PURE-ROLLING CAMS OR CRANK-CRANK MECHANISMS TO GENERATE ELLIPTICAL GEARS MOTION

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Abstract:–Elliptical gears are one of the most commonly used mechanical transmissions employed to produce a non-uniform speeds needed in some industrial applications. In such mechanical gear transmissions, friction and backlash are the major sources of energy and accuracy losses. Elliptical gears are more complicated in manufacturing than others, hence, backlash is more severe which leads to an accuracy loss. This work presents two alternative possibilities for replacing elliptical gears by a crank-crank or pure-rolling cam mechanisms.

1 INTRODUCTION

Elliptical types of mechanical gear transmissions are often used to produce a non-uniform speed in many industrial applications. For example, to damp the shaking effects occurring in other mechanisms when connected in series with them. Elliptical gears can also be used in special design tasks like turning a carrier with a small radius to actuate wheeled robots as introduced by Emura and Arakawa [1]. The largest sources of energy loss in this kind of mechanical gear transmissions are the friction and backlash. A large part of the friction comes from the sliding motion.

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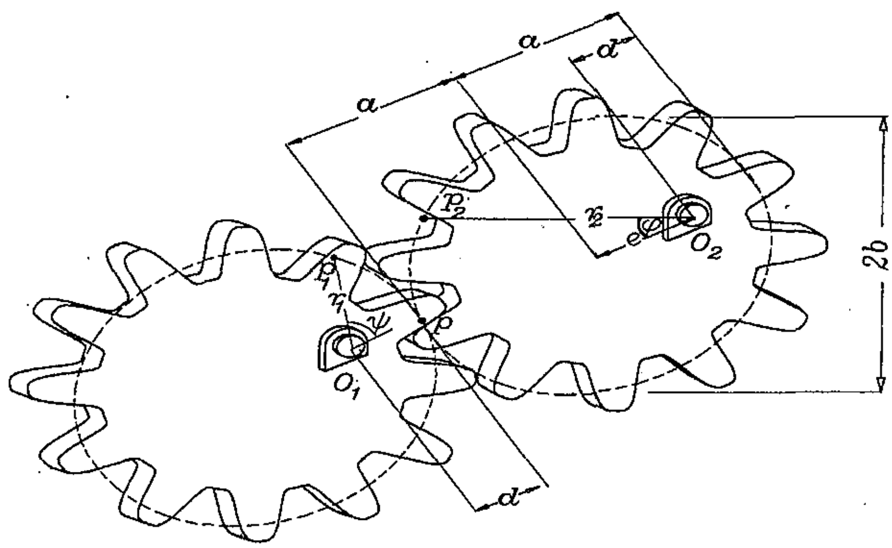

Figure 1: Identical elliptical gears

It is well known that, elliptical types of gears should be produced with a high accuracy machining as mentioned in [2]. Also, the machining process is known to be very complicated and the machining accuracy is generally not guaranteed. One of the big challenges for the elliptical gear designers is to obtain the desired non-uniform speed while using input-output intersecting axis.

A methodology for designing spherical cam mechanisms with a pure-rolling contact has been developed in [3]. The same methodology has been used in [4] to synthesize a cam mechanism for driving a shaking conveyor belt. The main benefit for using such cams is that they offer lower friction losses and negligible backlash, thereby leading to a high efficient transmission.

This work presents two alternatives to the elliptical gears: spherical crank-crank mechanism (SCC), and spherical cams with pure-rolling (SCPR).

A methodology for designing the spherical crank-crank mechanism to generate elliptical gears motion, based on using a suitable optimization technique to synthesize it with a suitable configurations is proposed.

The main benefit from using the spherical crank-crank mechanism to generate the desired motion is that the required machining process is more economic than the machining process of both elliptical gears and cams.

2 ELLIPTICAL GEARS MOTION RELATIONSHIP

Two identical elliptical gears are shown in Fig. 1. The two gears can correctly mesh at any instant under the conditions that the pitch point p be on the center line O_1O_2. As well as, the sum of the lengths from O_1 and O_2 to the contact point p between the two pith curves is constant and equals to the major ellipse diameter given by $(2a)$, as stated in [5-6].

