Menofia University Faculty of Engineering Shebien El-kom Academic Year : 2014-2015 Department : Basic Eng. Sci.	Minoufia University	Subject : Partial Differential Equations Time Allowed : 3 hours Date : 6 / 6 / 2015

Allowed Tables and Charts : None

Answer all the following questions: [100 Marks]

Question 1 [20 Marks]

A) For the following statements, state true or false and why?

1. A differential equation involving derivatives with respect to a multiple independent variables is called an *ordinary differential equation (ODE).*
2. A differential equation involving partial derivatives with respect to more than one independent variable is called *partial differential equations (PDE).*
3. The lowest order derivative involved in a partial differential equation is called the order of the partial differential equation.
4. The degree of a partial differential equation is the degree of the highest derivative which occurs in it.
5. The partial differential equation (PDE) is called quasi linear PDE if the equation is nonlinear in the highest order derivative but non-linear in other term.

B) Explain each of the following:

1. Boundary conditions (give an example)
2. Initial conditions (give an example)
3. Quasi-linear Partial differential equation (give an example)
4. Initial value problem (give an example)
5. Boundary value problem (give an example)

Question 2 [30 Marks]

Find the dependent variable of the heat equation of a thin rod

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + xe^{-t}, \qquad 0 < x < L, \qquad t > 0$$

With the boundary conditions:

$$u(0,t) = 1, \qquad u(L,t) = 2$$

And initial conditions:

$$u(x,0) = f(x)$$

Question 3 [30 Marks]

(A) For *the Sturm Liouville boundary value problem*, state true or false and discuss your answer.

1. All eigen values are real.
2. There exists an infinite number of the eigen values such that $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_1 < ..... < \lambda_n < \lambda_{n+1} < ....$
3. Corresponding to each eigen value λ_n, there is an eigen function, denoted $\varphi_n(x)$.
4. Eigen functions are orthogonal with respect to weight function $r(x)$ $\int_a^b r(x)\, \varphi_m(x)\, \varphi_n(x)\, dx = 0$ if $m \neq n \Rightarrow \lambda_n \neq \lambda_m$.
5. Any eigen function has exactly (n-1) zeros for $a \le x \le b$ without counting the end points.

(B) Solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < 1, \qquad t > 0$$

With the boundary conditions:

$$u(0,t) = 0, \qquad u(1,t) = 0$$

And initial conditions:

$$u(x,0) = f(x), \qquad \frac{\partial u(x,0)}{\partial t} = g(x).$$

Question 4 [20 Marks]

Find u of the wave equation or (finite string of length L)

$$\frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2} + x, \qquad 0 < x < L, \qquad t > 0$$

With the boundary conditions:

$$u(0,t) = 0, \qquad u(L,t) = 1$$

And initial conditions:

$$u(x,0) = f(x), \qquad \frac{\partial u(x,0)}{\partial t} = g(x)$$

This exam measures the following ILOs											
Question Number	Q1-a	Q4			Q2	Q3-b			Q1-b	Q3-a	
Skills											
	Knowledge &understanding skills				Intellectual Skills				Professional Skills		

With our best wishes

Associate Prof. Dr. Islam M. Eldesoky