MATHEMATICAL MODEL OF SYNCHRONOUS COMPENSATOR OPERATING IN CONJUNCTION WITH A RIGIDLY IMPULSED INVERTER

H.A.Abou-Tabl M.M.EL-Shamoty S.A.EL-Drieny

Department of Electrical Engineering, EL-Mansoura University,
EL-Mansoura,Egypt.

ABSTRACT

A synchronous compensator is recently used in conjunction with a rigidly impulsed inverter to supply an isolated AC load. In this case the inverter frequency is adjusted to be equal to the load frequency.In order to hold it constant, the feed-back control between the rotor speed and inverter frequency must be investigated. The system consumes its requirements of active power from the inverter through a DC-link which may be connected to an unconventional source of electrical power. Reactive power requirements will be supplied from compensator.In such a system, and after synchronizing the inverter with the DC-link, it is expected to have an oscilating behaviour which may force either the compensator to go out of stability or the inverter to have a failure.

The paper is concerned with the formulation of the quasi-steady behaviour of the foregoing through an adequate mathematical model. It will be based on the time varying effective value. The proposed model gives three approaches during formulation process of the problem. The computer results show the necessity of the system to be integrated with any conventional type of control.

LIST OF SYMBOLS

V := Voltage across the inverter AC side, load and compensator, r.m.s.
E := Voltage behind the compensator stator leakage reactance, r.m.s.
E_f := Excitation voltage, time-varying effective value.
V := Voltage across the inverter DC-side, time-varying effective value
V_D := Voltage across the DC-link behind the smoothing reactor, time-varying effective value.
I_D := Current in the DC-link, time-varying effective value.
I_N := The inverter current, time-varying effective value.
I_S := The compensator current, time-varying effective value.
I_L := The load current, time-varying effective value.
N := Constant of the inverter current = $\sqrt{6}/\pi$
R_S := Compensator resistance, in Ohms.
X_S := Synchronous reactance, in Ohms.
X_P := Compensator leakage-reactance, in Ohms.
a := p / ($J.\omega_0$)
p := Number of pole pairs.
X'_S := Compensator subtransient reactance, in Ohms.
X_E := Subtransient reactance of transformer may be connected with compensator, in Ohms.
Z_S := Compensator synchronous impedance, in Ohms
Z_L := Load-impedance, in Ohms.

J := Inertia coefficient of compensator rotor, kg-m^2.

P_{f+w} := Friction and windage loss of compensator, watts

w_o := Nominal rotor speed, which corresponds to the rigid inverter-frequency, in electrical rad./second.

w_r/w_o := Relative speed of E_f

w_v/w_o := Relative speed of V.

w_s/w_o := Relative speed of I_S

α := Arctan (R_S / X_S)

β := Advanced angle of the inverter, elec. rads.

δ := Load angle of the compensator, elec. rads.

γ := Commutator angle, elec. rads.

T_D := Time interval, in sec.

1.0 INTRODUCTION

Synchronous compensators are usually applied today across the AC side of inverter type convertors, which are used to permit the electrical energy to flow from a DC-link to an isolated AC load. So, the synchronous machine is suggested mainly to generate the AC voltage across the corresponding side of the inverter. This voltage is necessary to ensure voltage type commutation for the inverter. It is a simple method of commutation to be used. In addition to this job, the synchronous machine has to deliver the reactive power needed by the inverter and the load.

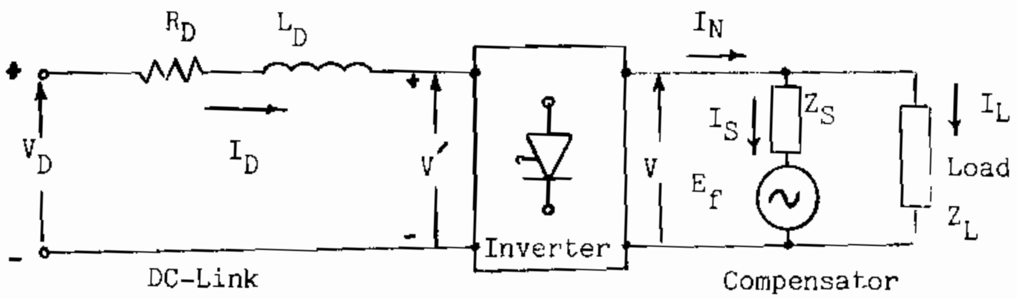

Figure 1 : Schematic Diagram of the Investigated System

Figure 1 shows the schematic diagram of the investigated system. It consists of a DC-link, a rigid impulsed inverter, a synchronous compensator and an isolated load. The inverter is suggested to be rigidly impulsed in order to hold the load frequency constant. Therefore, feed-back control between the compensator rotor speed and the impulse-frequency is not required. The input power to the DC-link may be supplied from a HVDC transmission line or from a renewable source of electrical power. The system can also be suggested to work in conjunction with the shaft alternators (4) .The synchronzation problem of the inverter with the DC-link had been discussed in reference (5).