The ratio of the instantaneous angular velocities of the two elliptical gears can be formulated to be inversely proportional to the distances from the centers of rotation to the pitch point p. Therefore, if two elliptical gears make contact at p_1 and p_2 as shown in Fig. 1, then the ratio of the instantaneous angular velocities can be written in the following form:

$$\frac{\dot{\phi}}{\dot{\psi}} = \frac{r_1}{r_2} \tag{1}$$

where $\dot{\psi}$ and $\dot{\phi}$ are the input and the output angular velocities. The pitch curve of the input gear can be expressed in polar coordinates as follows:

$$r_1 = \frac{b^2}{a(1+\cos\psi) - d\cos\psi} \tag{2}$$

where b is the minor ellipse radius, and d is equal to $(a-e)$, where e is the distance between the center of rotation (focal point) of the input gear and its geometrical center. We also have:

$$r_1 + r_2 = 2a \tag{3}$$

therefore from Eq. 2 and Eq. 3:

$$r_2 = \frac{d^2 + 2a(a-d)(1+\cos\psi)}{a(1+\cos\psi) - d\cos\psi} \tag{4}$$

where

$$(a-b)(a+b) = (a-d)^2 \tag{5}$$

The angular velocity $\dot{\phi}$ of the output gear corresponding to an angular velocity $\dot{\psi}$ of the input gear, is given by:

$$\dot{\phi} = \phi'(\psi)\dot{\psi} \tag{6}$$

By the same token, for a constant $\dot{\psi}$, the acceleration can be written in the following form:

$$\ddot{\phi} = \phi''(\psi)\dot{\psi}^2 \tag{7}$$

where $\phi'(\psi)$ is the *velocity ratio* which is equal to (r_1/r_2), and $\phi''(\psi)$ is the *acceleration ratio*. The *velocity ratio* $\phi'(\psi)$ can be formulated as follows:

$$\phi'(\psi) = \frac{b^2}{d^2 + 2a(a-d)(1+\cos\psi)} \tag{8}$$

By differentating Eq. 8 the *acceleration ratio* $\phi''(\psi)$ can be given by

$$\phi''(\psi) = \frac{2ab^2(a-d)\sin\psi}{[d^2 + 2a(a-d)(1+\cos\psi)]^2} \tag{9}$$

The output angular position ϕ can be given by integrating Eq. 8 using the following integration standard form

$$\phi = b^2 \int \frac{d\,\psi}{l_1 + l_2 \cos\psi} = \frac{2b^2}{\sqrt{l_1^2 - l_2^2}} \tan^{-1}\left(\frac{(l_1 - l_2)\tan(\psi/2)}{\sqrt{l_1^2 - l_2^2}}\right) \tag{10}$$

where

$$l_2 = 2a(a-d), \text{ and}$$
$$l_1 = l_2 + d^2$$

Hence,

$$\phi = \frac{2b^2}{d\sqrt{d^2 + 4a(a-d)}} \tan^{-1}\left(\frac{d\tan(\psi/2)}{\sqrt{d^2 + 4a(a-d)}}\right) \tag{11}$$

Here, the integral constant is equal to zero because $\phi = 0$ at $\psi = 0$. This equation and its derivatives give the desired objective functions.

3 SYNTHESIS OF SCC MECHANISM TO GENERATE ELLIPTICAL GEARS MOTION

The *spherical Crank-Crank mechanism* (SCC) of Fig. 2 is used to generate the elliptical gears motion, its parameters can be determined from the equation below [7]:

$$k_1 - k_2\cos\phi + k_3\cos\psi + k_4\cos\phi\cos\psi + \sin\phi\sin\psi = 0 \tag{12}$$

with the definitions:

$$k_1 \equiv \frac{\cos\alpha_1\cos\alpha_2\cos\alpha_4 - \cos\alpha_3}{\sin\alpha_2\sin\alpha_4}$$
$$k_2 \equiv \frac{\sin\alpha_1}{\sin\alpha_2}$$
$$k_3 \equiv \frac{\sin\alpha_1}{\sin\alpha_4}$$
$$k_4 \equiv \cos\alpha_1$$

in which ψ and ϕ are the input and output angles. In Eq. 12, it is assumed that $k_4 \leq 1$ and that all angles (i.e., $\{\alpha_i\}_{i=1}^4$) lie in the range of 0° to 180.0°.

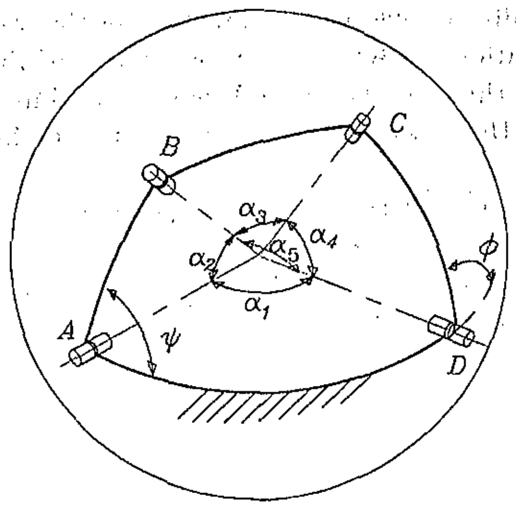

Figure 2: Spherical Crank-Crank Mechanism.

