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Nonlinear Modeling of Compression System and Stability with Recirculation

Aly El-Zahaby [1],El-Shenawy Abd-Elhameed[2], Zakarya Zyada[3], Khaled Sad Eldin[4] and Mohamed Salem [5]

1,2,3,4 Mechanical Power Engineering Department, Tanta University, Egypt
5 Mechanical Engineer., Gasco Company, Egypt
(Tel : +20-10-517-1393; E-mail: m_salem1969@yahoo.com)

ملخص البحث

النمذ جة اللاخطية والاستقرار لنظام الانضغاط بإعادة تدوير المانع

يستخدم النموذج الرياضي لنظام الانضغاط لتوضيح ظاهرتي عدم الاستقرار(تدهور الأداء والتموج)التي يتعرض إليها الضاغط أثناء تشغيله عند نقطة تشغيل معينة. في هذه المقالة نقدم النموذج الرياضي لنظام الانضغاط المقدم من **Moore and Greitzer (1986)** بعد تعديله لاستخدام تقنية رجوع المانع بعد خروجه من الضاغط وحقنة عند مدخل الضاغط للتغلب على مشاكل عدم الاستقرار , ويتم وضع النموذج في صورة مبسطة للحصول على الصورة المثلى القانون المستخدم للتحكم في تلك المشاكل . نتانج التمثيل الرياضي توضح فاعلية تلك التقنية في التحكم في تلك المشاكل.

Abstract

The model of compression system is used for capturing the dynamics of rotating stall and surge. In this paper, we introduce the modeling formulations of compression system after modifying the original work proposed by Moore and Greitzer (1986). Recirculation technique is used to eliminate the instabilities of compression system, where bled air from downstream compressor via bleed valves is re-circulated to injectors upstream of compressor inlet. The form of the modeling takes the four-state low order model which applied to design the nonlinear controller. The compressor works in stable regime by using recirculation technique. Lyapunov Control Function (LCF) is used to produce optimal control to stabilize the compressor operation by using recirculation technique, whereas simulation results show that it is a good technique. By using recirculation technique bleed valves requirements are reduced.

Keywords: Compression system, Compression system modeling with re-circulation, Surge control.

1. Introduction

Compressors are used, for example, as part of a gas a turbine for jet and marine propulsion or power generation, in superchargers and turbochargers for internal combustion engines, and in a wide variety of industrial processes. Rotating stall and surge are two aerodynamic instabilities in turbo compressors. It is clear that both phenomena can affect each other which give the simple fact that they occur in the same operating region of turbo-compressor. More specifically, surge can be regarded as the zeroth order eigenmode of the complete compression system while rotating stall is associated with higher order spatial modes inside the compressor [1]. The compression system model is used for capturing the dynamics of rotating stall and surge. Moore-Greitzer model [2] combined the partial differential equation (PDE) describing the flow field upstream of the rotor face and an ordinary differential equation (ODE) describing the overall pressure rise delivered by the compressor. The most common form of this model is a reduction of the PDE description to three coupled ODEs, the reduction is accomplished by assuming a Fourier series for the solution of the PDE and then truncating the series at a single mode. Modeling and active control of rotating stall and surge has been investigated by a number of researchers. Day [3] used the injection valves to damp out the stall cell after it has emerged. This gave an increase in stall margin of about six percent. In the case of surge, it was shown that this instability is preceded by a brief period of rotating stall, and by using the air injection scheme to eliminate the stall, and surge is also suppressed. The original Moore-Greitzer model [2] is developed by Behnken [4], and Yeung [5] to include air injection

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putting the nonlinear terms in their models. Injection can influence the overall behavior of the compressor in two ways. The first is simply the additional momentum added by the jets which increases the total pressure at rotor inlet. The second effect is a change in compressor performance which increases the pressure rise across the stage [6]. Implementing air injection is difficult because an independent source of compressed air is required. In the other side, Bleed valves present in modern compressors and no air supply required. Fontaine et al., [7] used Bernoulli's equation, Moore's model [8], and Moore-Greitzer model [2] proposed the model of compression system with bleed valves at inter- stage of an axial multistage compressor using Galerkin approximation. Yeung [5] used continuous air injection which serves to reduce the requirement of a bleed valves used for suppressing the rotating stall. Niazi [9] used self-recirculation casing treatment to alleviate the stall phenomenon, and extend the stable operating range by delaying the initiation of surge. In this paper we use recirculation technique for the following reasons: the first is that source of air injection doesn't depend on external source, secondly, instead of loosing the bled air, which is used in air injection, the third is that to keep the environment from pollution in case of using the compressor in pressurizing natural gas, and the fourth reason is that air injection doesn't need auxiliary system and consequently it needs low cost.

