in its joints, the number of degrees of freedom will be four. This means that there are four independent variables defining the position of a mechanism, namely the position of the driver and the coordinates defining the position of the frame, for example Θ_2, x_I, y_I and Θ_I. If a mechanism has one clearance, the number of degrees of freedom is increased by two, i.e. it becomes six-degrees of freedom system. It may be concluded that $F = 4 + 2\,n_c$.

Where F is the number of degrees of freedom.

n_c is the number of clearance joints in the mechanism.

If the input excitation is known for a mechanism having F degrees of freedom, a number of F equations must be integrated around similar number of independent variables. However if the input motion is known, the number of equations to be integrated will be reduced by one.

The main (3N + 2J) equations for a certain mechanism may be expressed in a matrix form as follows:

$$[A]\{B\} = \{C\} \qquad \text{.... XI}$$

where [A] is a square matrix (3N+2J) x (3N+2J) whose elements represent the coefficients of the unknown quantities, i.e. the forces, torques and accelerations.

{B} is a column matrix (3N+2J) whose elements represent the same unknowns.
{C} is a column matrix (3N+2J) whose elements represent the free terms of the equations.

It should be noted that the elements of matrix [A] and column matrix {C} include either constants, variables, time or first derivatives with respect to time.

To obtain the required equations for integration, the following steps need be considered.

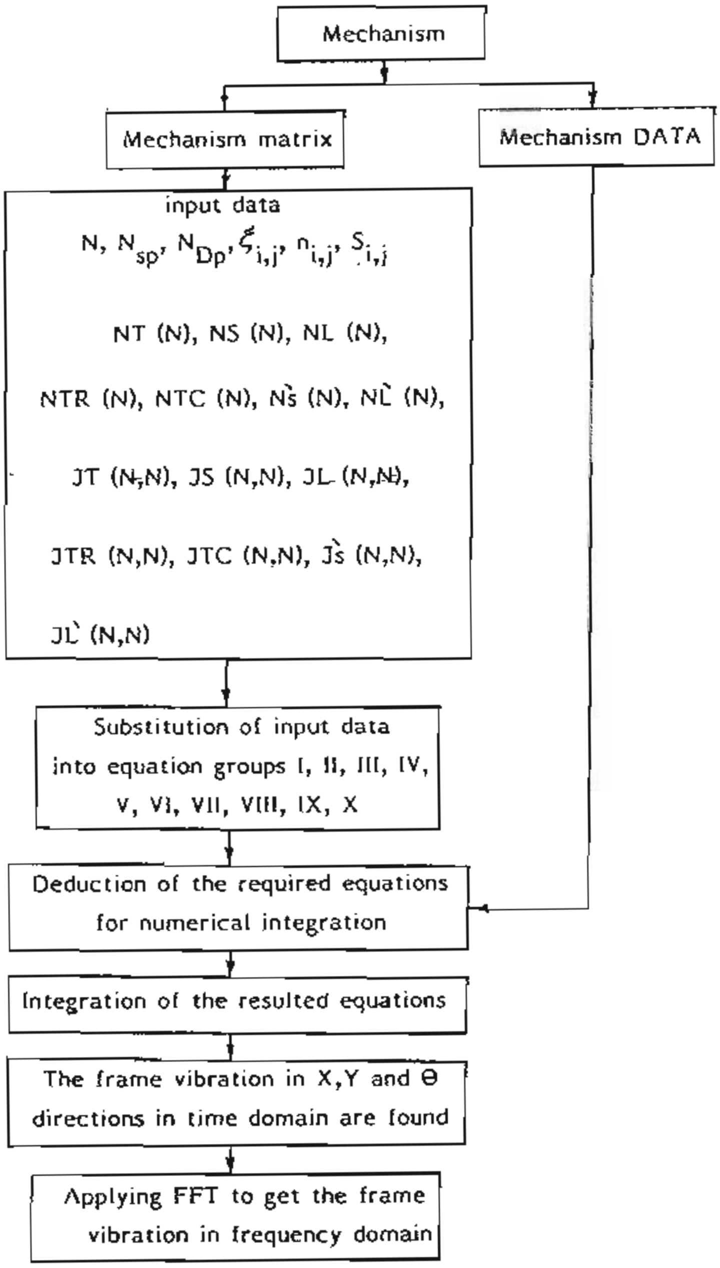


Fig. 5–Main flow chart

1- Define the independent variables to be integrated around according to the number of degrees of freedom of the mechanism and the type of known input. The independent variables are, generally, the frame displacement components, X_1, Y_1 and Θ_1 plus any additional displacement components for the mechanism links.

2- Use the auxiliary equations derived from the substructure of the mathematical model to express all the variables in matrix [A] and column {C} in terms of the independent variables and their first derivatives.

3- Multiply Equation XI by the inverse of matrix [A], i.e.

$$[A]^{-1}[A]\{B\} = [A]^{-1}\{C\}$$

$$\therefore \{B\} = [A]^{-1}\{C\} \qquad \text{XII}$$

In this case, all the unknown quantities are functions of the independent variables, their first derivatives, and time.

4- From the new set of equations, those with the second derivative of the independent variables in the left hand side will be the required equations for integration.

The differential equations are then solved numerically using Runge-Kutta-Jill method [20]. This method has the advantages of self-starting, variable step size, and high accuracy. The numerical solution requires the proper choice of the step size, h, which satisfies the requirements of minimum truncation error, minimum accumulated error, minimum round-off error, and minimum computer time. The main flow chart is shown in Fig.(5).

FREQUENCY ANALYSIS

The solution of the differential equations results in the determination of displacements, velocities, and accelerations of the center of gravity of the frame in the x , y, and Θ directions as functions of time. Then the frequency analysis is performed to break down the signals into their components at various frequencies. Fast Fourier Transform, FFT, is used to obtain the required spectra [21].

CONCLUSIONS

1- A mathematical model for obtaining the vibration spectra in x, y, and Θ directions of frames of wide range of planar mechanisms with arbitrary number and locations of clearances in the revolute pairs is constructed. The frames may also have arbitrary number and locations of supporting springs and dashpots.

2- The mathematical model presented may be considered as a base to construct a computer program which can provide the vibration spectra of any planar mechanism with arbitrary number and locations of clearance joints just by feeding its mechanism matrix with its data into the program.

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