

## INFLUENCE OF INLET VELOCITY ON THE PERFORMANCE OF RADIAL DIFFUSERS

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( Received July. 13, 1987, accepted Dec. 1987 )

تأثير سرعة دخول المائع على أداء الناشرات القطرية  
خلاصة: في الفترة الحالية هناك جهود ملموسة كرس لتطوير أداء الناشرات القطرية. هذا يتأتى بالحصول على التصميم المناسب. التحليل الذي اتبع في هذا البحث يعتمد أساساً على حل المعادلة التفاضلية لكمية الحركة للطبقة الجدارية المنضغطة المضطربة مع عمل بعض الفروض المحددة. فإنه من الممكن تمثيل معاملات انحدار السرعة على أنها تتوقف على عدد ثابت محدد والتي عليها يتوقف انفصال الطبقة الجدارية. هذا يساعد في إجراء حسابات الطبقة الجدارية ذات أقصى عجلة تقصيره وغير منفصله.

لهذه الحالة فإن الفروض التي اعتبرت فهي ان معاملات منحدر السرعة وهو رقم "أويلر"  $\Lambda$  له قيمة عددية للناشر القطري هي تقريباً 0.00094. كذلك سمك الازاحة الى معامل سمك كميعة الحركة  $\delta^*/\delta^{**}$  يتراوح بين 1.4 الى 1.6 لأقصى عجلة تقصيره. بذلك يمكن الحصول على معادلة رياضية لتوزيع السرعة ولكن يبين اهمية تطبيقها فقد تسورت ببعض النتائج العملية السابقة التي يمكن الحصول عليها. وقد بينت النتائج أنه يمكن الحصول على أقصى عجلة تقصيره كلما قل سمك كمية الحركة عند مدخل الناشر والذي يؤدي الى تحسين أدائه.

ABSTRACT - In recent years, considerable efforts have been devoted to improving the performance of radial diffusers. This can be achieved by obtaining a proper diffuser design.

The analysis followed in this paper is based upon solving the momentum equation of incompressible turbulent boundary layer under certain assumptions. It is possible to present the velocity profile parameters as dependent upon definite numbers, from which separation could result. This helps in solving attached boundary layers with maximum deceleration.

For the case in hand, the assumptions considered that the velocity profile parameter,  $\Lambda_{opt}$ , has a magnitude of about 0.00094, and  $\delta^*/\delta^{**}$  ranges between 1.4 to 1.6, for maximum deceleration [5].

To evaluate the mathematical model developed in this work, its results will be compared with previous available experimental data. This obtained results show that the maximum deceleration is obtained by decreasing the initial value of momentum thickness at diffuser entrance.

## NOMENCLATURE

$\bar{c}$	velocity at the outer edge of the boundary layer	(m/s)
$c_f$	local skin friction coefficient, $\tau_w / (1/2 \rho \bar{c}^2)$	
$c_\infty$	free stream velocity,	(m/s)
$c_x$	velocity of the fluid inside the boundary layer in x-direction,	(m/s)
$c_y$	velocity component inside the boundary layer in y-direction	(m/s)
$H_{12}$	boundary layer form parameter, $\delta^*/\delta^{**}$	
$Re_{\delta^{**}}$	momentum thickness Reynolds number, $\bar{c} \cdot \delta^{**} / \nu$	
x	coordinate in the direction of the wall,	(m)
y	coordinate normal to the direction of the wall,	(m)

$\delta$	boundary layer thickness,	(m)
$\delta^*$	boundary layer displacement thickness, $\int_0^{\delta} (1 - c_x/c) \cdot dy$	(m)
$\delta^{**}$	boundary layer momentum thickness, $\int_0^{\delta} (1 - c_x/\bar{c}) c_x/\bar{c} \cdot dy$	(m)
A	Euler number, $-\frac{1}{\bar{c}} \cdot \frac{d\bar{c}}{dx} \cdot \delta^{**}$	
$\nu$	kinematic viscosity of fluid,	(m <sup>2</sup> /s)
$\rho$	density of fluid,	(kg/m <sup>3</sup> )
$\tau_w$	wall shear stress,	(N/m <sup>2</sup> )

1- INTRODUCTION

Diffuser design has necessitated considerable efforts from engineers in order to increase pressure and reduce kinetic energy of flow in ducts. However, until 1950 diffuser design was a combination of art, luck, and a vast amount of empirical trial and error rules. Small changes in design parameters caused large changes in diffuser performance. That is increasing pressure in the diffuser is an unfavourable gradient. This causes the viscous boundary layers to break away from the walls and greatly reduces the diffuser performance.

Previous experimental work on radial diffuser contains information about diffuser efficiency and design [1:4].

An information about turbulent boundary layer by different roughnesses based on measurements on radial diffuser was given in [5]. This also was concerned with the dissipation of energy due to boundary layer growth in the diffuser. Moreover, diffuser efficiency by the different roughness was investigated.

In [6] a criteria for flow separation based on integrating the momentum equation of boundary layer was established.

