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"SUBSYNCHRONOUS RESONANCE PROBLEMS IN AN INTEGRATED POWER SYSTEM"

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ABSTRACT

The problem of subsynchronous resonance (SSR), which is not simple for an interconnected multi-machine power system, is studied in this paper. The analytical approach that has been used for such a study is described in detail, and numerical examples are given.

A mathematical approach has been derived which calculates the electrical natural frequencies in an interconnected power systems. It shows the effects of various degrees of series compensation that are possible, together with the effects of alternative system parameters.

This is necessary to determine the possibility of system natural interaction with torque resonance. Series compensation can be considered at any level or omitted altogether.

INTRODUCTION

The stability limit and hence the operational capability of long transmission lines are greatly improved by the use of series capacitor compensation, but unfortunate experiences of SSR with generator shaft torques have demonstrated the need for caution in the use of such series compensation [1,2,3,4]. Resonance can occur between the natural frequencies of oscillation inherent in the rotating masses of synchronous generators and prime movers coupled by shafts which are themselves elastic, and the natural frequencies of the electric system to which the generator is connected.

Sudden change of torque to the main turbine-generator coupling, produced by a transient variation of electric power, can excite torsional natural resonant frequencies. When series capacitors are used to compensate the reactance of the transmission system the torsional natural resonant oscillations in the turbine-generator shafts may be excited by the power system natural frequency. Self-sustaining torsional oscillations can shorten shaft life.

Previously studies have been made of the effects of long transmission line systems connecting generation at a remote point to the main power system [5,6]. It is necessary to calculate the electrical natural frequencies (ENF) to be able to locate the zone of torsional interaction which is around the points of peak of resonance (the points of coincidence of ENF'S with mechanical natural frequencies) [7]. The present work seeks to extend this to interconnected multi-machine systems with more than one set of series compensation, for this problem may be expected to be of greater importance as power systems continue to grow.

The mathematical approach has been developed as a general analysis of a closed ring or coupled rings of transmission lines connecting generators and loads, as it is clear that any power system, no matter how complex, can be reduced to a set of such rings. Each set can be studied in terms of the degree of series compensation, to evaluate the ENF and then to identify the zone of torsional interaction. The influence of different system parameters has been investigated, together with the effects of network topology.

THE MECHANICAL SYSTEM

The mechanical system of each prime mover and generator can be expressed for this purpose as a spring-mass model whose parameters are known. The matrices expressing the dynamic equation of motion include both the inertia matrix and the velocity damping matrix, and when coupled by the stiffness matrix to the applied torque vector permit the derivation of eigenvalues representing the mechanical modes of natural oscillations as well as the derivation of the eigenvectors of the mode shape. Appendix (I) describes the derivation of the model used and quotes typical values of data to indicate the use of the model.

THE ELECTRICAL NETWORK

The generalised form of a ring which can be part or the whole of the power system is shown in fig.(1-a). The possibility of series compensation in every interconnecting transmission line is considered but of course such compensation can be set at zero for as many lines as desired or at any required value whose resonance effect is to be determined .

The ENF'S are calculated by the following steps:

i) calculation of the single-phase equivalent circuit as shown in fig.(1-b).

ii) calculation of the inductance matrix L, which is given by

$$L = \begin{bmatrix} \Sigma L_{11} & L_{12} & \cdots & & L_{1n} \\ L_{21} & \Sigma L_{22} & \cdots & & \vdots \\ \vdots & \vdots & \Sigma L_{ii} & & \\ & & & \ddots & \\ L_{ni} & \cdots & & & \Sigma L_{nn} \end{bmatrix}$$

where, L_{ii} is the summation of inductances which are included in the loop i

L_{ij} is the inductance of the mutual link between loops i & j .

iii) calculation of the capacitance matrix C, assuming

$$1/c_i = S_i \, L_{ij} \quad , \quad i \, \& \, j = 1,2,\ldots\ldots, n$$

where,

c_i = the capacitance inserted in loop i

S_i = degree of compensation for that loop

L_{ij} = the inductance between the two directly linked buses i & j .

