Mansoura Engineering Journal Vol. 19, No. 3, September 1994

TRANSIENT PERFORMANCE ANALYSIS OF INVERTER-FED PERMANENT MAGNET SYNCHRONOUS MOTOR

F.M. Abdel-Kader   M.E. Eldeeb -   Ibrahim A. Amin
Electrical Eng. Dept., Faculty of Engineering, Suez Canal University
M.A. Ismaiel
Suez Canal Authority
VTHS, Ismailia, Egypt

تحليل الأداء العابر لمحرك تزامني ذو أقطاب دائمة يغذى من محول عكسي

الخلاصة: هذا البحث يقدم تحليل للأداء العابر لمحرك تزامني ذو أقطاب دائمة يغذى من محول عكسي مستخدما أسلوب العزوم المتزامنة والمخمدة ولقد طبقت طريقة ألجوريزم عددية للتنبؤ بالأداء العابر مستخدمين نموذج لاخطي مفصل لمحرك تزامني ذو أقطاب دائمة. وقد تم الحصول على مجموعة شاملة لنتائج التمثيل الرقمي للأداء العابر للمحرك مع استخدام أو بدون المحول العكسي. كما أمكن إيجاد وعرض الأشكال الموجية للأداء العابر عند حالات تشغيل مختلفة ومقارنتها لتوضيح الاختلافات الناتجة عند استخدام المحول العكسي. كما تم بحث تأثيرات تغير التردد نتيجة لاستخدام المحول العكسي على الأداء العابر للمحرك وكذلك المدى الأمثل للتردد.

ABSTRACT

This paper presents a comprehensive analysis of the transient performance of inverter-fed permanent magnet synchronous motor (PMSM) using damping and synchronizing torque technique. A numerical algorithm has been applied to predict the transient performance using detailed nonlinear model of permanent magnet synchronous motor. A set of results for the digital simulation of a motor with and without inverter at different modes of operations, are presented in a comparative form. This comparative form of study shows the differences in transient behaviour caused by introducing the inverter. The influence of frequency variation on motor transient performance is obtained and the optimum range of frequency is discussed.

1. INTRODUCTION

The prediction and analysis of transient performance of permanent magnet synchronous motor (PMSM) is very important, since many PMSMs are run-up from standstill by a direct connection with an ac supply with no need for dc field current.

Starting performance was first discussed by Honsinger [1]. He investigated the effects of rotor cage parameters on the run-up characteristics. Miller [2] developed a pull in criterion and studied the run-up characteristics of line start permanent magnet motors. Analytical expressions for the run-up conditions are given and provided by Rahman et al [3]. Osheba et al [4] presented the start-up performance, criteria for estimating balanced operation and maximum starting torque at a line frequency [4]. A comprehensive analysis of the transient performance using damping and synchronizing torques are defined by Osheba et al [5].

internal damping is modeled and optimum values of design parameters which improve transient performance are obtained.
The results of digital computer simulation of the transient performance including the effects of saturation are presented by Rahman et al [6].

However, these previous published works have not discussed the transient performance analysis of the inverter-fed PMSM when the motor is subjected to a large disturbance.

The object of this paper is to study the transient performance of the inverter-fed PMSM using damping and synchronizing torques technique. A comparison of transient performance when the motor is operated with and without inverter is presented. The effects of frequency variation on the motor performance and response are illustrated and the optimum range of operating frequencies are discussed.

2. MODELING OF MOTOR EQUATIONS

This section deals with the modeling of the transient performance of PMSM. During transient conditions, various disturbances can cause the armature and rotor fluxes to be changed both in magnitude and angular displacement as the rotor deviates from synchronous speed. These disturbances affect the transient currents, load angle changes, speed, slip and the electrical torque responses.