2. Model with recirculation

2.1 Moore - Greitzer model

The basic compression system in Fig (1) consists of inlet duct, inlet guide van, compressor, outlet duct, plenum, and throttle valve. From assumptions of Moore and Greitzer (1986) and applying Bernoulli's equation, the total-to-static pressure rise upstream of the compressor can be written as:

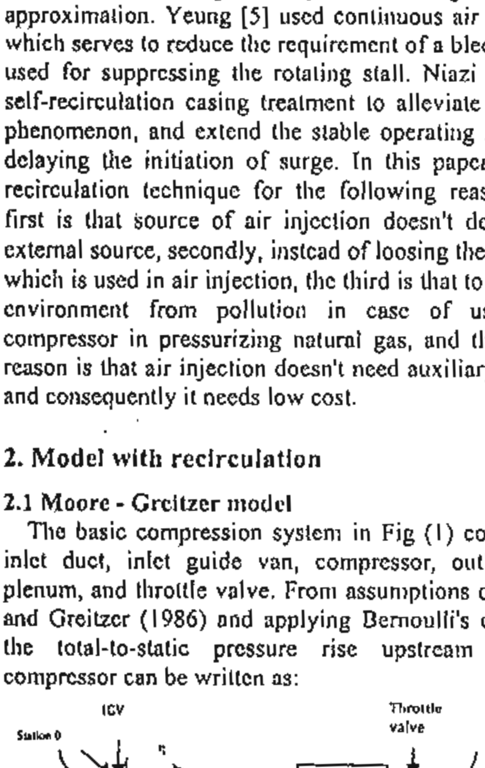

Figure (1): Scheme of compression system for modeling.

$$\frac{P_1 - P_T}{\rho u^2} = \frac{-\emptyset^2}{2} - l_I \frac{d\Phi}{d\zeta} - \frac{\partial \delta\emptyset}{\partial \zeta}, \quad (1)$$

$$\emptyset = \Phi + \delta\emptyset, \quad (2)$$

$$\zeta = tU/R. \quad (3)$$

Where P_i is the static pressure at station (i) where i=1,2,.. , P_T is the total pressure at ambient condition ,Φ is the non-dimensional mean axial velocity, ρ, is the density ,$\delta\emptyset$ is the non-dimensional axial velocity perturbation , $\emptyset = \frac{C_X}{U}$ is the non-dimensional local axial velocity ,ζ is the non-dimensional time , U is the Wheel speed at mean diameter , R is the mean rotor radius , t is the time , C_X is the local axial velocity, and $l_I = L_I/R$ is the non-dimensional length of inlet duct.

2.1.1 Compressor

The compressor is modeled as a semi-actuator disc such as:

$$\frac{P_2 - P_T}{\rho u^2} = F(\emptyset) - \lambda\frac{\partial\emptyset}{\partial\theta} - \mu\frac{\partial\emptyset}{\partial\zeta}, \quad (4)$$

$$\Psi_C(\emptyset) = F(\emptyset) - \frac{\emptyset^2}{2}. \quad (5)$$

where F ($\emptyset$) is the nonlinear pressure rise map for the compressor as a function of the axial flow, θ is the circumferential angle, μ is the fluid inertia parameter, λ is the rotor inertia parameter.

Downstream dynamics of compressor is given as:-

$$\frac{P_3 - P_2}{\rho u^2} = -l_E\frac{d\Phi}{d\zeta} - (m-1)\frac{\partial\delta\emptyset}{\partial\zeta}. \quad (6)$$

where $l_E = L_E/R$ is the non-dimensional length of exit duct, m is compressor – duct flow parameter, m = 2 (for long enough exit duct), and m = 1 (for a very short one). By defining:

$$\Psi = \frac{P_3 - P_T}{\rho u^2}. \quad (7)$$

where Ψ is plenum pressuer rise coefficient.