In order to ensure that the linkage is of the spherical crank-crank-link type, the constraints (conditions for full-mobility) below must be imposed [7]. For an input crank, we must have

$$(k_4 - k_2)^2 - (k_3 + k_1)^2 \geq 0 \tag{13a}$$

$$(k_4 + k_2)^2 - (k_3 - k_1)^2 \geq 0 \tag{13b}$$

while, for an output crank:

$$(k_3 + k_4)^2 - (k_1 - k_2)^2 \geq 0 \tag{13c}$$

$$(k_3 + k_4)^2 - (k_1 + k_2)^2 \geq 0 \tag{13d}$$

A complete study of these parameters is included in the Appendix to formulate the output angular position ϕ, the velocity ratio $\phi'(\psi)$ and the acceleration ratio $\phi''(\psi)$ which must satisfy the desired objective functions. The output angular velocity ($\dot{\phi}$) and the output angular acceleration ($\ddot{\phi}$) can take the same form as in Eqs. 6 and 7. Moreover, the input/output (I/O) relation given by Eq. 12 can be rewritten in the following form:

$$f(\psi, \phi, \mathbf{k}) = k_1 - k_2 \cos\phi + k_3 \cos\psi + k_4 \cos\phi \cos\psi + \sin\phi \sin\psi \tag{14}$$

The previous equation, for m I/O pairs, can be rewritten into a common form, namely $\mathbf{Sk} = \mathbf{b}$. Where, S is the $m \times n$ synthesis matrix and $\mathbf{b}$ is m-dimensional vector. The design errors can be defined, using the latter equation, to be the Euclidean norm of the associated design error vector $\mathbf{d}$, and can be written as:

$$\mathbf{d} = \mathbf{b} - \mathbf{Sk} \tag{15}$$

The *transmission defect* is the complement of the *transmission quality* in the sense that the two quantities add up to 1.

The transmission quality measures the goodness of the transmission angle throughout the operation. The *transmission defect* of SCC mechanisms should be considered through the optimization procedures. For spherical crank-crank linkages, the transmission defect δ takes the form [8]:

$$\delta = (\frac{v_1 v_2}{v_3})^2 (v_4^2 + v_5^2 - 2v_4 v_5 - 0.5 v_6^2) \tag{16}$$

where the coefficients v_i, for $i = 1, 2, .., 6$, are only functions of the spherical four-bar linkage parameters ($k_i, i = 1, 2, 3, 4$), and are given by:

$$v_1 = \sqrt{\frac{1}{2}\left(1 - \frac{k_2 k_3 k_4 - k_1(1 - k_4^2)}{\sqrt{(1 + k_2^2 - k_4^2)(1 + k_3^2 - k_4^2)}}\right)} \tag{17a}$$

$$v_2 = \sqrt{1 + k_3^2 - k_4^2} \tag{17b}$$

$$v_3 = \sqrt{\left(1 - \frac{[k_2 k_3 k_4 - k_1(1 - k_4^2)]^2}{(1 + k_2^2 - k_4^2)(1 + k_3^2 - k_4^2)}\right)(1 - k_4^2)} \tag{17c}$$

$$v_4 = \frac{k_2 k_4}{\sqrt{1 + k_2^2 - k_4^2}} \tag{17d}$$

$$v_5 = \frac{[k_2 k_3 k_4 - k_1(1 - k_4^2)]k_3}{\sqrt{1 + k_2^2 - k_4^2(1 + k_3^2 - k_4^2)}} \tag{17e}$$

$$v_6 = \frac{1 - k_4^2}{\sqrt{1 + k_2^2 - k_4^2}} \tag{17f}$$

For minimizing the *transmission defect*, a class of linkages known as *zero-mean linkages* was introduced by Gosselin and Angeles [9]. For zero-mean linkages, Eq. 16 can be written, in the following form as stated in [10]:

$$\delta = \frac{\left(1 - [k_2 k_3 k_4 - k_1(1 - k_4^2)]\sqrt{(1 + k_2^2 - k_4^2)\lambda}\right)\lambda}{4(1 + k_2^2 - k_1^2)} \tag{18}$$

in which $\lambda \equiv 1 + k_3^2 - k_4^2$
As well as, for zero-mean linkages:

$$k_2 k_4 + k_1 k_3 = 0 \tag{19}$$

Furthermore, for zero-mean linkages, full-mobility conditions can be written in the following form as introduced by [10]:

$$k_4^2 \leq 1, \qquad k_2^2 \leq k_1^2, \qquad k_3^2 \leq k_4^2 \quad \text{and} \quad k_1^2 \leq k_4^2 \tag{20}$$

The optimum design problem can be formulated as a nonlinear least-square minimization under equality constraints. The optimization problem can be stated as mentioned in [10], as : Finding an approximate solution of $\mathbf{f}(\mathbf{k}) = \mathbf{0}_m$ such that the objective function z, as defined below, is minimum:

$$z(\mathbf{k}) = \frac{1}{2}\mathbf{f}^T\mathbf{W}\mathbf{f} \Longrightarrow \min_{\mathbf{k}} \tag{21}$$

