VELOCITY PROFILES IN THE TRANSITION LENGTH OF A RECTANGULAR TUBE

منحنيات السرعة فى منطقة الدخول الهيدروديناميكى للمواسير ذات القطاعات المستطيلة

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ملخص : يتناول هذا البحث تقنية حديثة لحساب منحنيات السرعة فى منطقة الدخول الهيدروديناميكى للمواسير ذات القطاعات المربعة والمستطيلة، عن طريق حل معادلة نافيه – ستوكس وهذه التقنية سهلة التطبيق والحسابات. وتتلخص هذه التقنية فى عمل تعويض عن المقطع المستطيل بمقطع دائرى مكافئ ، ثم رسم منحنيات السرعة للمقطع الدائرى المكافئ ، وبعدها يتم عمل نقل مع حفظ الزوايا لمنحنيات السرعة للمقطع الدائرى المكافئ للحصول على منحنيات السرعة للمقطع المستطيل.

ABSTRACT :A new ad hoc technique for solving the Navier - Stokes' equation and determining the velocity profiles through a rectangular tube transition length, is introduced. This technique is very simple in application and in calculations.

NOMENCLATURE

x,y,z	= Rectangular coordinates.
p	= Static pressure.
g	= Gravitational acceleration.
a	= Width of rectangular section.
b	= Height of rectangular section.
ρ	= Density of the liquid.
υ	= Kinematics viscosity of the fluid = μ/ρ.
μ	= Dynamic viscosity.
u	= Velocity
u_r	= Fully developed velocity in rectangular section..
u_c	= Fully developed velocity in equivalent circular section.
u_{co}	= Entrance velocity.
L_T	= Transition length.

INTRODUCTION

When an incompressible fluid flows in a pipe, the wall shear stress is very large at the entrance and generally decreases in the direction of flow to a fixed value The velocity profile also changes, then it reaches a fixed profile. When the wall shear stress, and the velocity profile reach constant conditions, the flow is called fully developed flow

In order to obtain the velocity profiles in the transition zone, the Navier-Stokes' equation must be solved, which is a non-linear partial differential equation, no general exact solution is obtainable for it Exact solution have been obtained for only a limited number of laminar flow situations for which some terms of the Nervier-Stokes' equation are neglected, the integration is then possible

Langhaar [1], solved the Navier-Stokes' equation for the case of the steady flow in the transition length of straight circular tube, by means of a linearizing approximation. Langhaar defined the family of velocity profiles by Bessel functions. Sabry [3], presented a high complicated, mathematical concepts to study the flow field in the entrance zone of a rectangular duct, by using regular reductive integral equations method therefore, the objective of the present study is to fined a new technique to study the flow patterns in the entrance zone of a rectangular tube by means of solving the Navier - Stockes' equation

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This new technique makes it possible to obtains the velocity profile at any section of the rectangular tube throughout the transition length by a very simple mathematical tools.

FORMULATION AND SOLUTION OF THE PROBLEM

The Navier-Stokes' equation is :

$$\frac{\partial u}{\partial t} = g_z - \frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \quad \ldots (1)$$

For a flow field in the entrance zone of a rectangular tube, the following boundary conditions can be consedered

$u = 1$	at	$z = 0$
$u = 0$	at	$y = \pm b$
$u = 0$	at	$x = \pm a$
$u = U_r$	at	$z = L_T$

It is required to obtain the velocity profiles through the different sections along the transition length. A new technique was developed, this technique can be summarized in the following steps

1- Substituting the rectangular section to an equivalent circular section,

2- Drawing the velocity curves for the equivalent circular section,

3- Determining the fully developed velocity for the rectangular section, and

4- Conformal mapping.

1 - Substituting the rectangular section to an equivalent circular section :

Substitute the rectangular section by an equivalent circular section having the same hydraulic radius and Reynolds number.

2 - Drawing the velocity curves for the equivalent circular section :

The velocity curves for the equivalent circular section can be easily drawn along the tube at different section, for example see [1] and [4].

3 - Determining the fully developed velocity for the rectangular section :

For the case of steady flow $\partial u/\partial t = 0$. In the region in which flow is fully developed $\partial p/\partial z$ is constant and the term $\partial^2 u/\partial z^2$ can be neglected.