When the machine is running at a speed ω different than synchronous speed ω the equations for polyphase PMSM when the reference frame is fixed in the rotor [7,8] are :

$$v_d = -V \sin\delta = P\lambda_d + r_1 i_d - \lambda_q \omega_r \tag{1}$$

$$v_q = V \cos\delta = P\lambda_q + r_1 i_q + \lambda_d \omega_r \tag{2}$$

$$0 = P\lambda_D + r_D i_D \tag{3}$$

$$0 = P\lambda_Q + r_Q i_Q \tag{4}$$

$$T = \lambda_d i_q - \lambda_q i_d \tag{5}$$

With a fixed magnet strength, excitation may be represented by an equivalent field current i_f, the axes fluxes linkage may be given as:

$$\lambda_d = L_d i_d + L_{ad} i_D + L_{ad} i_f \tag{6}$$

$$\lambda_q = L_q i_q + L_{aq} i_Q \tag{7}$$

$$\lambda_D = L_D i_D + L_{ad} i_d + L_{ad} i_f \tag{8}$$

$$\lambda_Q = L_Q i_Q + L_{aq} i_q \tag{9}$$

Where δ is the torque angle which is given as :

$$\delta = \omega t - \omega_r t$$

From the equations (1) to (9) the i_d and i_q can be solved.

3. MODELING OF TORQUE COMPONENTS

Deviations in electrical torque at any frequency of rotor motion, can be decomposed into two components; (i) a synchronizing component in phase with the rotor displacement and (ii) a damping component in phase with the rotor speed. A numerical algorithm to model both damping and synchronizing torques is presented [8]. Synchronizing torques based on a time domain analysis of nonlinear response.
Following a small load change, the mechanical equation can be written as:

$$\Delta \dot{\omega}_M - (\Delta T_M - \Delta T_1) / J \tag{10}$$

The change in electrical torque, T_M can be decomposed as

$$\Delta T_M (t) = T_D (t) + f_S (t) \tag{11}$$

The above torque components can be represented by the damping torque coefficient K_D and the synchronizing torque coefficient K_S. Therefore ;

$$\Delta T_M (t) = \dot{K}_D \, . \, \Delta\omega(t) + K_S \, . \, \Delta\delta (t) \tag{12}$$

For a stable permanent magnet motor, the torque component coefficients K_D and K_S must be positive. The prediction of these coefficients provides quantative assessment of transient performance under different operating conditions. For the purpose of computing these coefficients, the error between the actual torque deviation and that obtained by summing both the damping and synchronizing components can be defined in the time domain from Eq. (12) as [7, 8] :

$$E(t) = [\Delta T_M (t) - (K_D \; \Delta\omega (t) \; + \; K_S \; \Delta\delta(t))] \tag{13}$$

The summation of errors squared over a period of time t can be written as follows :

$$\sum^{M} [E (t)]^{2} = \sum^{M} [\Delta T_M(t)] - (K_D \, \Delta\omega(t) + K_S \, \Delta\delta(t))]^{2} \tag{14}$$

Where t = N.T (N is the number of iterations and T is the sampling interval). Minimization of the summation of errors squared, Eq. (13) leads to the following algorithm ;

$$\sum^{M} \Delta T_M(t) . \Delta\delta(t) = K_s \sum^{M} [\Delta\delta(t)]^2 + K_D \sum^{M} \Delta\omega(t) . \Delta\delta(t) \tag{15}$$

$$\sum^{M} \Delta T_M(t) . \Delta\omega(t) = K_D \sum^{M} [\Delta\omega(t)]^2 + K_s \sum^{M} \Delta\omega(t) . \Delta\delta(t) \tag{16}$$

Solving the above equations gives the time invarient values of torque coefficients K_p and K_s which can be solved numerically with digital simulation.

4. PULSE - WIDTH MODULATION INVERTER CONTROL

The system under consideration is shown in Fig. (1). A pulse-width modulation inverter utilizes a 3 phase controlled rectifier to generate the required d.c. voltage level for the desired volts/frequency ratio. The voltage is then filtered by a large d.c. link reactor and capacitor before being inverted to the required frequency for the desired motor speed. The frequency of the output voltage correspond to the required speed is obtained by the transistors or thyristors in the inverter sections. This type of inverter produces voltage with high level of harmonic content. The resulting harmonic currents depend on the load impedance at the harmonic frequency.