From the upstream, compressor and downstream dynamic the following PDE is obtained:

$$\Psi = \Psi_C(\Phi + \delta\emptyset) - l_c\frac{d\Phi}{d\zeta} - \lambda\frac{\partial\delta\emptyset}{\partial\theta} - \mu\frac{\partial\delta\emptyset}{\partial\zeta} - m\frac{\partial\delta\emptyset}{\partial\zeta}, \quad (8)$$

$$l_c = l_E + l_I + \mu. \quad (9)$$

where l_I, l_E, l_T, l_c are length of entrance ,exit, throttle, and effective length of setup divided by wheel radius respectively.

Choosing:-

$$\delta\emptyset = A_1(\zeta)\sin(\theta - \zeta). \quad (10)$$

where A_1 is the amplitude function of first-harmonic angular disturbance and computing the annulus-average, sine and cosine moments of equation (8) gives rise to a set of equations describing the time evolution of Φ, $J = A_1{}^2$ and r these equations are as follows

$$\frac{d\Phi}{d\zeta} = \frac{1}{l_c}\left[\Psi_C(\Phi) - \Psi + \frac{J}{4}\frac{\partial^2\Psi_C(\Phi)}{\partial\Phi^2}\right], \quad (11)$$

$$\frac{dJ}{d\zeta} = \frac{2}{m+\mu}J\left[\frac{\partial\Psi_C(\Phi)}{\partial\Phi} + \frac{J}{8}\frac{\partial^2\Psi_C(\Phi)}{\partial\Phi^2}\right], \quad (12)$$

$$\frac{dr}{d\zeta} = \lambda(m+\mu). \quad (13)$$

which describe that the stall of compressor rotor is at a constant speed.

2.1.2 Plenum Mass Balance

Balancing the mass going Into, leaving and being stored In the plenum gives:

$$\frac{d\Psi}{d\zeta} = \frac{1}{4l_cB^2}(\Phi - \gamma\sqrt{\Psi}), \quad (14)$$

$$B = \frac{U}{2a_S}\sqrt{\frac{V_P}{A_cL_c}}. \quad (15)$$

where B denotes the Greitzer parameter, a_S is the speed of sound, V_P is the volume of the plenum, A_c is the compressor duct area, $Lc = l_c R$ is the aerodynamics length of the system, and γ is the throttle coefficient.

2.1.3 Throttle valve

The throttle characteristic is taken as:

$$\Phi_T = \gamma\sqrt{\Psi}. \quad (16)$$

Where Φ_T is the throttle flow coefficient.

2.1.4 Characteristics

The compression system characteristics are taken form [2]. The usual 3rd order polynomial steady compressor is employed:

$$\Psi_c(\Phi) = \Psi_{c0} + H\left(1 + \frac{3}{2}\left(\frac{\Phi}{W} - 1\right) - \frac{1}{2}\left(\frac{\Phi}{W} - 1\right)^3\right). \quad (17)$$

where the parameters Ψ_{c0}, H and W are defined in [2]. The compressor characteristics can also be plotted as a family of curves. One line for each rotational speed, with the surge line passing through the local maximum of the constant speed lines. Using the non-dimensionalizing employed here and when the compressor is in rotating stall the characteristics is given by,

$$\Psi_s(\Phi) = \Psi_{c0} + H\left(1 - \frac{3}{2}\left(\frac{\Phi}{W} - 1\right) + \frac{5}{2}\left(\frac{\Phi}{W} - 1\right)^3\right). \quad (18)$$

2.1.5 Final form of the model

Collecting the equations (11), (12), and (14), the following 3-state system is used as a low order model for stall and surge.

$$\dot{\Phi} = \frac{1}{l_c}\left[\Psi_c(\Phi) - \Psi + \frac{1}{4}\frac{\partial^2\Psi_c(\Phi)}{\partial\Phi^2}\right]. \quad (19)$$

$$\dot{\Psi} = \frac{1}{4l_c B^2}(\Phi - \Phi_T), \quad (20)$$

$$\dot{J} = \frac{2}{m+\mu}J\left[\frac{\partial\Psi_c(\Phi)}{\partial\Phi} + \frac{J}{8}\frac{\partial^3\Psi_c(\Phi)}{\partial\Phi^3}\right]. \quad (21)$$

2.2 Moore - Greitzer Model with Air Injection

There are two primary effects caused by adding air injection to compression system [4], the first one is a shift in the steady-state compressor performance characteristics and the second is momentum addition effects. The physical effect for adding air injection into the model as a shift to the steady-state compressor characteristics based on experimental results is presented by D'Andrea et al., [10], where air injection actuators substantially shifted the compressor performance characteristics. The idea which was proposed by Gysling et al., [11] is that an actuator could affect the pressure rise delivered by the compressor. On the Caltech rig, the shift is used primarily to model the swirl (angular momentum) added by the air injection actuators. In addition to the swirl component, air injection also adds mass and momentum to the compressor inlet. In Fig. (2), the full order model can be approximated by a set of ordinary differential equations using Galerkin's method.

