



**Answer all the following questions:** [100 marks]

**Question 1** [30 marks]

**A Find the general solution of the differential equations:** [20 marks]

(i)  $\frac{dy}{dx} = \frac{2x \sinh \frac{y}{x} + 3y \cosh \frac{y}{x}}{3x \cosh \frac{y}{x}}$

(ii)  $\frac{dy}{dx} = \frac{y}{2y \ln y + y - x}$

(iii)  $\frac{dy}{dx} = \frac{2x + y - 1}{4x + 2y + 5}$

(iv)  $\frac{dy}{dx} + y \sec x = \cos^2 x$

**B Find the general solution of the differential equations:** [10 marks]

(i)  $\left(\frac{dy}{dx}\right)^2 + 2\left(\frac{dy}{dx}\right)y \cot x - y^2 = 0$

(ii)  $\left(\frac{dy}{dx}\right)^2 - 2x \frac{dy}{dx} + y = 0$

**Question 2** [10 marks]

**A** Deduce the general solution of the equation  $(x^2 D^2 + 3x D - 3)y = x$ , then check your result.

**B** If the acceleration distribution of the planer motion of a moving particle is represented by the two components:  $\ddot{x} = 3x + 4y$  and  $\ddot{y} = -x - y$ . Determine the parametric displacements in  $x$  and  $y$  direction, then deduce the Cartesian equation of the path of the particle, and finally check your results.

**Question 3** [20 marks]

**A Test and find Laplace transform of:**

(a)  $t^3 e^{3t}$ , (b)  $\frac{1 - e^{-2t}}{t(1 + e^{-t})}$ , (c)  $\int_0^t \frac{\sin t}{t} dt$ , (d)  $f''(t)$  where  $f(t) = \cos 3t + \sin 2t$ .

**B** Test and find the inverse Laplace transform of  $\frac{1}{s^4 + 2s^3 + s^2}$  using convolution theorem

**C** Using Laplace transforms solve and check the initial value problem

$$\ddot{y} + 3\dot{y} + 2y = e^{-t}$$

subjected to the initial conditions  $y(0) = 0$  and  $\dot{y}(0) = 0$ .

**Question 4**
**25 marks**

- A** If the sequence  $\left\{\frac{3}{2}, \frac{5}{4}, \frac{7}{6}, \dots\right\}$  is bounded between the two numbers  $a$  and  $b$ , find the general term  $u_n$ , then calculate each number.
- B** Test the series  $\left[1 + \frac{1}{1 \times 2} + \frac{1}{1 \times 2 \times 3} + \frac{1}{1 \times 2 \times 3 \times 4} + \dots\right]$  for convergence.
- C** Test the series  $\left[\frac{1}{3} + \left(\frac{2}{5}\right)^4 + \left(\frac{3}{7}\right)^6 + \left(\frac{4}{9}\right)^6 + \dots\right]$  for convergence
- D** 4. Test the series  $\left[\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots\right]$  for convergence
- E** Find the interval of convergence of the series  $\left[3x + \frac{9}{2}x^2 + 9x^3 + \dots\right]$ .

**Question 5**
**15 marks**

- A** Determine the mass  $m$  and the Cartesian coordinates of the center of mass  $(\bar{x}, \bar{y})$  of a thin plate bounded by the parabola  $y = 2x - x^2$  and the straight line  $y = x$ , if the mass density of the plate is  $\rho(x, y) = 12x$ .
- B** Calculate the mass of a thin plate of density  $\rho = 3\sqrt{x^2 + y^2}$ . The plate has the shape of a circular annulus bounded by the two concentric circles  $x^2 + y^2 = 4$ ,  $x^2 + y^2 = 9$ .
- C** Evaluate the mass of the solid volume bounded by the coordinate planes and the planes:  $x = 1$ ,  $y = 1$ ,  $z = 1$ , where the density of the solid material  $\rho = x^2y$ .

This exam measures the following ILOs

Question Number	Q2-A	Q2-B	Q3-A	Q3-B	Q4-E	Q5-B	Q1-B		Q2-B	Q3-C	Q5-C
Skills	Q4	Q1-A									
	Knowledge & understanding skills				Intellectual Skills				Professional Skills		

*With our best wishes*
**Dr. Bilal Ali Maher**
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