The pulse-width modulation inverter is controlled to produce a variable-voltage, variable-frequency output. The output voltage is controlled by varying the width of the pulses symmetrically [9]. In some applications the number of pulses per half-cycle is kept fixed at all voltages. More complex schemes allow for increasing the number of pulses per half-cycle at lower output voltage to reduce the harmonic contents of the waveform. However the relative pulse-width is defined as :

$$rpw = \beta / \tau$$

Where

τ : is the theoretical maximum pulse-width ($\tau = \pi/3m$)

β : is the variable pulse-width

$m = 1, 2, 3, \ldots$

The amount of harmonic content in the output voltage waveform of inverter-fed PMSM is of concern to the machine designer because of the impact on torque pulsations, noises, losses and efficiency [10-12]

However, when the PMSM fed from pulse-width modulation inverter the voltage applied to the motor is the summation of harmonic contents C_n, obtained from Fourier series as follows :

$$V(\omega t) = \sum_{n=1}^{\infty} C_n \sin(n\omega t + \phi_n)$$

A numerical integration procedure may be used to determine the constants C_n in the Fourier series expansion

$$C_n = (a_n^2 + b_n^2)^{1/2}$$

Where

$$a_n = [2E_{d.c.} / n\pi] \sum_{k=0}^{2m-1} [\cos n((\pi/6) + (K\pi/3m)) - (\cos n((\pi/6) + (K\pi/3m) + \beta)] \quad , \text{for } n = 1, 3, 5, \ldots$$

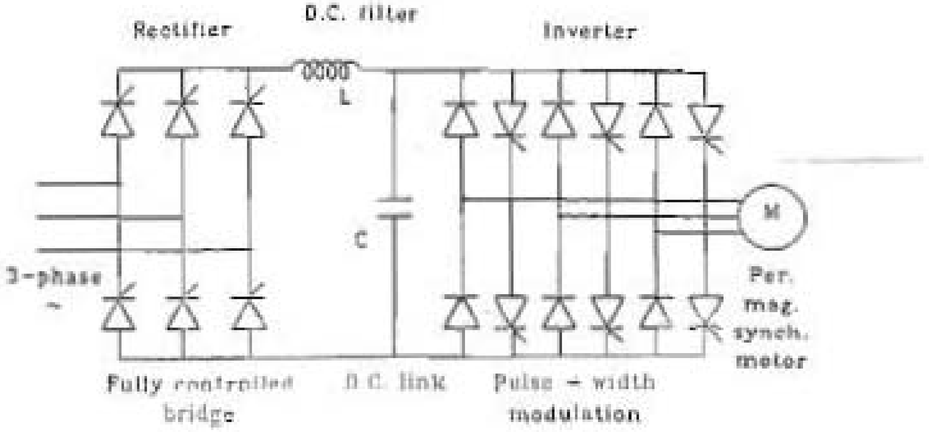


Fig. (1) Variable-voltage variable-frequency control of permenant magnet synchronous motor

Table (1) Voltage values for rpw = 1.0

V	V1	V5	V7	V11	V13
1.0915	1.0529	0.2105	0.1504	0.0957	0.0810

Table (2) Voltage values for rpw = 0.75

2m	V	V1	V5	V7	V11	V13
2	0.9316	0.8059	0.3891	0.1151	0.1768	0.1496
4	0.9005	0.7937	0.1813	0.1527	0.2075	0.1738
6	0.8317	0.7915	0.1674	0.1270	0.1010	0.1048
8	0.8244	0.7907	0.1631	0.1203	0.0852	0.0782

$$b_n = (2\, E_{d.c.} / n\pi) \sum_{k=0}^{2m-1} [\sin n \{ (\pi/6) + (K\pi/3m) + \beta \} - \sin n \{(\pi/6) + (K\pi/3m)\}] \quad , \text{ for } n = 1,3,5, \ldots$$

The p.u. values of harmonic contents of the output voltage waveforms for rpω = 1.0, 2m = 2 are shown in ~~Table~~ (1). Table (2) illustrates the values of harmonic contents of the voltage waveforms for rpω =0.75 and at different values of number of pulses per half-cycle (2m=2, 4, 6, 8).

From these tables it is noticed that as the number of pulses per half-cycle increases the ratio of the harmonics to the fundamental approaches the value for an equivalent square wave, i.e. for the fifth harmonic it approaches one-fifth for the harmonic content waveforms at rpω=1.0. When rpω is increased, the fundamental component is increased, while the harmonic contents are decreased, due to decreasing of turn off period.