Then the Navier-Stokes' equation becomes :

$$\rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 u_r}{\partial x^2} + \frac{\partial^2 u_r}{\partial y^2}\right) = 0 \quad \ldots (2)$$

Owing to the symmetry about x and y axes. It is easier to consider one quadrant. If the equation is dimensionless, the solution will be applied to all geometrically similar cross sections

A coordingly, it is important to introduce the following non dimensional variables :

$$x_1 = \frac{x}{b/2} \qquad (3\text{-a})$$

$$y_1 = \frac{y}{b/2} \qquad (3\text{-b})$$

$$v = \left(\frac{\mu}{(b^2/4)(\rho g_z - \partial p/\partial z)}\right) u_r \qquad (3\text{-c})$$

Then by substituting x, y, and u_r as given above into eq. (2) yields

$$\frac{\partial^2 v}{\partial^2 x_1^2} + \frac{\partial^2 v}{\partial^2 y_1^2} = -1 \qquad (4)$$

Subjected to the boundary conditions :-

$0 \le x_1 \le a/b$

$0 \le y_1 \le 1$

Finite element method will be employed to solve equation (4) by using FORTRAN computer program and the velocity (v_{ij}) at any point (i,j) will be determined. To determine, the fully developed velocity ($u_{r_{ij}}$) at any point (i,j) the value ($\rho g_z - \partial p/\partial z$) is needed, and this term can be evaluated

from : $$\rho g_z - \frac{\partial p}{\partial z} = \frac{4\mu}{b^2} \frac{\bar{u}_r}{\bar{v}} \qquad (5)$$

where :

$\bar{u}_r$*: average velocity = flow rate/area.*

$\bar{v}$ *: average* (v) over the cross sections.

Finally, knowing ($\bar{u}_r$) ,and ($\bar{v}$) we can get $\rho g_z - \frac{\partial p}{\partial z}$ from equation (5). And from equation (3-c) we can get $u_{r_{ij}}$.

4 - CONFORMAL MAPPING

Since the entrance velocity (u_{co}) for the two sections (rectangular and circular) are equal, fully developed velocity for the two sections (rectangular and circular), and transition length were calculated, the rectangular velocity curves can be obtaianed from circular velocity curves by transformation mapping of the Z-plane (circular axial velocity) to the W-plane (rectangular axial velocity). These transformation is illustrated in figure (1).

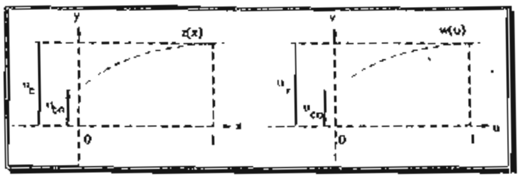

Figure (1) : Conformal Mappings

where :

$$Z(x) = x + y\,i \quad \text{..........} \quad (6)$$

$$w = az + b \quad \text{..........} \quad (7)$$

$$w = u + iv = (a_x + a_y\, i)(x + y\, i) + (b_x + b_y\, i) \quad \text{..........} \quad (8)$$

By substituting (a_x, a_y, b_x, b_y) and (x, y) in equation (8) the velocity curves can be easily drown along the rectangular tube at any point of section. After that, the velocity profiles can be gotten at any section along the tube by driving it from the velocity curves results pervious. Figures from (2) to (9) illustrate the velocity profiles for different rectangular sections (calculated by the new technique).

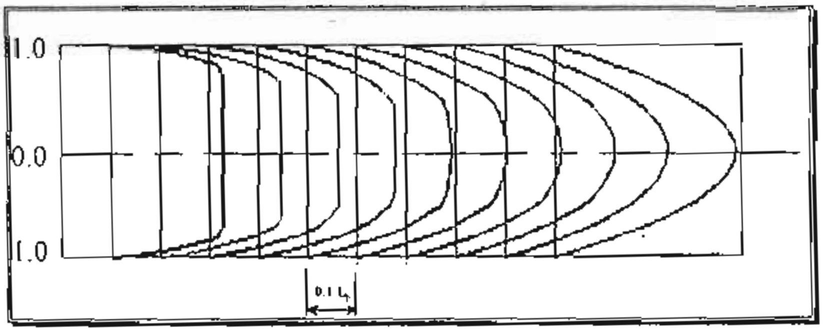

Figure (2) : Velocity profiles computed by the new technique, for (a/b = 4)

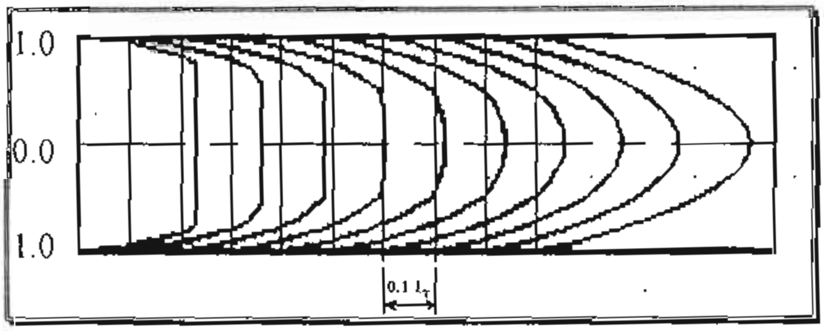

Figure (3) : Velocity profiles computed by the new technique, for (a/b = 3)