The differential equations describing the total and static pressure based on a potential flow solution in inlet duct may be given as follows;

$$\left[\left(\frac{2}{n} + \mu\right)\frac{\partial}{\partial\zeta} + \lambda\frac{\partial}{\partial\theta}\right]\left(\delta\emptyset_1 + \frac{\delta L_j}{L_a}\Phi_j\right) = \Psi_{c(effective)} - \Psi - l_c\frac{d\Phi}{d\zeta}, \quad (22)$$

$$\Psi_{c(effective)} = \Psi_c\left(\Phi + \delta\emptyset_1 + \frac{\delta L_j}{L_a}\Phi_j\right) + \left(\Phi_j - \Phi + \delta\emptyset_1\right)\frac{\delta L_j}{L_a}\Phi_j - \frac{1}{2}\left(\frac{\delta L_j}{L_a}\Phi_j\right)^2. \quad (23)$$

$$\delta\emptyset_1 = \emptyset - \Phi = \sum_{n=1}^{\infty} a_n\cos(n\theta) + b_n\sin(n\theta). \quad (24)$$

where $\delta\emptyset_1$ denotes the asymmetric component of $\emptyset$, a_n and b_n are the Fourier coefficients of $\emptyset$.

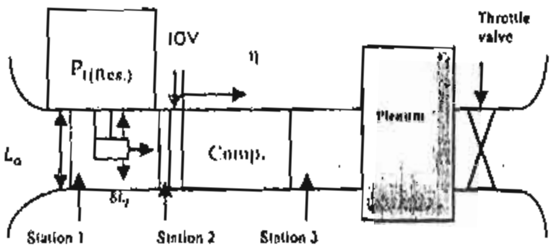

Figure (2): Scheme of compression system with air injector

The solution is first written as the linear combination of once more assumed basis functions and substituted into the partial differential equation. The resulting residual error ($\in$) becomes:

$$\epsilon = \dot{a}\cos\theta + \dot{b}\sin\theta - \lambda(a\cos\theta - b\sin\theta) + \Psi_{c(effective)}. \quad (25)$$

This residual error is minimized by making it orthogonal to each of the assumed basis functions. For the rotating stall problem, the flow field's spatial Fourier coefficients are convenient basis functions for the projection consider the first spatial harmonic (n=1). The residual is minimized by making it orthogonal to each basis function $\cos(\theta)$, *and* $\sin(\theta)$.

The Galerkin approximation is calculated using the weight functions

$$h_1 = 1, h_2 = \sin\theta, h_3 = \cos\theta, \quad (26)$$

$$\langle\epsilon|h_i\rangle = \frac{1}{2\pi}\left[\int_0^{2\pi}[\epsilon h_i]\,d\theta\right]. \quad (27)$$

Calculating the moments

$$M_i = \langle\epsilon|h_i\rangle = 0 \quad for\ i = 1,2,3 \quad (28)$$

The four-state Galerkin model can be written as

$$\dot{\Phi} = \frac{1}{2\pi L_c}\left[\int_0^{2\pi}\left[\Psi_{c(effective)} - \Psi\right]d\theta\right], \quad (29)$$

$$\dot{\Psi} = \frac{1}{4l_c B^2}\left(\Phi + \frac{\delta L_j}{L_a}\Phi_j - \Phi_T\right). \quad (30)$$

$$\dot{a} = \frac{1}{m+\mu}\left[-\lambda b + \frac{1}{\pi}\int_0^{2\pi}\left[\Psi_{c(effective)}\cos\theta\, d\theta\right]\right], \quad (31)$$

$$\dot{b} = \frac{1}{m+\mu}\left[\lambda a + \frac{1}{\pi}\int_0^{2\pi}\left[\Psi_{c(effective)}\sin\theta\, d\theta\right]\right]. \quad (32)$$