Meanwhile, the following system of nonlinear constraints should be maintained:

$$\mathbf{g}(\mathbf{k}) = \mathbf{0}_p \tag{22}$$

In the previous equations, $\mathbf{f}$ and $\mathbf{g}$ are m- and p-dimensional nonlinear vectors, respectively. As well as, $\mathbf{0}_m$ and $\mathbf{0}_p$ are the m- and p-dimensional zero vectors. Furthermore, $\mathbf{W}$ is an $m \times m$ weighting matrix. Here, QUADMIN package [11] can be used for the solution of optimum design problems using a least-square approach. Now Eq. 14 can be used to define the jth component of $\mathbf{f}$, i.e., $f_j(\mathbf{k})$, corresponding to the jth I/O pair, for $j = 1, ..., m$. Furthermore, a slack variable k_5 can be introduced to equate Eq. 18 to k_5^2. Hence, a new component for $\mathbf{f}$, i.e., f_{m+1} can be obtained in the following form:

$$f_{m+1}(\mathbf{k}) \equiv \lambda[\sqrt{\zeta\lambda} - k_2k_3k_4 - k_1(1 - k_4^2)] - 4k_5^2(1 + k_2^2 - k_1^2)\sqrt{\zeta\lambda}$$

where

$$\zeta = 1 + k_2^2 - k_4^2$$

To minimize δ, the constraint below can be imposed:

$$f_{m+2}(\mathbf{k}) \equiv k_5 = 0$$

By the same token f_{m+3} is defined in the form

$$f_{m+3}(\mathbf{k}) \equiv k_4 = 0$$

The first constraint corresponding to the first component of $\mathbf{g}$ is obtained from Eq. 19 to be:

$$g_1(\mathbf{k}) \equiv k_2k_4 + k_1k_3 = 0$$

By introducing four slack variables $\{k_i\}_{i=6}^{9}$, the constraints given by Eq. 20 can be re-expressed in the following form:

$$\begin{aligned} g_2(\mathbf{k}) &\equiv k_4^2 - 1 + k_6^2 = 0 \\ g_3(\mathbf{k}) &\equiv k_2^2 - k_1^2 + k_7^2 = 0 \\ g_4(\mathbf{k}) &\equiv k_3^2 - k_4^4 + k_8^2 = 0 \\ g_5(\mathbf{k}) &\equiv k_1^2 - k_4^4 + k_9^2 = 0 \end{aligned}$$

Hence, $\mathbf{k}$, $\mathbf{f}$ and $\mathbf{g}$ take on the forms:

$$\begin{aligned} \mathbf{k} &\equiv [k_1, ..., k_9]^T \\ \mathbf{f} &\equiv [f_1, ..., f_m, f_{m+1}, f_{m+2}, f_{m+3}]^T \\ \mathbf{g} &\equiv [g_1, ..., k_5]^T \end{aligned}$$

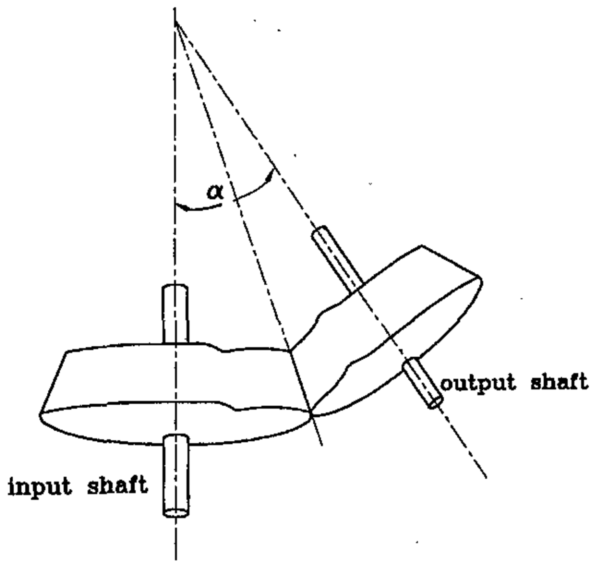

Figure 3: Spherical cam Mechanism

4 SYNTHESIS OF (SCPR) TO GENERATE ELLIPTICAL GEARS MOTION

The synthesis methodology of spherical cams with a pure-rolling (SCPR) based on the Aronhold- Kennedy Theorem [12] was used. It states that, when three bodies are in relative motion, the three different instant screw axes (ISAs) share one common perpendicular. According to the previous theorem, the ISA of the follower with respect to the cam, when the axes of rotation of both intersect, passes through the intersection of those two axes and is coplanar with them. At points located on the ISA, which becomes now an instant axis of relative rotation, no slip occurs. In [3], the position vectors $\mathbf{r}_2$ and $\mathbf{r}_3$ that define the cam and follower profiles of a spherical cam mechanism, as shown in Fig. 3, were found to be