This paper is concerned with developing a mathematical model for the velocity distribution in radial diffuser. This is based on the integration of the momentum equation of incompressible boundary layer under certain assumptions. The assumptions regards that the velocity profile parameter has an optimum value,  $\Lambda_{opt}$ , with a magnitude of about 0.00094, and the form parameter,  $H_{12} = \delta^*/\delta^{**}$ , ranges between 1.4 and 1.6 for maximum deceleration. To evaluate the mathematical model developed in this work, its result was compared with the only previous result available in hands, that given in [5]. The obtained results show that better diffuser performance was obtained by reducing initial velocity at diffuser inlet, or maximum deceleration by smaller initial value of momentum thickness.

2- GOVERNING EQUATIONS

The governing equation of mean motion for incompressible turbulent flow is the momentum integral equation [7] given as:

$$\frac{d\delta^{**}}{dx} + (2 + \frac{\delta^*}{\delta^{**}}) \frac{\delta^{**}}{\bar{c}} \frac{d\bar{c}}{dx} = \frac{\tau_w}{\rho \bar{c}^2} = \frac{C_f}{2} \quad \dots (1)$$

Equation (1) is normalized By using,

$$x^* = \frac{x}{x_0} ; \bar{c}^* = \frac{\bar{c}}{\bar{c}_0} ; \Delta = \frac{\delta^{**}}{x_0} ,$$

$x_0$  and  $\bar{c}_0$  being appropriate reference quantities. Thus,  $x_0$  would be the length from the center of the radial diffuser to the beginning of the parallel walls ;  $\bar{c}_0$  the component of velocity at the inlet by  $x_0$  at the edge of boundary layer. Fig. (1) illustrates the coordinate system used for the radial diffuser.

Equation (1) becomes:

$$\frac{d\Delta}{dx^*} + (2 + H_{12}) \frac{\Delta}{\bar{c}^*} \frac{d\bar{c}^*}{dx^*} = \frac{c_f}{2} \quad \dots (2)$$

The optimum value of Euler number,  $\Lambda_{opt} = -\frac{\Delta}{\bar{c}^*} \frac{d\bar{c}^*}{dx^*}$ , was found to be close to 0.00094

[5] and so one adopts this constant value of  $\Lambda$ , for the analysis presented here. Moreover, by this value of  $\Lambda$ , the local skin friction coefficient,  $c_f$ , is very small and diminishes, and the form parameter,  $H_{12} = \delta^*/\delta^{**}$ , has a magnitude of 1.4 to 1.6 [5], so that equation (2) can be integrated with  $c_f \rightarrow 0$  and  $\Lambda_{opt}$  and  $H_{12}$  are constants. Equation (2) therefore yields

$$\frac{d\Delta}{dx^*} + (2 + H_{12}) (-\Lambda_{opt}) = 0 \quad \dots (3)$$

Separating the variables and integrating, this equation (3) one gets:

$$\Delta = (2 + H_{12}) \Lambda_{opt} \cdot x^* + C, \quad \dots (4)$$

where  $C = \Delta_0$ , since  $\frac{\delta^*}{\delta^{**}} = \text{const}$  and  $c_f \rightarrow 0$  when the first term on the right hand side of equation (4) is equal to zero. Then the solution of Eq. (4) is given as

$$\frac{\delta^{**}}{x_0} = (2 + \frac{\delta^*}{\delta^{**}}) (\Lambda_{opt}) \frac{x}{x_0} + \frac{\delta_0^{**}}{x_0} \quad \dots (5)$$

Recalling the integral definition of equation (1), and substituting for  $c_f = 0$ , this yields

$$\frac{d\delta^{**}/x_0}{d\bar{c}/\bar{c}_0} = - (2 + \frac{\delta^*}{\delta^{**}}) \frac{d\bar{c}/\bar{c}_0}{dx/x_0} \frac{\delta^{**}}{x_0} \quad \dots (6)$$

Rearranging equation (6) by separating the variables and integrating one gets,

$$\ln \left( \frac{\delta^{**}}{x_0} \right) = - \frac{(2 + \frac{\delta^*}{\delta^{**}})}{A} \ln \frac{\bar{c}}{\bar{c}_0} + \ln C \quad \dots (7)$$

or

$$\ln \left( \frac{\delta^{**}}{x_0} \right) = - C \ln \left( \frac{\bar{c}}{\bar{c}_0} \right)^A, \quad \dots (7')$$

where  $C = \frac{\delta_0^{**}}{x_0}$ , since  $\bar{c} = \bar{c}_0$  at  $x = x_0$

Therefore  $\frac{\delta^{**}}{x_0} = \left( \frac{\bar{c}}{\bar{c}_0} \right)^{-A} \cdot \left( \frac{\delta_0^{**}}{x_0} \right) \quad \dots (8)$

Equation (5) and (8) are two independent expressions for  $\delta^{**}/x_0$ . Combining these two equations (5), (8) and one gets a simple equation,

$$\left( \frac{\bar{c}}{\bar{c}_0} \right)^2 = \left[ 1 + (2 + H_{12}) \Lambda_{opt} \frac{x/x_0}{\delta^{**}/x_0} \right]^{-\frac{2}{(2 + H_{12})}} \quad \dots (9)$$

using a mean value for  $H_{12} = 1.5$  and  $\Lambda_{opt} = 0.00094$  and substituting in equation (9), the following form for the velocity distribution, or the recovery factor, is

$$\left( \frac{\bar{c}}{\bar{c}_0} \right)^2 = \left[ 1 + 0.0033 \frac{x/x_0}{\delta^{**}/x_0} \right]^{-0.5714} \quad \dots (10)$$

Equation (10) is used in the analysis to obtain the effect of the initial velocity on the diffuser performance.