In this paper, a mathematical model of the above described system will be derived to represent its quasi-steady behaviour after synchronzation. Natural oscillations in the compensator speed, due to the absence of adapting the rotor speed with the triggering frequency, will be expected. They may force the synchronous machine to go out of stability or may lead to an inverter failure. The interaction between the synchronous, as the only source of voltage and reactive power, and the inverter, as the only source for current and active power plays a great role in the overall system stability. Accordingly, the system may be assisted by any type of conventional controls in order to ensure more stable operation. For example this will be achived by applying continous control to the compensator excitation or by controlling continously the DC level behind the DC link.

The proposed mathematical model describes the system through three approaches having the following features:

1- The first approach does not consider any of the above mentioned controls
2- The second approach considers only the excitation control in order to maintain constant terminal voltage.
3- The third approach adds to the excitation control the control of the voltage behind the DC-link.

The additional type of control suggested in the latter approach aims mainly to hold the advance angle β of the inverter constant at a proper value and away from its two extreme limits. The commutation angle γ is taken to be the lower limit. The advance angle β must be greater than the commutation angle. As a maximum limit, the angle β is not allowed to be equal or greater than $\pi/2$, else the power flow will be reversed . The foregoing both limits lead to inverter failure (6).

2.0 MATHEMATICAL MODEL

As the synchronous compensator is supplied from the inverter, the speed of the rotating field established by the stator currents will be proportional to the triggering frequency of the inverter impulses. The rotor speed will be in turn equal to that of the rotating field. Under steady state ,the balance of the power supplied to the system must be exist such as :

1-The active power supplied by the inverter must be equal to the active power consumed by both compensator and load.
2- The reactive power supplied by the compensator must be equal to the reactive power consumed by the inverter and the load.

The proposed mathematical model is based on small perturbations in the rotor speed, w_r. They are due to lacks of the proper damping and control. Theses small perturbations in the rotor speed are caused by the attempt of both power source in the system to get the power balance. Therefore ,the rotor speed will oscillate in air-gap around its nominal value w_o and the relative rotor speed (w_r/w_o) can be taken as a measure of the expected oscillations. Accordingly, the phase shift ($\beta + \delta$) as shown in figure 2, between

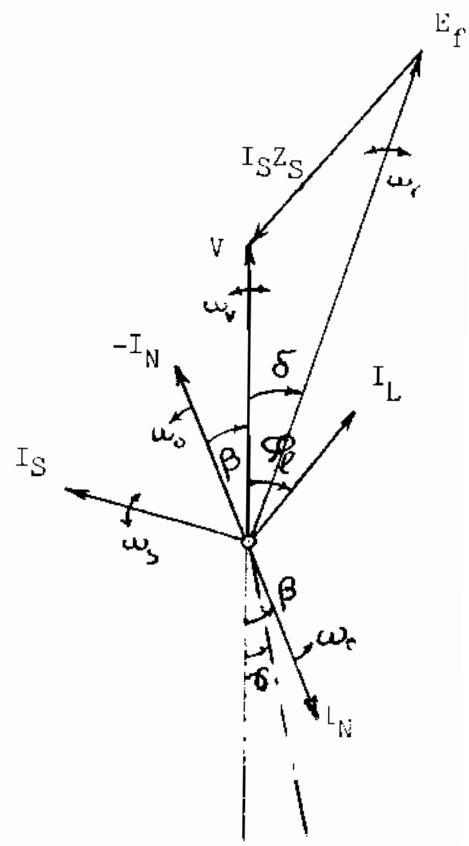

Fig. 2: Quasi-Steady Complexor Phasor Diagram.

the rigid frequency inverter current I_N and the oscillating excitation voltage E_f. This variation is due to both variations in β and δ. The variation in δ is naturally caused by the rotor oscillation while the variation in β is caused by the variations in the DC link conditions : V, I_D, V_D. As the rotor oscillation are assumed to be small, the system can be considered to be operating under quasi-steady state conditions. Therefore, the system of differential equations describing this mode of operation can be written in term of the ' time varying effective value' of the concerned variables. Accordingly, the initial conditions for these equations at each time interval can be obtained with the help of the complexor phasor diagram of Figure 2 taking into considration the variaions gained in both β and δ from the corresponding differential equation.