$$\mathbf{r}_2 = v \begin{bmatrix} -\sin\psi \quad \sin\vartheta \\ -\cos\psi \quad \sin\vartheta \\ \cos\vartheta \end{bmatrix} \tag{23a}$$

and

$$\mathbf{r}_3 = v \begin{bmatrix} -\sin\phi \quad \sin(\vartheta-\alpha) \\ -\cos\phi \quad \sin(\vartheta-\alpha) \\ cos\ (\vartheta-\alpha) \end{bmatrix} \tag{23b}$$

where v denotes the thickness of the contact surfaces, α is the angle between

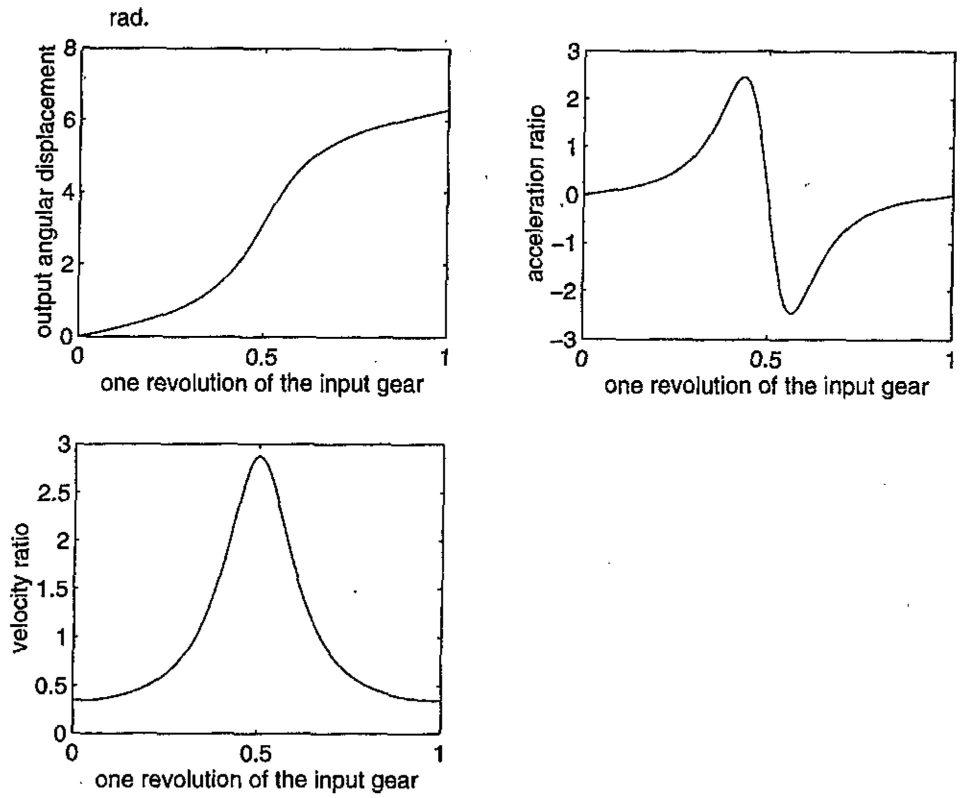

Figure 4: The desired motion of the elliptical gears

intersecting input and output axes and ϑ is computed from:

$$\tan\,\vartheta = \frac{\phi' \,\sin\alpha}{\phi' \,\cos\alpha - 1} \tag{24}$$

Moreover, the pressure angle can be computed as,

$$\mu = \tan^{-1}\left(\frac{\phi'\sqrt{\phi'^2 - 2\phi'\cos\alpha + 1}}{\phi''}\right) \tag{25}$$

where ϕ' and ϕ'' are the *velocity* and *acceleration* ratios.

5 RESULTS

5.1 *Desired Motion*

Figure 4 illustrates variables ϕ, ϕ' and ϕ'' of the output elliptical gear of Fig. 1, with respect to one revolution of the input gear using the numerical values ($a = 0.12, b = 0.105$ and $d = 0.0619$). Now, the crank-crank and pure-rolling cam mechanisms can be synthesised to generate the previous variables (ϕ, ϕ' and ϕ'') of the elliptical gears as the desired objective function.