5. TRANSIENT PERFORMANCE

This section is concerned with the prediction and analysis of the transient performance of PMSM with and without voltage-frequency control when the motor is subjected to a large disturbance.

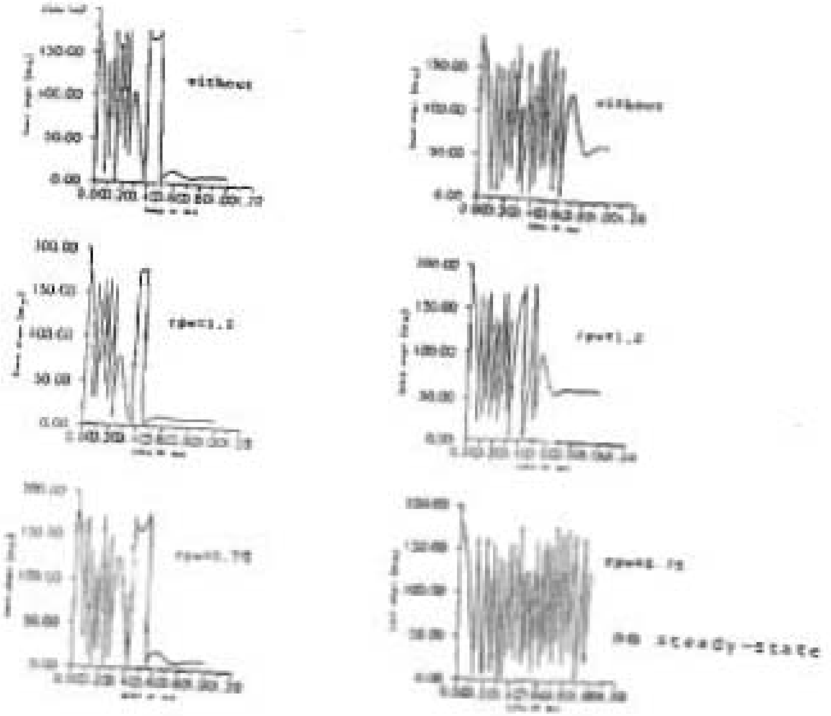
The object of this comparative form of study is to define the influence of inverter on the motor transient performance. The p.u. values of the parameters of the motor and its data are given in the appendix with their base quantities. The waveform of the time-invariant of load angle, speed and electrical torque are presented. The performance is carried out for two operating conditions, no-load and full-load with rpω=1.0 and 0.75, which have been proved to be the most efficient and suitable values.

5.1. Time Response

The permanent magnet synchronous motor is operated at no-load and full load conditions with and without inverter (rpω=1.0 and 0.75, 2m=2) at its nominal synchronous speed. The effect of a large load disturbance on load angle, rotor speed, motor torque and slip responses are determined and discussed.

5.1.1- Load Angle Characteristics

Fig. (2a,b) shows the load angle response when the motor operates at no-load and full load with and without inverter at rpω=1.0 and 0.75. The response almost gives similar conclusions and provides well damped and minimum settling time. It may also be observed that compared with the case of without inverter for rpω=1.0 the steady state value is decreased, but for rpω =0.75 the steady state value is increased. It is clear that either at no-load or full-load, the load angle for rpω=1.0 is smaller than of without inverter meaning that at the same load angle, the motor could give more output torque and increase the power limit.



(a) (b)