2.1 DERIVATION OF THE BASIC EQUATIONS

In addition to the above mentioned assumption, the following assumptions are also considered to develop the differential equations:

1- Saturation is neglected in the synchronous compensator. Thus the machine is assumed to operate in the linear part of the magnetisation curve, In this case, the excitation voltage control is proportional to the field control.

2- Higher harmonics are neglected. The first harmonic is only taken into consideration. This assumption is valid for both the compensator and the inverter.

3- The electrical load is assumed to be passive and balanced load. This assumption enables to represent the system by its phase equivalent circuit.

4- Damper-winding parameters are referred to the stator-side. Induction in the field-circuit as well as the saliency-effect are neglected.

5- The inverter-thyristor units are assumed to possess ideal resistances in both forward and reverse directions. Also, it is assumed that there is no effect of the commutation on the rectangular shape of the inverter current.

6- The smoothing-reactor is assumed to be a proper one and is enough to eliminate the ripples in both the DC voltage and current.

The above assumptions are practically reasonable as the system is operating under quasi-steady conditions.

So, the differentional equations representing the system can he given now:

$$d(w_r / w_o)/dt = a\ (TS + TAS - TD) \qquad (1)$$

$$d(\beta + \delta)/dt = w_o(1 - w_r/w_o) \qquad (2)$$

$$d(I_D)/dt = V_D/L_D - I_D . R_D/L_D - (3.N/L_D).V.\cos\beta \qquad (3)$$

The first equation is derived mainly with the help of the dynamic equation of the synchronous compensator which relates the electric developed synchronous torque, TS, to all other torques exerted inside the machine;

$$TS = TI + TD - TAS \qquad (4)$$

where

$$TS = (V.\ E_f.\sin(\delta + \alpha)/Z_S - (E_f/Z_S)^2.R_S).(p/w_o) \qquad (4\text{-a})$$

$$TI = J \cdot d(w_r)/dt = (d(w_r/w_o)\ /dt\ )\ /\ a \qquad (4\text{-}b)$$

$$TD = TD_o \cdot (w_r/w_o) \qquad (4\text{-}c)$$

$$TAS = (E \cdot X_S' / (X_E + X_S'\ ))^2\ /\ R_K \cdot (p/w_o) \cdot (1 - w_r/w_o) \qquad (4\text{-}d)$$

Where R_K is damper resistance referred to stator side
The synchronous torque,TS, depends mainly on the excitation voltage E_f and the load angle δ which could be positive or negative under oscillating conditions. The inertia torque, TI, is a function, in the first derivative of (w_r/w_o). The damping torque corresponds to the friction and windage losses inside the machine. It depends on the rotor relative speed where TD_o is the damping torque measured at synchronous speed w_o. The asynchronous torque TAS is exerted by the damper winding and depends on the air-gap voltage . It will be, accelerating torque for $(w_r/w_o) < 1$, positive slip, and it will be, retarding torque for $(w_r/w_o) > 1$, negative slip. It is also evident that under steady state for $w_r = w_o$, TS =TD as it is well known for compensator mode of synchronous machine operation. The second equation, eqn. 2,is derived by relating the variation $\Delta(B + \delta)$ with the time, to both frequencies w_o and w_r. Equation 3 is developed from the DC-link mesh where V_D is given by

$$V_D = I_D \cdot R_D + L_D \cdot d(I_D)/dt + V' \qquad (5)$$

Where

$$V' = 3 \cdot N \cdot V \cdot \cos \beta \qquad (6)$$

V' is the inverter DC voltage
Equation 5 makes the necessary coupling between the compensator terminal voltage, which also the load terminal voltage and the DC-link variables, including the advance angle β of the inverter. Equation 3 forms a non-linear system. It can't be written in the form of a state equation.Therefore, the initial conditions for each time interval, while solving the system numerically by a fourth order Runge-Kutta's method, will be essential to continue the solution.

2.2 INITIAL CONDITIONS

1-At the starting point, t =0 ,we have the following initial conditions:

(i)Load : the load power (KVA), load power factor $\cos\varphi_l$ and its type whether lagging or leading, and the rated terminal voltage ,V, of the compensator or the inverter.

(ii)Inverter: the impulse frequency which determines the load or compensator frequency. This frequency remains constant. In addition, the initial value of the advance angle β_o of the inverter must be known. This angle must be greater than the expected commutation angle and too far less than $\pi/2$.

(iii)Compensator: the rated mechanical losses are measured at the synchronous speed,w_o, which determines the rated damping torque TD_o. In addition, the ratio of the total active power to the apperent power (kw/kva) at rated excitation must be known.