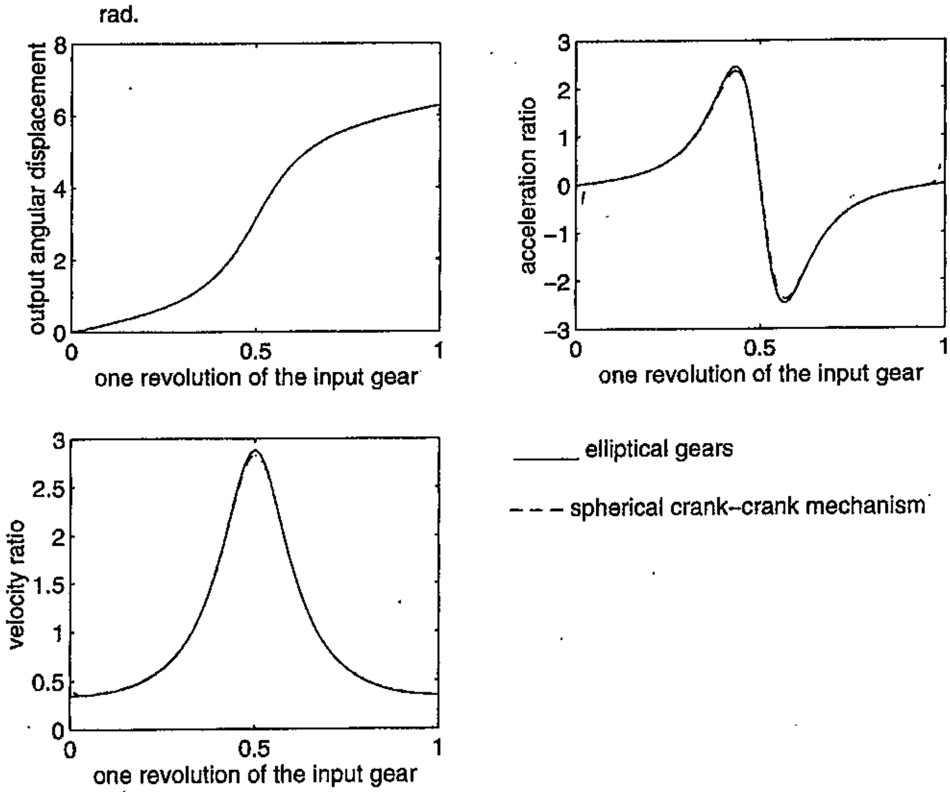

Figure 5: The desired motion of the elliptical gears generated by using SCC mechanism

5.2 *Synthesised Spherical Crank-Crank Mechanism Parameters*

The optimum coefficients k_i, for $i = 1, 2, 3, 4$, of the spherical drag-link mechanism that were produced using the QUADMIN package [11] were found to be $k_1 = -0.6243$, $k_2 = 0.0052$, $k_3 = 0.00525$ and $k_4 = 0.62435$. As well, the transmission defect δ was found to be equal to 0.1.

Figure 5 shows the variables ϕ, ϕ' and ϕ'' of the output of the elliptical gear and the corresponding synthesised values of the spherical crank-crank mechanism, with respect to one revolution of the input gear. This figure illustrates that, the desired ϕ and synthesised one are approximately the same. But, for ϕ' and ϕ'', there is a slight difference.

5.3 *Synthesised Pure-Rolling Cam Mechanism Parameters*

Figure 6 presents a Silicon Graphics solid model of the pure-rolling cam mechanism to generate the desired motion of the elliptical gears. Figure 7 shows the pressure angle of the synthesised pure-rolling cam mechanism.

When the pressure angle is smaller than 90 degrees, the force transmission is from the cam to the follower, which we call positive action (PA). And