After integration, the modeling of compression system with air injection is developed in the following form:-

$$\dot{\Phi} = \frac{1}{l_c}\left[[\Psi_c(\Phi) + \frac{1}{4}\Psi_c^{''}(\Phi)(a^2+b^2) - \Psi] + [\Psi_c^{'}(\Phi) + \frac{1}{4}\Psi_c^{'''}(\Phi)(a^2+b^2) + \Phi_j - \Phi]\frac{\delta L_j}{L_a}\Phi_j\right], \quad (33)$$

$$\dot{\Psi} = \frac{1}{4l_cB^2}\left(\Phi + \frac{\delta L_j}{L_a}\Phi_j - \Phi_T\right), \quad (34)$$

$$\dot{a} = \frac{1}{m+\mu}\left[-\lambda b + [\Psi_c^{'}(\Phi) + \frac{1}{8}\Psi_c^{'''}(\Phi)(a^2+b^2)]a + \frac{\delta L_j}{L_a}\Phi_j a(\Psi_c^{''}(\Phi)+1)\right], \quad (35)$$

$$\dot{b} = \frac{1}{m+\mu}\left[\lambda a + [\Psi_c^{'}(\Phi) + \frac{1}{8}\Psi_c^{'''}(\Phi)(a^2+b^2)]b + \frac{\delta L_j}{L_a}\Phi_j b(\Psi_c^{''}(\Phi)+1)\right]. \quad (36)$$

where $\Psi_c^{'}(\Phi), \Psi_c^{''}(\Phi), \Psi_c^{'''}(\Phi)$are the first, second , and third derivative of $\Psi_c(\Phi)$ with respect to Φ.

2.3 Moore-Greitzer Model with bleed valve

Full order model of compression system with bleed valve according to Fontaine [7] is given as follows

$$\dot{\Phi} = \frac{1}{l_c}\left[\Psi_c(\Phi) + \frac{1}{4}\Psi_c^{''}(\Phi)(a^2+b^2) + \Phi\Phi_b - \Psi + \frac{1}{2}(aC_a + bC_b)\right]. \quad (37)$$

$$\dot{\Psi} = \frac{1}{4l_cB^2}(\Phi - \Phi_b - \Phi_T), \quad (38)$$

$$\dot{a} = \frac{1}{m+\mu}\left[-\lambda b + (\Psi_c^{'}(\Phi) + \frac{1}{8}\Psi_c^{'''}(\Phi)(a^2+b^2) + \Phi_b)a + \Phi C_a\right], \quad (39)$$

$$\dot{b} = \frac{1}{m+\mu}\left[\lambda a + (\Psi_c^{'}(\Phi) + \frac{1}{8}\Psi_c^{'''}(\Phi)(a^2+b^2) + \Phi_b)b + \Phi C_b\right]. \quad (40)$$

2.4 Modeling of recirculation

As indicated in Fig. (3), air is re-circulated from the compressor downstream to injector at upstream of compressor. In this case, we assume $\Phi_b = \Phi_j$, $\frac{\delta L_j}{L_a} = 1$, from section (2-2), and section (2-3), the low order model for air recirculation is developed in the form :

$$\dot{\Phi} = \frac{1}{l_c}\left[\Psi_c(\Phi) + \frac{1}{4}\Psi_c^{''}(\Phi)(a^2+b^2) - \Psi + (\Psi_c^{'}(\Phi) + \frac{1}{4}\Psi_c^{'''}(\Phi)(a^2+b^2) + \Phi_b)\Phi_b + \frac{1}{2}(aC_a + bC_b)\right], \quad (41)$$

$$\dot{\Psi} = \frac{1}{4l_cB^2}(\Phi - \Phi_T), \quad (42)$$

$$\dot{a} = \frac{1}{m+\mu}\left[-\lambda b + (\Psi_c^{'}(\Phi) + \frac{1}{8}\Psi_c^{'''}(\Phi)(a^2 + b^2) + 2\Phi_b + \Phi_b(\Psi_c^{''}(\Phi)))a + \Phi C_a\right]. \quad (43)$$

$$\dot{b} = \frac{1}{m+\mu}\left[\lambda a + (\Psi_c^{'}(\Phi) + \frac{1}{8}\Psi_c^{'''}(\Phi)(a^2 + b^2) + 2\Phi_b + \Phi_b(\Psi_c^{''}(\Phi)))b + \Phi C_b\right]. \quad (44)$$