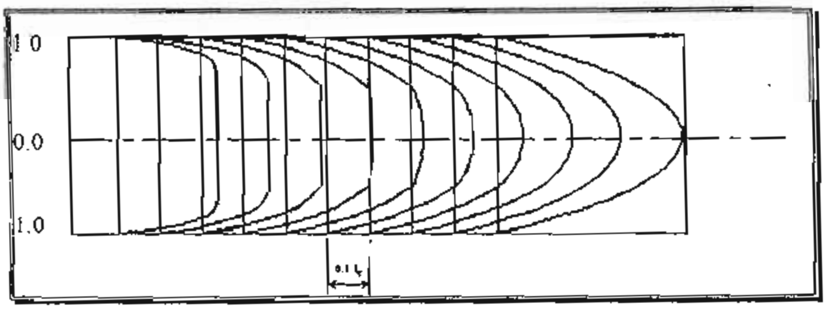

Figure (4) : Velocity profiles computed by the new technique, for (a/b = 2)

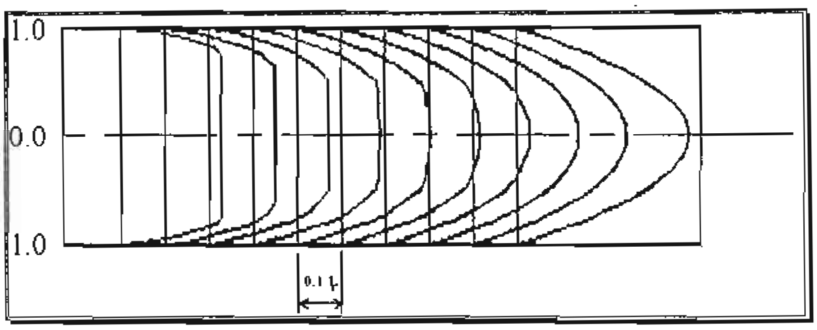

Figure (5) : Velocity profiles computed by the new technique, for (a/b = 3/2)

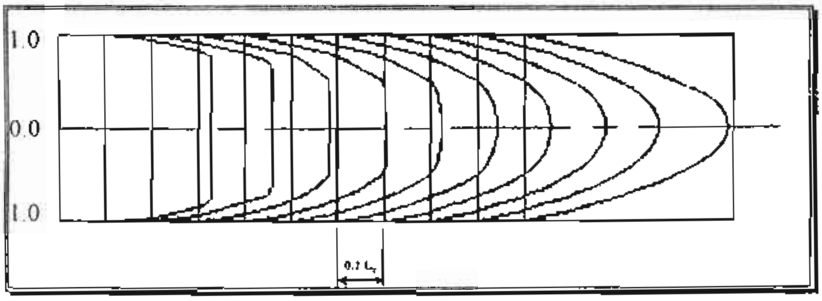

Figure (6) : Velocity profiles computed by the new technique, for (a/b = 4/3)

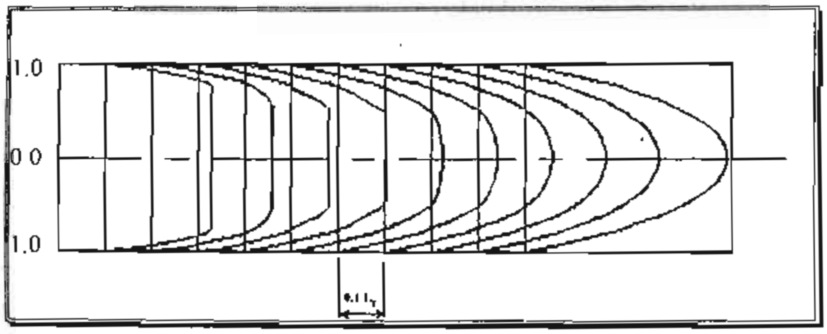

Figure (7) : Velocity profiles computed by the new technique, for (a/b = 1)