Then, the capacitive reactance matrix F for fig.(1-b) is:

$$F = \begin{bmatrix} 1/c_1 & & & & 1/c_1 \\ & 1/c_2 & & O & 1/c_2 \\ & & 1/c_3 & & 1/c_3 \\ & O & & \ddots & \vdots \\ 1/c_1 & 1/c_2 & 1/c_3 & \cdots & \sum_{i=1}^{n} 1/c_i \end{bmatrix}$$

and, the capacitance matrix is

$$C = F^{-1}$$

iv) the electrical natural frequencies ENF'^{s} , of a network are those that arise from its configuration, without applied e.m.f'^{s} [8]. They can be found as the square roots of the reciprocal of the eigenvalues of the P matrix where,

$$P = L \, C$$

and those values are viewed from the stator side. Alternatively, they can be viewed from the rotor side, where

$$f_r = f_n \pm f_s$$

if f_r is the rotor freq., f_n is the natural freq., f_s is the synchronous freq., +ive sign for supersynchronous freq., and -ive sign for subsynchronous frequency.

The steps (i ⟶ iv) can be applied for two or more coupled rings as shown in fig.(2- a&b) . In the two cases, the matrices are seen to have interesting properties. A "base" matrix "B" can be derived, which is fixed and constant for all degrees of compensation, as long as these are constant throughout the network. If different degrees are used only the diagonal elements of this base matrix are affected. It verifies the relation

$$C = F^{-1} = k \, B$$

where, k is a scaler positive value, and equal to c_n . The matrix B for the first case, fig.(1), is given by

$$B = \begin{bmatrix} b_{11} & 1 & 1 & \cdots & 1 & \cdots & -1 \\ 1 & b_{22} & 1 & \cdots & 1 & \cdots & -1 \\ 1 & 1 & b_{33} & \cdots & 1 & \cdots & -1 \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ -1 & -1 & -1 & \cdots & -1 & \cdots & b_{nn} \end{bmatrix}$$

and,

$$b_{ii} = (c + k)/k \;, \quad i = 1 \;, 2 \;, \ldots\ldots\ldots, n-1$$

$$b_{nn} = 1$$

$$b_{ni} = b_{in} = -1 \quad , \quad i = 1 \;, 2 \;, \ldots\ldots\ldots, n-1$$

$$b_{ij} = 1 \quad , \; i\&j = 1 \;, 2 \;, \ldots\ldots\ldots, n-1$$

With increase of the coupling between the different buses in the network as shown in the second case, fig.(2), a slight difference has been noticed for constructing the base matrix B which is given by

$$B = \left[\begin{array}{cc|cc|c|c|c|c} b_{11} & b_{12} & -1 & -1 & \cdots & -1 & b_{1,n-1} & 1 \\ b_{21} & b_{22} & -1 & -1 & \cdots & -1 & b_{2,n-1} & 1 \\ \hline -1 & -1 & b_{33} & b_{34} & \cdots & b_{3,n-2} & -1 & 1 \\ -1 & -1 & b_{43} & b_{44} & \cdots & b_{4,n-2} & -1 & 1 \\ \vdots & \vdots & -1 & -1 & \cdots & \vdots & \vdots & 1 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ & & & & & \cdots & 1 & -1 \\ -1 & -1 & b_{n-2,3} & b_{n-2,4} & \cdots & b_{n-2,n-2} & 1 & -1 \\ \hline b_{n-1,1} & b_{n-1,2} & -1 & -1 & \cdots & 1 \cdots & b_{n-1,n-1} & -1 \\ 1 & 1 & 1 & 1 & \cdots & -1 & \cdots -1 & b_{nn} \end{array}\right]$$

Here, for every internal set, e.g. the set of loops 1,2,n-1 and the set of loops 3,4,n-2 , ... ext. as shown in fig.(2-b), the matrix elements which relate the loops of each set are constructed by the same relations of the matrix B in the first case, where

$$b_{ii} = (c_i + k) / k = \text{on-diagonal element,}$$

but the off-diagonal elements b_{ij} & $b_{n-1,n-1}$ for the first set or $b_{n-2,n-2}$ for the second set are equal to $(c_{n-1} + k) / k$ or $(c_{n-2} + k) / k$, respectively, instead of the unity value in the first case and with the same sign.

The matrix elements which represent the coupling between the loop no. n and all loops which are directly coupled with it,(e.g. loop no. n-1, n-2, ...ext.) are constructed exactly as in the first case without any difference. Therefore, the matrix B can be partitioned into different parts, each part representing a set of coupled loops with the same configuration as in the first simple case. The off-diagonal elements between these parts are equal to -1 , except those in the last row and column are equal to 1 . For clarity, this is illustrated numerically in example 2 below.

The base matrix is multiplied by a positive scaler factor to identify the effect of various degrees of compensation. Thus the transmission lines can be considered as compensated with the same degree or different degrees of compensation without complication, one such degree being zero if required. Omission of compensation decreases the number of eigenvalues and the number of natural frequencies.