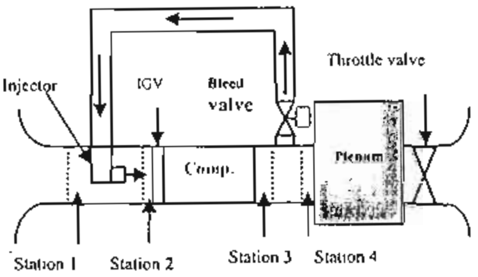

Figure (3): Scheme of compression system with recirculation

2.4.1 Surge subsystem and stall subsystem

We refer to equations (41)-(42) as the surge subsystem .In the surge subsystem, the characteristic $\Psi_c(\Phi)$ is the locus of equilibrium values for the mass flow parameter Φ and the pressure rise parameter Ψ. A desired equilibrium is selected by adjusting the throttle coefficient γ. Decreasing the value of γ corresponds to closing the throttle valve, which decreases the mass flow and increases the pressure rise. The cubic characteristic $\Psi_c(\Phi)$ in fig. (4) has one local maximum (peak). To the right of the peak, the equilibrium is locally stable without feedback. To the left of the peak, the equilibrium is unstable. The surge subsystem describes the fluid in plenum as a nonlinear spring. Under certain conditions, compressor enters a surge limit cycle and undergoes large oscillations in Φ and Ψ that often include negative values of Φ. In the stall subsystem equations (43)-(44), to the left of the peak, the slope of the characteristic is positive. The resulting instability is known as "rotating stall" involves oscillations in a, and b which are damaging to the engine and reduce the pressure rise across the compressor.

3. Stability and control design

3.1 Design objective

When the mass flow rate of the compressor is reduced due to closing the throttle valve, this causes two flow instabilities (rotating stall and surge).Without an effective feedback controller, the effect of these instabilities may drive the compressor into a region from which it does not return to a desired steady–state operation. The main task of a feedback controller is to enlarge the region of the asymptotically stable equilibrium which represents the desired steady–state compressor operation. Our desired equilibrium is the highest pressure rise at the peak of the characteristic. We design linear and nonlinear control laws to assess the controller's ability to maintain this equilibrium. By using these controllers we can move the stall-surge inception point to lower value of (Φ), thereby increasing the operating range of the compressor and consequently optimizing steady–state performance.

3.2 Lyapunov stability analysis

This section illustrates the Lyapunov stability proof for the arbitrary order Moore - Greitzer model and shows how controllers can be derived using this theory. Consider the state error vector $e = x - x_{eq}$ where x_{eq} is the desired equilibrium point. A simple candidate Lyapunov function (V) in the absence of actuator is stated. This function measures the total perturbation kinetic and potential energy stored in the compression system. When the actuator is included, this Lyapunov function measures the perturbation energy in an "equivalent compressor" that accounts for the effects of actuator on the compressor state. The perturbation energy is guaranteed to go to zero if the perturbation power, $\dot{V}$ is negative definite.

3.3 Basic controllability

Let us consider the issue of controllability. In finite dimensional control theory, a system is said to be controllable if for every two points $x_0, x_1 \in X$ and every two real numbers $t_0 < t_1$, there exists a control function (u) such that the unique solution of the equation:

$$\dot{x} = f(x, u), x(t_0) = x_0, \qquad (45)$$

Satisfies $x(t_1) = x_1$

3.4 Linearization

There are many ways to design control laws to achieve $\dot{V} < 0$ in the vicinity of the equilibrium point to enlarge the domain of attraction, and to make $\dot{V}$ more negative to increase robustness to modeling errors. One way to impose a negative definite $\dot{V}$ is to cancel the dynamics introduced by the compressor characteristic and replace them with some desired linear dynamics. The form of linear system takes the following form:

$$\dot{x} = Ax + B. \qquad (46)$$

Where A, and B are the matrices of the linearization of equations (41-44) appear in a decoupled block diagonal structure,

$$A = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}, B = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \qquad (47)$$

This allows for decomposition into two subsystems. axisymmetric control variable Φ_b controls Φ, and Ψ in the linearized surge subsystem where

$$A_1 = \begin{bmatrix} \frac{\Psi_c(\Phi_e)}{l_c} & \frac{-1}{l_c} \\ \frac{1}{4l_cB^2} & \frac{\gamma}{8l_cB^2\sqrt{\Psi_e}} \end{bmatrix}, B_1 = \begin{bmatrix} \frac{\Psi_c'(\Phi_e)}{l_c} & 0 \\ 0 & 0 \end{bmatrix}, \qquad (48)$$

Surge subsystem alone is controllable at all equilibrium points on the compressor characteristic.