Fig. (2) Load angle response.
(a) no load (b) full load

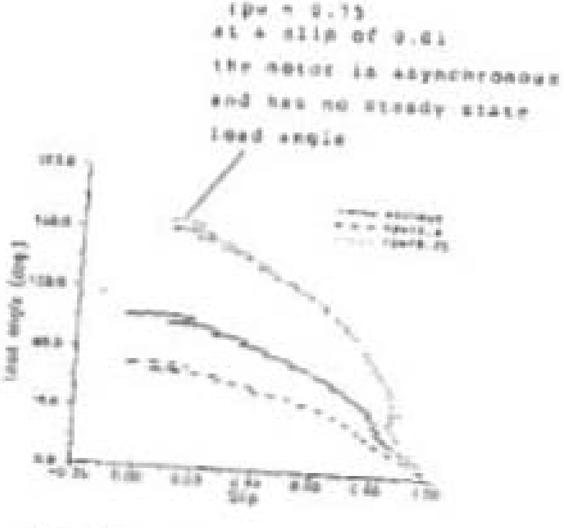


Fig. (3) Load angle versus slip.
full load

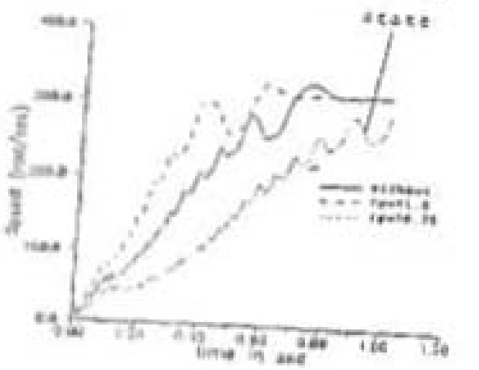


Fig. (4) Speed response.
full load

The load angle-slip characteristics when the motor operates at full load torque with and without inverter are shown in Fig. (3). When the motor uses an inverter with $rp\omega=1.0$, the load angle response has been improved and minimize the final or steady state load angle, while for $rp\omega=0.75$, the load angle values has been increased if compared with the case of without inverter. If the number of load angle-slip cycles is decreased, this means well damped and stable operation as shown in the case $rp\omega=1.0$.

5.1.2 Starting Characteristics

Fig.(4) gives the starting speed-time response at full load with and without inverter at $rp\omega=1.0$ and 0.75. It is shown that the motor reached its steady-state speed for without and with case (inverter $rp\omega=1.0$ faster, while for $rp\omega=0.75$, the steady-state value reached in a longer time). For $rp\omega=1.0$ the instability region gives a large value of speed, but for $rp\omega=0.75$ gives oscillatory region. These results show an improve of speed performance with the inverter if the $rp\omega=1.0$ w.r.t the case without inverter.

5.1.3 Torque Response

The comparison of electrical torque response at full load torque with and without inverter at $rp\omega=1.0$ and 0.75 are shown in Fig.(5). For motor operates with inverter at $rp\omega=1.0$ the transient period has been reduced and the settling time is decreased if compared with that of without inverter. For $rp\omega=0.75$, the settling time has not been reached and the oscillation continued. Therefore the motor runs asynchronously with large pulsating torques due to the large values of harmonic content from our point of view.

Fig. (6) shows the electrical torque-slip characteristics of the PMSM at no-load. When the motor operates through inverter for $rp\omega=1.0$, the electrical torque-slip cycles have been reduced, while for $rp\omega=0.75$, which is the worst mode of operation, the cycles are increased if compared with the case of without inverter.

Fig. (7) shows the electrical torque-slip characteristics when the motor operates at full load torque with and without inverter. The number of electrical torque-slip cycles is increased for full load torque if compared with the case of no-load torque. The first electrical torque-slip cycle simulates the stable operation at lower values of slip. For $rp\omega = 0.75$ the instability occurs in the motor performance. To reduce oscillatory, the load torque must be reduced. Generally, the influence of using inverter with $rp\omega = 1.0$ not only controlling the speed, but also improve the transient performance and increase the stability limit of the motor, which in fact prove the validity of using the selected inverter with this type of motor. .

6. EFFECT OF FREQUENCY VARIATION ON TRANSIENT CHARACTERISTICS

In the previous sections it has been proved that the selections of the right type of inverter improve the transient performance while controlling the motor speed. This section concern with the study of motor

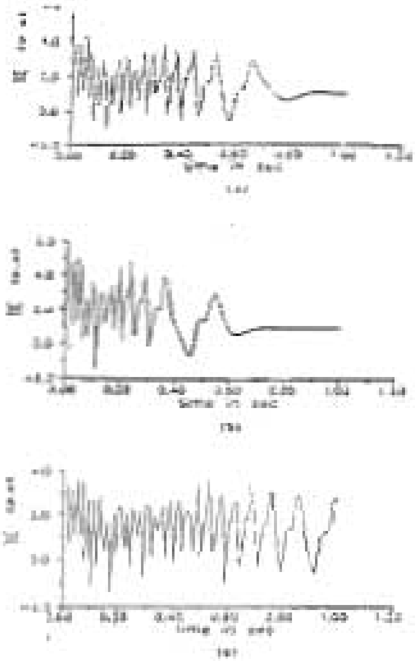


Fig (5) Comparison of elect. torque
responses at full load
(a)without (b)rpw=1.0
(c)rpw=0.75