The following numerical examples are given to demonstrate the above two cases (one ring & two coupled rings).

<u>EXAMPLE 1</u> :

For the studied system shown in fig.(3-a&b) [9], the inductance matrix is

$$L = \begin{bmatrix} 0.459 & 0.101 & 0.000 & 0.000 & 0.000 & 0.085 \\ 0.101 & 0.543 & 0.350 & 0.000 & 0.000 & 0.092 \\ 0.000 & 0.350 & 0.760 & 0.240 & 0.000 & 0.170 \\ 0.000 & 0.000 & 0.240 & 0.660 & 0.322 & 0.101 \\ 0.000 & 0.000 & 0.000 & 0.322 & 0.576 & 0.072 \\ 0.085 & 0.092 & 0.170 & 0.101 & 0.072 & 0.681 \end{bmatrix}$$

and the base matrix B when S_i is equal for all links is :

$$B' = \begin{bmatrix} 2.89 & 1 & 1 & 1 & 1 & -1 \\ 1 & 2.75 & 1 & 1 & 1 & -1 \\ 1 & 1 & 1.95 & 1 & 1 & -1 \\ 1 & 1 & 1 & 2.6 & 1 & -1 \\ 1 & 1 & 1 & 1 & 3.24 & -1 \\ -1 & -1 & -1 & -1 & -1 & 1 \end{bmatrix}$$

where, the scaler factor k varies with equal S as :

S %	k	S %	k
10	62.11	50	12.42
20	31.06	60	10.36
30	20.70	70	8.87
40	15.46	80	7.76

Accordingly, the electrical natural frequencies can be calculated at each case by calculating the square roots of the reciprocal of the eigenvalues of the matrix

$$P = L\,C = k\,L\,B$$

and the frequency map is as seen in fig.(4), which identifies the cross-over points where the network natural frequencies coincide with those of the mechanical system. Of course, the horizontal lines representing the mechanical modes of oscillation will be moved upwards or downwards for other different turbines shafts in the system, giving different intersection-points, with different values of inertia constants and shaft stiffness as shown in fig.(5).

The inductance matrix changes with load, taking account of the range of expected loads, has been studied as shown in fig.(6). It is seen that, for a fixed compensation, the natural frequencies viewed from the rotor side for the loops (which include the loads reactances) are increased with increasing the load, while for the other loops, the frequencies are constant.

In case of using $S_1 = 15\,\%$, $S_2 = 20\,\%$, $S_3 = 30\,\%$

$S_4 = 15\,\%$, $S_5 = 25\,\%$, $S_6 = 40\,\%$

the base matrix B is given by

$$B = \begin{bmatrix} 2.58 & 1 & 1 & 1 & 1 & -1 \\ 1 & 4.5 & 1 & 1 & 1 & -1 \\ 1 & 1 & 2.26 & 1 & 1 & -1 \\ 1 & 1 & 1 & 5.26 & 1 & -1 \\ 1 & 1 & 1 & 1 & 4.58 & -1 \\ -1 & -1 & -1 & -1 & -1 & 1 \end{bmatrix}$$

and $k = 15.53$, where both of the diagonal elements of "B" and the value of k are dependent on the different combinations of different S_i's .

EXAMPLE 2 :

Taking into consideration the same studied system in example 1, with adding a line between buses 5 & 9 with an inductance equal to 0.18 p.u., the equivalent L-C circuit is shown in fig.(7) to study the effect of alternative line configurations (network topology). It has been seen that

$$L = \begin{bmatrix} 0.460 & 0.101 & 0.000 & 0.000 & 0.000 & 0.085 & 0.000 \\ 0.101 & 0.543 & 0.350 & 0.000 & 0.000 & 0.092 & 0.000 \\ 0.000 & 0.350 & 0.760 & 0.240 & 0.000 & 0.170 & 0.000 \\ 0.000 & 0.000 & 0.240 & 0.663 & 0.322 & 0.000 & 0.101 \\ 0.000 & 0.000 & 0.000 & 0.322 & 0.576 & 0.000 & 0.072 \\ 0.085 & 0.092 & 0.170 & 0.000 & 0.000 & 0.527 & 0.180 \\ 0.000 & 0.000 & 0.000 & 0.101 & 0.072 & 0.180 & 0.514 \end{bmatrix}$$