The stall subsystem by linearization:

$$A_2 = \begin{bmatrix} \frac{\Psi_c'(\Phi_e)}{m+\mu} & \frac{-\lambda}{m+\mu} \\ \frac{\lambda}{m+\mu} & \frac{\Psi_c'(\Phi_e)}{m+\mu} \end{bmatrix}, B_2 = \begin{bmatrix} \frac{\Phi_e}{m+\mu} & 0 \\ 0 & \frac{\Phi_e}{m+\mu} \end{bmatrix}. \qquad (49)$$

Is controlled by C_a, and C_b, see ref. [7].

3.5 Surge control design

Compressor's steady- state performance is determined by the operating point on the characteristic $\Psi_c(\Phi)$. The highest pressure rise Ψ, and hence, the best steady- state performance is achieved at the peak of $\Psi_c(\Phi)$. However, the peak is not a stable equilibrium of the uncontrolled system. Our design objective is to stabilize the peak and provide as large a region of attraction as possible.

3.5.1 Linear control Design: LQR Control

The control that minimizes the cost function $J = \int_0^\infty (x^TQx + u^TRu)\, dt$, is given by LQR design in ref.[12].

$$u = -R^{-1}B_1^TP_1^T\tilde{x}. \qquad (50)$$

Where P_1 is the positive definite solution of the Algebraic Ricatti Equation (ARE)

$$A_1^TP_1 + P_1A_1 - P_1B_1R^{-1}B_1^TP_1 + Q1 = 0, \qquad (51)$$

The solution P1 exists because the pair {A1 B1} is controllable for all operating points on the compressor characteristic $\Psi_c(\Phi)$. Thus, the LQR design results in local asymptotic stability of the surge subsystem.

3.5.2 Nonlinear control design

Many nonlinear control designs are introduced in ref. [13], and [14]. Because of simplicity of a nonlinear controller design, similar to [7], it is applied to produce the current surge control with recirculation which take the form

$$\dot{x}_1 = f_1 + g_1\Phi_b. \qquad (52)$$

Where

$$f_1 = \left[\frac{1}{l_c}(\Psi_c(\Phi) - \Psi) \quad \frac{1}{4l_cB^2}(\Phi - \Phi_T)\right], \qquad (53)$$

$$g_1^T = \begin{bmatrix} \frac{\Psi_c'(\Phi)}{l_c} & 0 \end{bmatrix}, \qquad \dot{x}_1^T = [\Phi \quad \Psi]. \qquad (54)$$

We now design a simple nonlinear controller to increase the negativity of $\dot{V}(x)$ for the same Lyapunov function $V(x) = x^TP_1x$. The region $\dot{V}(x) < 0$ is increased by selection of the control law [7] as follows:

$$u = -R^{-1}g_1^TP_1^T\tilde{x}_1. \qquad (55)$$

4. Simulation results

4.1 Surge model

In this section it, is illustrated that the developed model is capable of simulating surge. Results are presented from simulations of the compressor system when driven into surge by a drop in mass flow at $\gamma = 0.5$ which move the throttle characteristic to the left of the top of steady state characteristic as indicated in figure (4). Simulation parameters are shown in table (1) which are taken from [15]. The compressor under this condition undergoes deep surge with oscillations in mass flow, and pressure rise, as shown in figure (5), where the fluctuation of (Φ, Ψ) with respect to time, and the phase plan (Φ, Ψ), where surge cycle exist. The surge has oscillation with a period $\zeta \approx 500$. This is corresponding to surge frequency of about 4.4 Hz

4.2 surge control

By using the nonlinear control law the bleeding air is re-circulated to be injected at compressor's upstream. The compressor can operate in a stable mode even to the left of the surge line. In figure (6) mass flow drops as in the surge simulation is introduced at $t = 0.18$s, after that the compressor remains stable. This figure explains the stabilization of (Φ, Ψ) with respect to time and the phase plan (Φ, Ψ) where the operating point converges to equilibrium point.