## 3- PRESENTATION OF RESULTS AND DISCUSSIONS

The two equation formerly obtained ;

$$\frac{\delta^{**}}{x_0} = \frac{\delta_0^{**}}{x_0} + 0.0033 \frac{x}{x_0} \quad (5)$$

and

$$\left(\frac{\bar{c}}{\bar{c}_0}\right)^2 = \left[ 1 + 0.0033 \frac{x/x_0}{\delta_0^{**}/x_0} \right]^{-0.5714} \dots (10)$$

are evaluated and plotted in figures 2 and 3. Equation (5) emphasises that the momentum thickness increases linearly. In figure (2) indicates the linear increase in momentum thickness which obtained from equation (5).

Equation (10) for the velocity distribution shows that the velocity at the boundary layer edge decreases according to a power function. This relation given by equation (10) is illustrated in figure (3). Moreover, figure (3) shows the marked effect of the initial value of momentum thickness (suffix 0), while smaller thicknesses of boundary layers results in values of higher deceleration than these corresponding to thicker values.

## CONCLUSION

The conclusion to be drawn from the analysis is that the growth of the momentum thickness of boundary layer at the inlet of the radial diffuser has a marked influence on its performance. That is, maximum deceleration is obtained by decreasing the initial value of momentum thickness.

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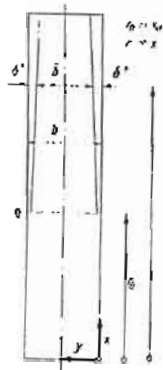


FIG. 1 Illustration of coordinate system on radial diffuser

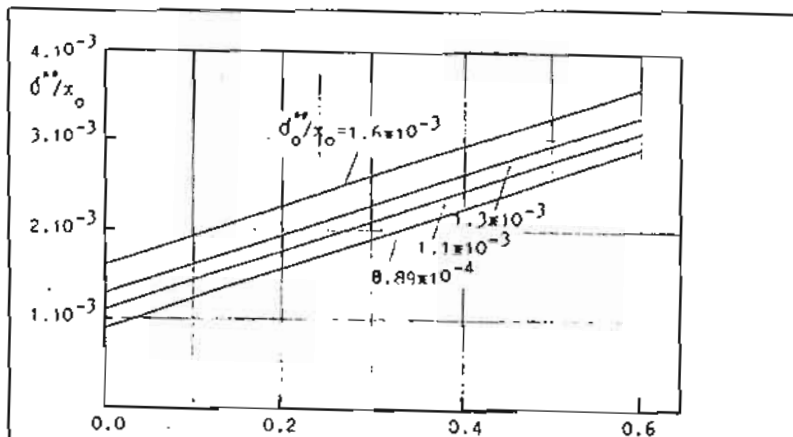


FIG. 2 Incompressible turbulent boundary layer with  $c_f \rightarrow 0$ , momentum thickness versus  $x/x_0$

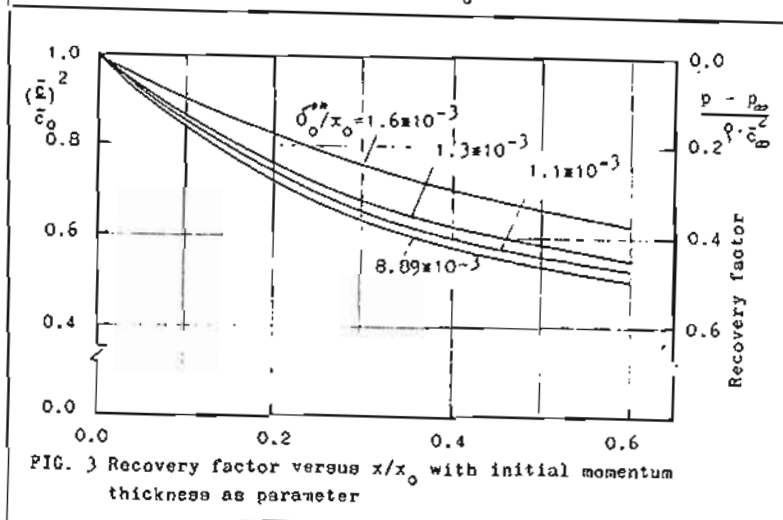


FIG. 3 Recovery factor versus  $x/x_0$  with initial momentum thickness as parameter