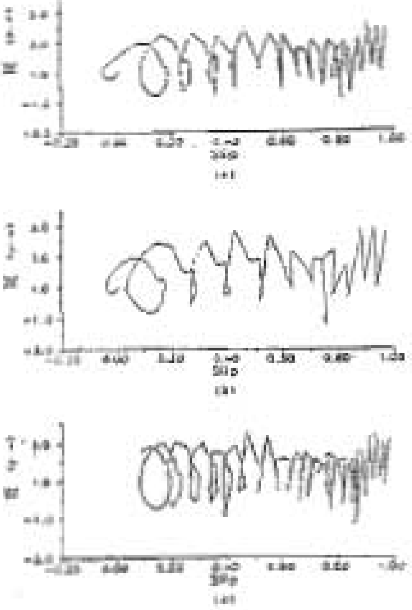


Fig.(7) Elect. torque versus slip a
full load
(a)without (b)rpw=1.
(c)rpw=0.75

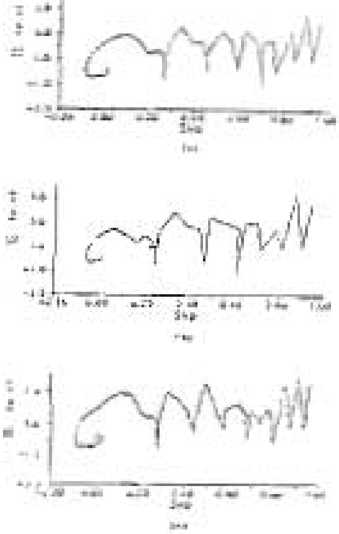


Fig.(8) Elect. torque versus slip at
no load
(a)without (b)rpw=1.0
(c)rpw=0.75

transient performance during different range of speed control by varying the line frequency. In order to maintain approximately constant flux density, the line voltage should also be varied directly with the supply frequency. The maximum torque then remains very nearly constant, while for increasing the frequency over optimum range decreases the available torque due to reduction of air-gap flux.
When the permanent magnet synchronous motor operates at low frequency,the instability problem has occurred. Increasing supply voltage and frequency increase both motor damping and stability are reserved. The starting performance is improved with the use of voltage-frequency control.

The effect of frequency variation on speed-time response when the motor is subjected to half-load and full-load torque are shown in Fig.(8). These curves are obtained when the motor is operated from an inverter with rpw=1.0 and at constant value of V/F ratio. It is noticed that, for increasing the frequency, the speed steady state value occurs in small time as shown at 75 and 100 HZ. For low frequency the oscillations occurred as shown at 25 HZ. The optimum lower range of frequency is obtained at 35 HZ with stable operation when the motor operates at half-load torque, while for full load torque the oscillation is increased and the settling time occurs over one second.
It is also shown that, when the motor operates at 25 HZ this simulates the unstable operation due to the lack of synchronism. To reduce the oscillating in the speed response at low frequency, the load torque must be reduced. The maximum value of the optimum range of frequency can be increased over 85 HZ, when the motor operates at starting or at transient disturbance.

The effect of frequency variation on electrical torque response when the inverter fed PMSM operates at half-load torque is shown in Fig.(9). It is noted that, for the mode of operation at 25 HZ the motor has not reached the steady state operation meaning that the motor is oscillatory and rotates asynchronouly. For 35 HZ mode of operation the settling time is small and the motor operates in stable mode. The results conclude that, when the frequency range increased a faster response is occurred, and the transient performance is improved as shown in the cases of 75 HZ and 100 HZ.

Fig.(10) shows the effect of frequency variation on electrical torque response when the inverter-fed PMSM operates at full load torque. It shows that, at 25 Hz the time response is unstable and the motor operates out of a synchronism as induction motor due to the starting cage. At 35 Hz the motor reach steady-state operation with a large settling time. It is also noted that, when the motor operates at 75 and 100 Hz, it has a faster response to reach the steady-state value operation. It has been concluded that, for the motor at low frequency the load torque must be reduced in order to reach the steady-state operation.