Table, (1) numerical values simulations

Sym.	value	Sym.	value	Sym.	Value
R	0.1m	l_c	13.3	Ψ_{Co}	0.3
H	0.18	B	1.8	U	215m/s
W	0.25	W	0.25	γ	0.5

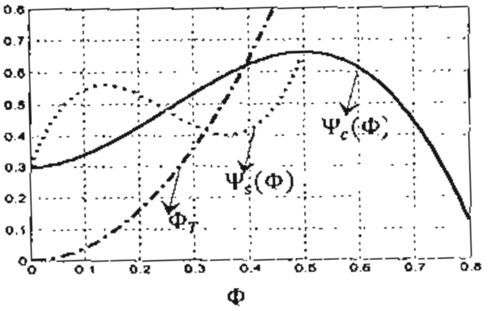

Figure (4): Compression system characteristic.

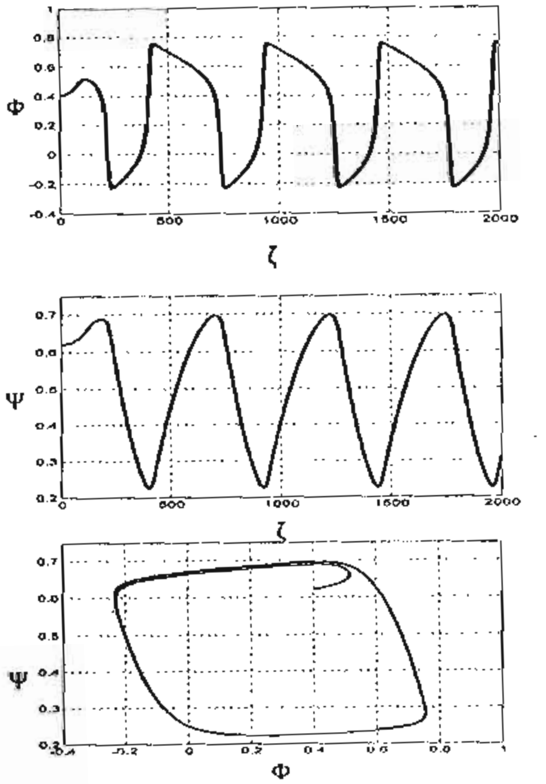

Figure (5): Simulation of sub-surge system, at B=1.8.

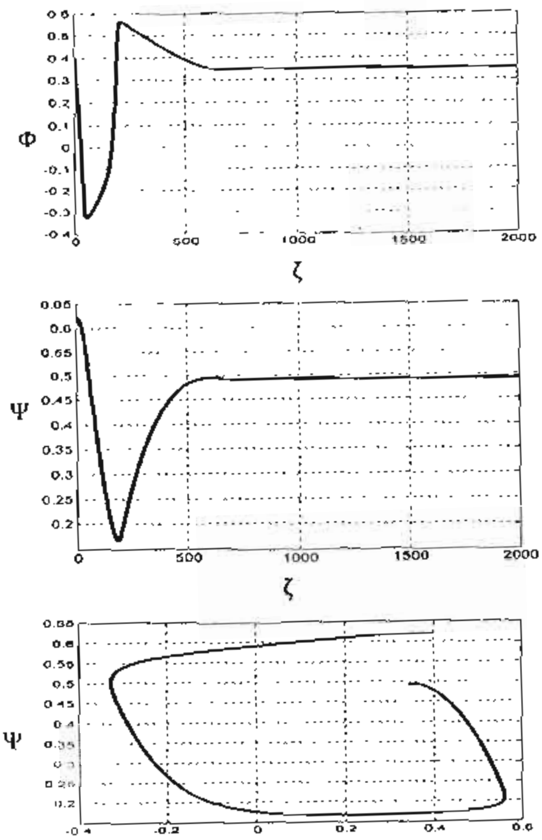

Figure (6): Stabilization of compressor using nonlinear control

5. Conclusions and future work

Moor-Greitzer compression system with re-circulation technique has been developed. Lyapunov function is applied to introduce linear and nonlinear optimal control to stabilize the compressor when it undergoes surge. Simulation results show that re-circulation technique is powerful for stabilizing the compressor at higher Greitzer parameter (B), where the operating point lies in the unstable regime. Future work includes: 1) use of Lyapunov function to introduce linear and nonlinear control design to stabilize the compressor when it undergoes a rotating stall, 2) investigate re-circulation technique for stabilizing the compressor and validating the results experimentally, 3) modeling of higher harmonics in rotating stall, which includes more terms in the Fourier series of flow coefficient fluctuation applying re-circulation technique.

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