and $C = F^{-1} = k\,B$, where k is at the same values as in example 1 &

$$B = \begin{bmatrix} 3.79 & 1.89 & 1.89 & -1 & -1 & -1.89 & 1 \\ 1.89 & 3.65 & 1.89 & -1 & -1 & -1.89 & 1 \\ 1.89 & 1.89 & 2.84 & -1 & -1 & -1.89 & 1 \\ -1 & -1 & -1 & 2.59 & 1 & 1 & -1 \\ -1 & -1 & -1 & 1 & 3.24 & 1 & -1 \\ -1.89 & -1.89 & -1.89 & 1 & 1 & 1.89 & -1 \\ 1 & 1 & 1 & -1 & -1 & -1 & 1 \end{bmatrix}$$

The natural frequencies cen then be calculated. It has been found that the natural frequencies for the loops which are at the same condition of coupling in the first case are not changed. However, the natural frequencies for the loops which are more coupled are changed and become higher when they are viewed from the rotor side.

Thus, improved coupling has the same effect upon the natural frequencies as does increased the loads.

From the above numerical studies, the frequency map at 50 % loading as shown in fig.(6-b), can be redrawn with changing either the inertia or shaft constants to move the mechanical modes upward or downward in order to determine the shortest possible range of series compensation at which the occurence of torsional interaction is possible, e.g. by choosing the shaft stiffness values as shown in fig.(5-b-i), the frequency map at load = 50 % is shown in fig.(8), where the range of series compensation corresponding to the zone of torsional interaction is as short as possible.

CONCLUSIONS

Most published studies of SSR problems in power systems have been considered with a system configuration where remote generation has been coupled through long transmission lines to a system large enough to be considered as an infinite system.

Such simplification is not always possible and it is expected that circumstances will arise where more extensive use will be made of series compensation within that "infinite" system.

A method has been defined for the consideration of this much more difficult problem. It leads to the identification of the maximum permissible compensation in a given transmission line or lines within a power system. It also includes consideration of the load effect. Inertia and shaft stiffness effects are also included. The results are given in graphical form with practicable numerical illustrations.

It is interesting to note that increasing loads have a similar effect to improving coupling in increasing the natural frequencies.

This paper shows that a full design study is necessary when series compensation is under consideration, and indicates a practicable method for such a study.

APPENDIX (I) :

Natural Frequencies and Mode Shapes of the Turbine-Shafts :

The system is assumed to be represented by the spring-mass model shown in fig.(9), and assumes an unforced and undamped mechanical system. The data of the turbine shaft studied are from a real example [10] and the values of the stiffness are k_{12} = 29.437 , k_{23} = 62.241 , k_{34} = 73.906 , k_{45} = 5.306

Stage inertia constants :

HP	stage	= 0.0649	sec.
IP	"	= 0.2552	"
LP	"	= 1.5390	"
Generator rotor		= 0.98	"
Exciter rotor		= 0.0298	sec.

(all values are in p.u. except t in sec. & θ in rad.)

Applying the eigenvalue conventional method [7] , the mechanical natural frequencies are :

$f_{m1} = 20$, $f_{m2} = 27$, $f_{m3} = 30$, $f_{m4} = 45$ c.p.s.

and the mode shapes are shown in fig.(10).