7. CONCLUSION

A comparison of the transient performance of the permanent magnet synchronous motor with and without inverter using damping and synchronizing torques technique has been presented. The results include the load angle, speed and electrical torques responses.

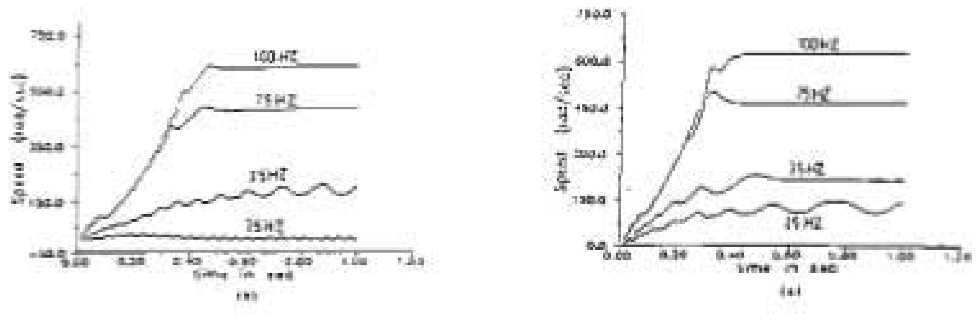


Fig. (8) Effects of frequency on speed response.
(a) half load (b) full load

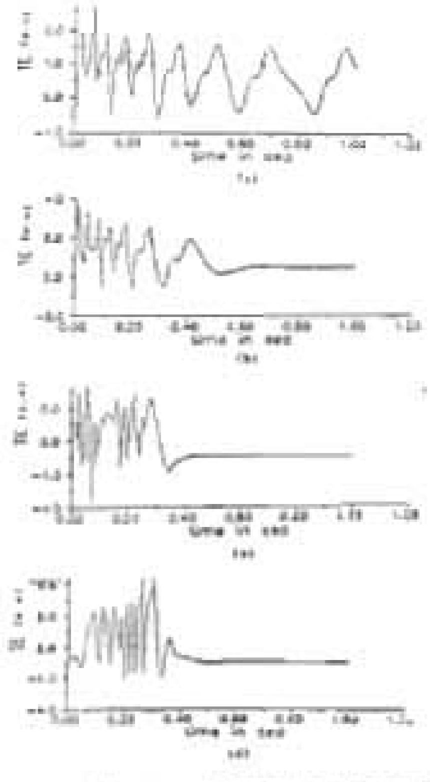


Fig.(9) Effects of frequency on elect. torque at half load
(a)f=25 HZ (b)f=35 HZ
(c)f=75 HZ (d)f=100 HZ

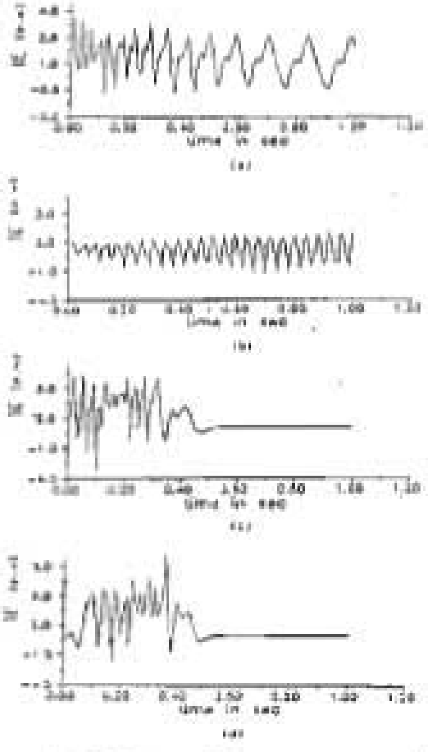


Fig.(10) Effects of frequency on elect. torque at full load
(a)f=25 HZ (b)f=35 HZ
(c)f=75 HZ (d)f=100 HZ

The characteristics of load angle and torque versus slip were investigated at the nominal voltage and frequency. Motor operations from a variable-voltage, variable-frequency, and the effects of frequency on motor transient performance were presented.