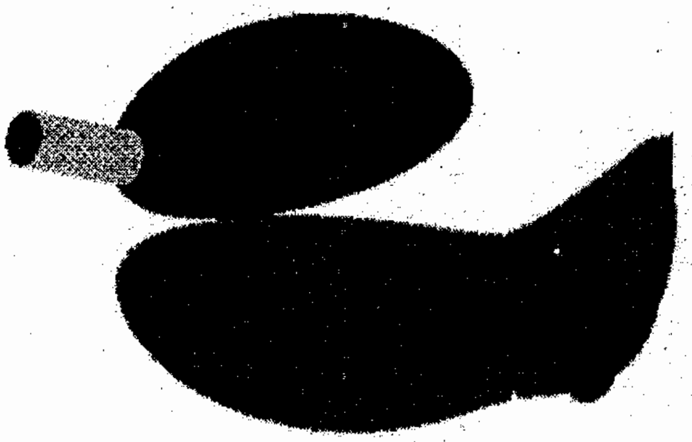

Figure 6: Silicon Graphics solid model of Pure-rolling cam mechanism

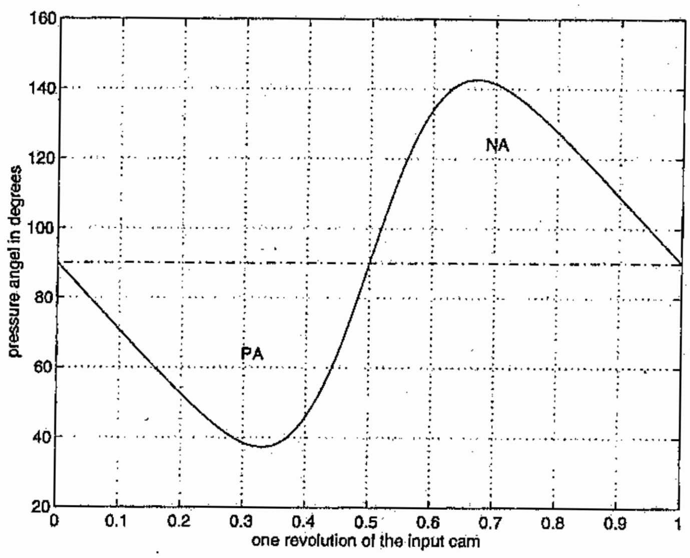

Figure 7: Pressure angle

when the pressure angle is bigger than 90 degrees, the force transmission is from the follower to the cam, which is called negative action (NA). (see Fig. 7). In order to eliminate the occurrence of a negative action, the Pure Rolling Indexing Cam Mechanisms (PRICAM) which was introduced by [3] could be used here.

6 CONCLUSIONS

In order to circumvent the friction, backlash and accuracy losses problems occurring when using elliptical gear transmissions, especially with intersecting input output axes, we showed that we can replace them by spherical crank-crank or pure-rolling cam mechanisms to generate the same desired motion.

The manufacturing of the spherical crank-crank mechanism is economic, while the output of the spherical cams with pure-rolling is more accurate in the desired motion.

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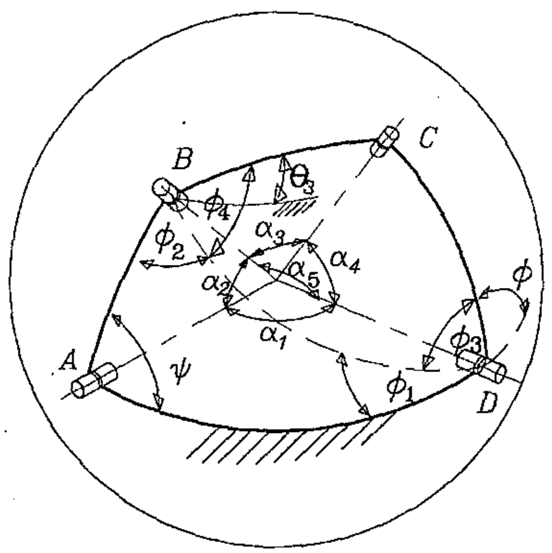

Figure 8: Geometry of spherical four bar-linkage

APPENDIX

6.1 *Angular Position of the Output Link of SCC Mechanism*

Expressions for the angular position of the output link, ϕ, are derived here as functions of the angular position of the input link, ψ, and the inner angles, as shown for spherical mechanisms in Fig. 8.

By taking into account that the input link pulls the output link when the latter moves counterclockwise, the angular position of the output link ϕ can be written as

$$\phi = \begin{cases} \pi - \phi_1 - \phi_3 & \text{if} \quad 0 \le \psi < \pi \\ \pi + \phi_1 - \phi_3 & \text{if} \quad \pi \le \psi \le 2\pi \end{cases}$$

Also, by taking into account that the input link pushes the output link when the latter moves counterclockwise, ϕ becomes:

$$\phi = \begin{cases} \pi - \phi_1 + \phi_3 & \text{if} \quad 0 \le \psi < \pi \\ \pi + \phi_1 + \phi_3 & \text{if} \quad \pi \le \psi \le 2\pi \end{cases}$$

where the inner angles ($\phi_i \le \pi$, $i = 1, 2, 3, 4$), for spherical mechanisms can be written as shown below:

$$\phi_1 = \tan^{-1}\left(\cos\zeta_1 \sec\zeta_2 \cot(\frac{\psi}{2})\right) - \tan^{-1}\left(\sin\zeta_1 \ \mathrm{cosec}\ \zeta_2 \cot(\frac{\psi}{2})\right)$$

$$\phi_2 = \tan^{-1}\left(\cos\zeta_1 \sec\zeta_2 \cot(\frac{\psi}{2})\right) + \tan^{-1}\left(\sin\zeta_1 \ \mathrm{cosec}\ \zeta_2 \cot(\frac{\psi}{2})\right)$$

$$\phi_3 = 2\tan^{-1}\sqrt{\frac{\sin(\alpha_s - \alpha_4)\sin(\alpha_s - \alpha_5)}{\sin(\alpha_s - \alpha_3)\sin\alpha_s}}$$

$$\phi_4 = 2\tan^{-1}\sqrt{\frac{\sin(\alpha_s - \alpha_3)\sin(\alpha_s - \alpha_5)}{\sin(\alpha_s - \alpha_4)\sin\alpha_s}}$$

$$\alpha_5 = \cos^{-1}(\cos\alpha_1 \cos\alpha_2 + \sin\alpha_1 \sin\alpha_2 \cos\psi)$$

$$\alpha_s = \frac{\alpha_3 + \alpha_4 + \alpha_5}{2}, \quad \zeta_1 = \frac{\alpha_1 - \alpha_2}{2}, \quad \text{and} \quad \zeta_2 = \frac{\alpha_1 + \alpha_2}{2}$$