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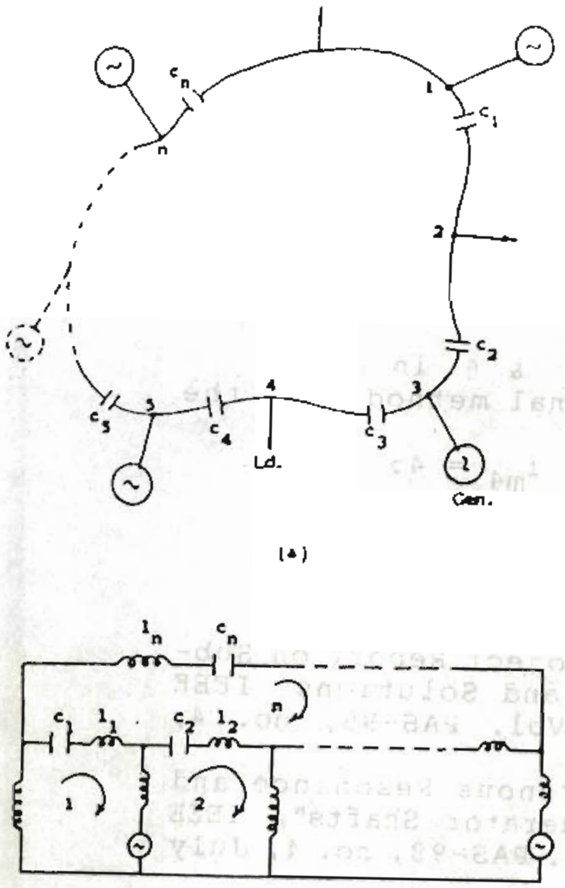
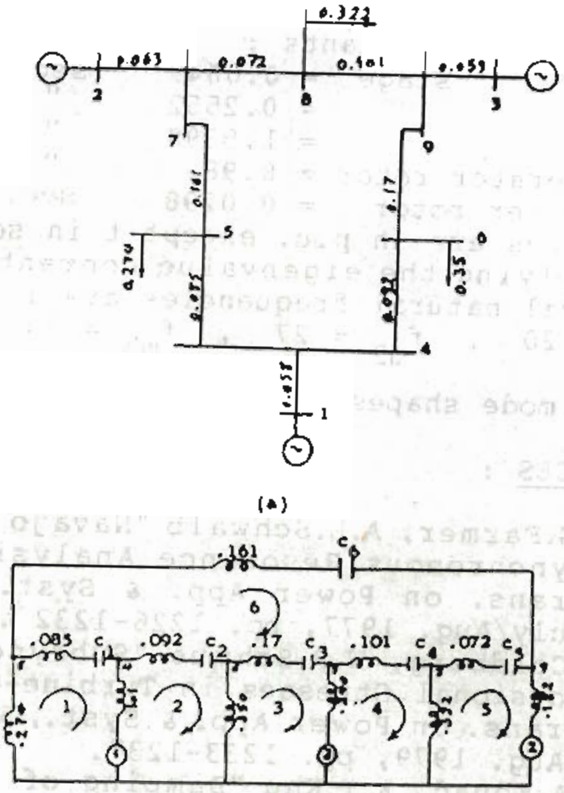


Fig.(1): a- General loop configuration for an interconnected system.
b- The equivalent L - C loops of the system.



Fig(3): a- The single phase equivalent circuit of the system studied.
b- The equivalent L-C loops of the system.

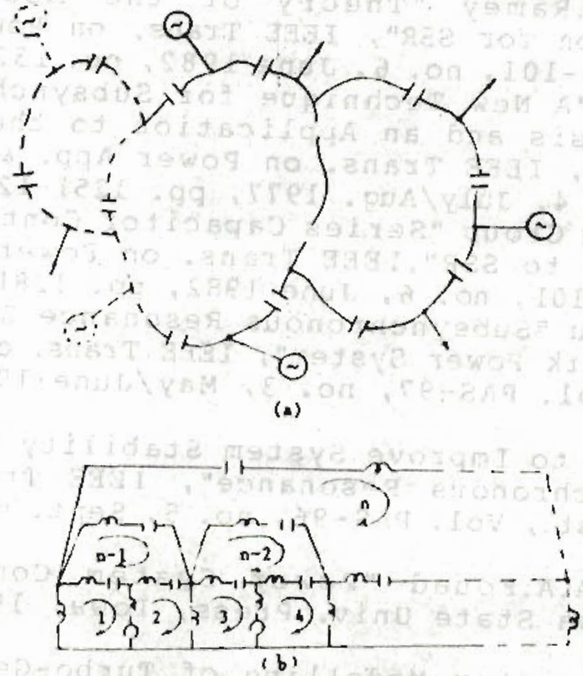


Fig.(2) : a - General loop configuration for an interconnected system considered as coupled rings.
b - The equivalent L-C loops of the system.