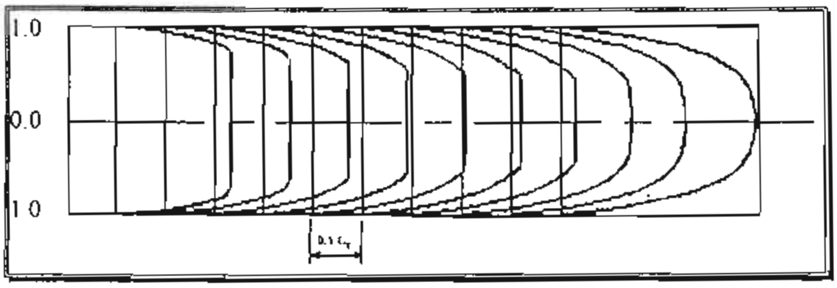

Figure (8) : Velocity profiles computed by the new technique, for (a/b = 2/3)

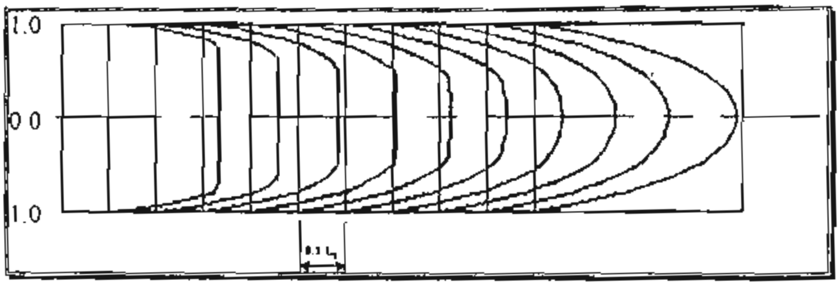

Figure (9) : Velocity profiles computed by the new technique, for (a/b = 1/3)

TECHNIQUE VERIFICATION

Comparing the results of the new technique with the results of Sabry's technique, the velocity curves match. Figures (10-a), (10-b) show a comparison between velocity curve samples for both techniques.

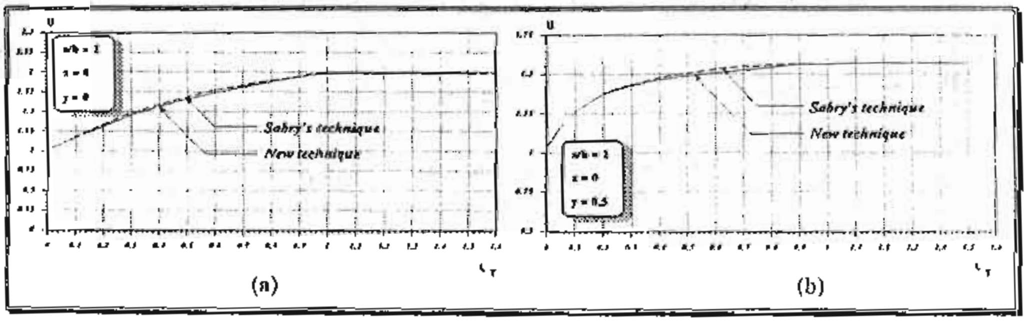

Figure (10) : Comparison of velocity curves between new technique and Sabry's technique

CONCLUSION

The introduced technique studies the flow field in the entrance zone of a straight rectangular tube. The velocity profiles at any section through the entrance zone were obtained, using a very simple well-known mathematical tools.

The results obtained agree with the results of Sabry [3], in which very complicated mathematical concepts were used.

As a future research proposal, it is suggested to apply the suggested new technique for studying the flow field in the entrance zone of irregular tubes.

REFERENCES

1- Langhaar, H. L., "Steady flow in the Transition length of a straight tube.", Journal of Applied Mechanics, A : 55 - 58, June, 1942.

2- Morse, P M. & Fes Hbach, H., "Methods of theoretical physics.", Mc Growhill, New-York, Toronto, London, 1978.

3- M. N Sabry, "On an integral equation method for a linearized version of the Navier-Stokes equation.", Mansoura Eng. Journal, Vol. 13, No. 2 pp, M88 - M97, 1988.

4- M. N Sabry, G M Atia and El-Sedeek, "A generalized transformation of the Navier-Stokes' equation and its solution by an integral equation method ", Mataria Eng Journal, Vol. I, pp. 32-48, 1990

5- Niels Saabye Ottosen and Hans Petersson, "Introduction to the finite element method.", Prentice Hall International (UK) Ltd, 1992.

6- V. Vemuri, and Walter J. Karpins. "Digital computer treatment of partial differential equations". Prentice-Hall, Inc , Engle wood cliffs, New Jersey,1981.

7- Yougren, G K. & Acrivos, A., "Stokes flow a particle of arbitrary shape a numerical 7method of soultion ", Journal of Fluid Mechanics, Vol. 69, part 2, pp.337 - 403, 1975.