6.2 Velocity Ratio $\phi'(\psi)$ of SCC Mechanism

By differentiation of both sides of Eq. 12 with respect to time, we obtain

$$\begin{aligned} \dot{\phi} k_2 \sin\phi - \dot{\psi} k_3 \sin\psi &= k_4(\dot{\psi}\cos\phi\sin\psi - \dot{\phi}\sin\phi\cos\psi) \\ &\quad -\dot{\psi}\sin\phi\cos\psi - \dot{\phi}\cos\phi\sin\psi \end{aligned} \tag{26}$$

Upon solving for $\dot{\phi}$, we get:

$$\dot{\phi} = \left(\frac{k_3\sin\psi + k_4\cos\phi\sin\psi - \sin\phi\cos\psi}{k_2\sin\phi - k_4\sin\phi\cos\psi + \cos\phi\sin\psi}\right)\dot{\psi} \tag{27}$$

and hence, the velocity ratio is the coefficient of $\dot{\psi}$ in Eq. 27 above, namely:

$$\phi'(\psi) = \frac{k_3\sin\psi + k_4\cos\phi\sin\psi - \sin\phi\cos\psi}{k_2\sin\phi - k_4\sin\phi\cos\psi + \cos\phi\sin\psi} \tag{28}$$

6.3 Acceleration Ratio $\phi''(\psi)$ of SCC Mechanism

By differentiation of both sides of Eq. 26 with respect to time, we get:

$$\begin{aligned} k_2\ddot{\phi}\sin\phi + k_2\dot{\phi}^2\cos\phi &= k_4\dot{\psi}^2\cos\phi\cos\psi - 2k_4\dot{\phi}\dot{\psi}\sin\phi\sin\psi + k_4\dot{\phi}^2\cos\phi\cos\psi \\ &\quad +k_4\ddot{\phi}\sin\phi\cos\psi - 2\dot{\phi}\dot{\psi}\cos\phi\cos\psi + \dot{\psi}^2\sin\phi\sin\psi \\ &\quad -\ddot{\psi}(k_3\sin\psi + k_4\sin\psi\cos\phi - \cos\psi\sin\phi) \\ &\quad -\ddot{\phi}\cos\phi\sin\psi + \dot{\phi}^2\sin\phi\sin\psi + k_3\dot{\psi}^2\cos\psi \end{aligned} \tag{29}$$

where

$$\ddot{\phi} = \left(\frac{\hat{\nu}_1 + \hat{\nu}_2 + \hat{\nu}_3}{k_2 \sin\phi - k_4 \sin\phi \cos\psi + \cos\phi \sin\psi} \right) \dot{\psi}^2 + [k_3 \sin\psi + k_4 \sin\psi \cos\phi - \cos\psi \sin\phi)] \ddot{\psi} \qquad (30)$$

and

$$\hat{\nu}_1 = \phi'^2(\psi)[-k_2 \cos\phi + k_4 \cos\phi \cos\psi + \sin\phi \sin\psi]$$
$$\hat{\nu}_2 = 2\phi'(\psi)[-2k_4 \sin\phi \sin\psi - 2\cos\phi \cos\psi]$$
$$\hat{\nu}_3 = k_3 \cos\psi + k_4 \cos\phi \cos\psi + \sin\phi \sin\psi$$

The acceleration ratio is the coefficient of $\dot{\psi}^2$ in the above equation, namely:

$$\phi''(\psi) = \frac{\hat{\nu}_1 + \hat{\nu}_2 + \hat{\nu}_3}{k_2 \sin\phi - k_4 \sin\phi \cos\psi + \cos\phi \sin\psi} \qquad (31)$$

آلية كامات لها خاصية التشغيل بالدوران الخالص بدون انزلاق أو آلية رباعية لتوليد نفس حركة التروس البيضاوية

تعتبر التروس البيضاوية من العناصر شائعة الاستخدام فى نوا قل الحركة الميكانيكية لتوليد سرعات غير منتظمه التى تكون مطلوبة فى بعض التطبيقات الصناعية.
وفى حالة استخدام التروس كوسيلة لنقل الحركة يكون الاحتكاك والتداخل من أكبر مصادر فقد الطاقة ونقص الدقة.
ومن المعروف أن تصنيع التروس البيضاوية معقد جدا وفقد الطاقه عن طريق الاحتكاك و التداخل أثناء تشغيلها كبير مقارنة بأنواع التروس الأخرى، ولهذا قد قدم هذا البحث طريقين لاستبدال التروس البيضاوية بتوليد نفس الحركة المطلوبة من التروس البيضاوية عن طريق آلية رباعية كروية أو آلية كامات لها خاصية التشغيل بالدوران الخالص بدون انزلاق مما يقلل جدا الاحتكاك وبذلك يقل فقد الطاقة.