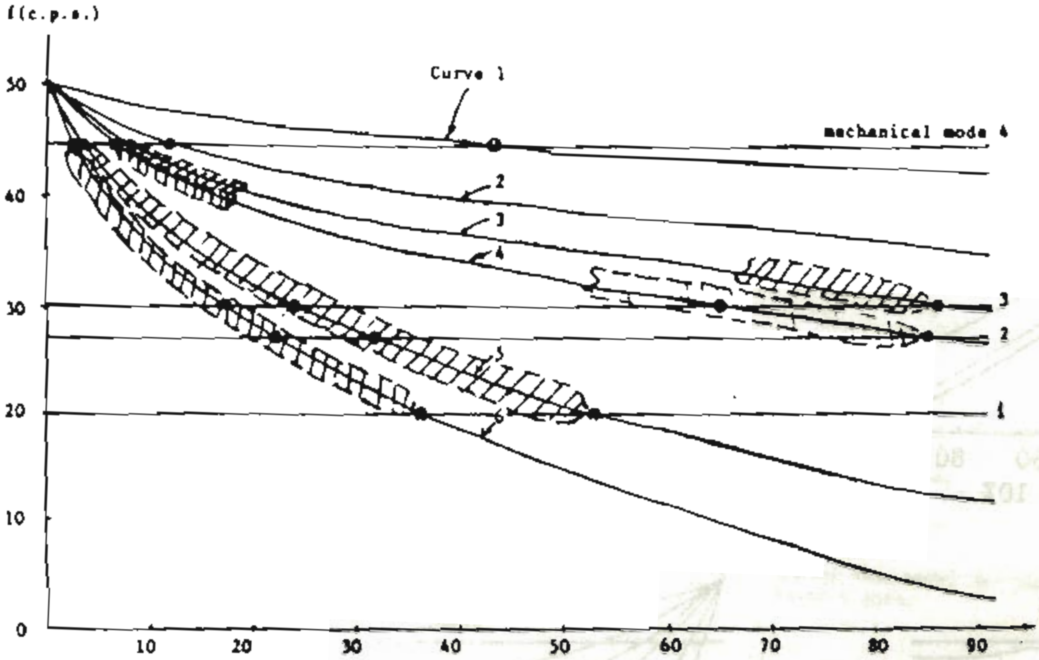
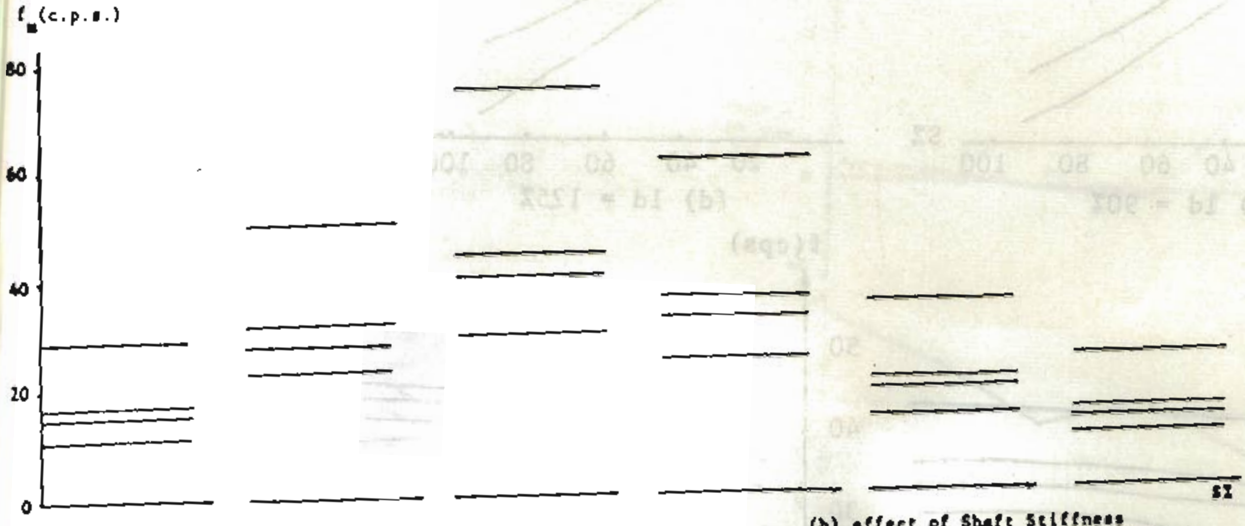


Fig.(4) : The frequency map at load = 100% - at each degree of series compensation, there are six ENF which are represented by curves 1, 2,..., 6
(● points of peak of resonance and the hatched areas are the zones of torsional interaction)



(a) effect of inertia

HP	0.1947	0.0649
IP	0.7627	0.2552
LP	4.683	1.5390
Gen.	2.94	0.48
Exc.	0.0894	0.0298
Tot.	8.678	2.369
	(i)	(ii)

(b) effect of Shaft Stiffness

k_{12}	66.314	44.21	14.737	7.368
k_{23}	140.04	93.362	31.121	15.560
k_{34}	166.28	110.860	36.950	18.477
k_{45}	11.93	7.959	2.653	1.3265
	(ii)	(i)	(iii)	(iv)

Fig.(5) : (a) The mechanical modes at different values of inertia constants.
(b) The mechanical modes at different values of shaft stiffness.