For voltage-frquency control, the harmonic content may be decreased by increasing the number of pulses per half-cycle. To obtain a maximum fundamental component the relative pulse-width (rpw) must be increased. When the motor operates through inverter at two pulses per half-cycle and rpw=0.75, this mode of operation simulates the worst mode of operation.

It is found that, increasing the voltage and frequency increase both motor damping and stability reserve.
However, transient performance at low frequency increase the motor instability and causes pulsation torques. To overcome this problem at low frequency the load must be reduced at low frequency with the same rate. The optimum range of frequency for transient operation can be increased more than 85 Hz.

REFERENCES

[1] V.B Honsinger " Permanent magnet machines, asynchronous operation " IEEE Trans. on Power Appar. and systems, Vol. PAS-99, No.4 PP. 1510-1518, 1980.

[2] T.J.E. Miller "Synchronization of line-start permanent magnet AC motors " IEEE Trans. on Power Appar. and systems, Vol. PAS-103, No.7, PP. 1822-1828, 1984.

[3] M.A. Rahman and A.M. Osheba " Performance of large line start permanent synchronous motors " IEEE Trans. Energy Convers., Vol.5, No.1, PP.211-217, March, 1990.

[4] A.M. Osheba and M.A. Rahman " Performance of line -Start single-phase permanent magnet synchronous motors " IEEE Industry Applications Conf. Atlanta, GA, 18-23, Oct., 1987.

[5] S.M. Osheba and F.M. Abdel-Kader " Performance analysis of permanent magnet synchronous motors. Part II: operation from variable source and transient characteristics " IEEE Trans. Energy Convers. " Vol.6, No.1, PP.83-89, March, 1991.

[6] M.A. Rahman and T.A. Little " Dynamic performance analysis of permanent magnet synchronous motors " IEEE Trans. on Power Appar. and Systems, Vol. PAS-103, No.6, PP.1277-1282, 1984.

[7] F.M. Abdel-Kader and S.M.Osheba " Performance analysis of permanent magnet synchronous motors ", PartI : Dynamic performance " IEEE Trans. Energy Conver's., Vol. 5, no.2, PP. 366-373, June, 1990.

[8] R.T. H. Alden and A. A. Shaltout " Analysis of damping and synchronizing torques " IEEE Trans.on Power Appar. and Systems, Vol. PAS-98, PP. 1696-1708, 1979.

[9] H. A. Ismail " Performance analysis of inverter-fed permanent magnet synchronous motor" M.Sc. Thesis Suez-Canal University, 1994.

[10] Hasmuckh S. Patel and Richard G. Hoft ", Generalized techniques of harmonic elimination and voltage control in thyristor inverter : Part I-Harmonic elimination " IEEE Trans. Appl., Vol.IA-9, PP.310-317, May/June, 1973.

[11] Hasmuckh S. Patel and Richard G. Hoft," Generalized techniques of harmonic elimination and voltage control in thyristor Inverter : Part II-voltage control technique " IEEE Trans. Ind. App.Vol IA-10, PP. 666-673, Oct, 1974.

[12] S.B. Dewan and A. Straughen " Power semiconductor circuits " John Willey & Sons, Inc 1975.

List of Symbols

$E_{d.c}$: D.c. voltage, p.u.

i : Current, P.u.
J : Motor Inertia
K_D : Damping torque coefficient

K_s : Synchron. torque coefficient

L : Inductance
p : Differential operator
r : Resistance, P.u.
rpw : Relative pulse-width

t : Time in Second

T : Torque , P.u.
V : Voltage , P.u.
w : Angular velocity, rad./sec.

δ : Load angle

λ : Flux Linkage

APPENDIX

Motor parameters in per unit values of a 4 h.p. 3-phase, 60 Hz, 230 v, permanent magnet synchronous motor are [6]:

$r_1 = 0.0173$
$r_D = 00054$
$r_Q = 0.108$
$i_f = 1.817$
$X_D = 0.608$
$X_Q = 1.151$

$X_d = 0.543$
$X_{ad} = 0.478$
$X_q = 1.086$
$X_{aq} = 1.021$
$H = 0.251$ Sec.