Accordingly to the mentioned principle of power balance, and the complex-or phasor diagram shown in Fig. 2, the following equations can be written:

$$I_S \cdot \sin \varphi_s = I_N \cdot \sin \beta - I_L \cdot \sin \varphi_\ell \quad (7\text{-a})$$

$$I_S \cdot \cos \varphi_s = I_N \cdot \cos \beta - I_L \cdot \cos \varphi_\ell \quad (7\text{-b})$$

$$E_f \cdot \cos \delta = V - I_S . Z_S \cdot \cos (\varphi_s + \theta_s) \quad (8\text{-a})$$

$$E_f \cdot \sin \delta = I_S \cdot Z_S \cdot \sin (\varphi_s + \theta_s) \quad (8\text{-b})$$

The manipulation with the above conditions and the equations 7 and 8 give the initial values of all AC voltages and currents and the corresponding phase shifts referred to the terminal voltage V. The voltage E can be obtained with equations similar to that written for E_f. On the DC side, the current I_D can be determined by

$$I_D = I_N / N \quad (9)$$

The steady-state form of equation 5 yields the initial value of the voltage behind the DC link. Now with the help of the equation 4 and for an initial value of (w_r/w_o), equation 3 is ready to start the solution.

2- Initial conditions for any instant $t > 0$

The determination of these conditions depends on the results gained at the previous interval : $(\beta + \delta)_{n-1}$, $(w_r/w_o)_{n-1}$, and $(I_D)_{n-1}$; where n is the interval order. To get the rest of the initial conditions included in the differential equations, the separation between β and δ must be processed taking into considration the relation :

$$(I_N)_{n-1} = N \cdot (I_D)_{n-1} \quad (10)$$

This separation is directed by the applied approach and depends in turn on its variables.

2.3 SOLUTION ALGORITHM

It follows now the salient directions and manipulation required for the separation process. It will be noticed that φ_ℓ is assumed to be constant in all three approaches :

2.3.1 First Approach

This approach doesn't consider the application of any control concepts. This result in :

$$V_D = \text{constant} \quad \text{and} \quad (E_f)_{n-1} = E_{fo} \cdot (w_r/w_o)_{n-1}$$

Although the excitation isn't controlled, its voltage $(E_f)_{n-1}$ differs from that calculated at t =0, E_{fo}, by the rotor relative speed So, substituting

$$(\delta)_{n-1} = (\beta + \delta)_{n-1} - (\beta)_{n-1} ,$$

The following variables can be declared as knowns while manipulating with equations 7 and 8 : $(V_D, I_N, \varphi, E_f, \delta)_{n-1}$

The manipulation yields the following results.

$$\beta = \text{Arc Tan } (K_1 / K_2) \tag{11}$$

Where

$K_1 = \text{Sin } (\beta + \delta) - AK_2.I_D - AK_3.\text{Cos } (B + \delta) - AK_1.AK_3.I_D$

$K_2 = \text{Cos } (\beta + \delta) + AK_1.I_D + AK_3.\text{Sin } (\beta + \delta) - AK_2 .AK_3.I_D$

$AK_1 = N. R_S / E_f$

$AK_2 = N. X_S / E_f$

$AK_3 = -AK_4 / AK_5$

$AK_4 = (R_S. \text{Sin}\varphi_\ell + X_S. \text{Cos } \varphi_\ell) / Z_L$

$AK_5 = 1 + (R_S.\text{Cos}\varphi_\ell - X_S.\text{Sin } \varphi_\ell) / Z_L$

Having β and δ , the terminal voltage can be determined as:

$$V = (E_f. \text{Cos } \delta + K_3.I_D)/ K_4 \tag{12}$$

Where

$K_3 = (R_S.\text{Cos } \beta - X_S .\text{Sin } \beta) . N$

$K_4 = AK_5$

2.3.2 Second Approach

An excitation control is applied,here, to hold the constraint of having the terminal voltage V to be constant. The manipulation with eqns. 7 and 8 in this case implies that

$$\beta_{n-1} = (\beta + \delta)_{n-1} - \delta_{n-1}$$

So, the following variables can be declared as knowns :

$$(V_D, I_N, V_o, I_L, \beta, \varphi_\ell)_{n-1}$$

Accordingly , the δ relation can be derived as

$$a. \text{Sin}^2\delta + b . \text{Sin}\delta + c = 0 \tag{13}$$

Its solution yields

$$\text{Sin} = (-b \pm \sqrt{b^2 - 4.a.c}) / 2a \tag{14}$$

Where $a = (-R_S. I_L. \text{Sin } \varphi_\ell - X_S.I_L. \text{Cos } \varphi_\ell)^2$

$+ (V_o - X_S.I_L.\text{Sin}.\varphi_\ell) + R_S. I_L. \text{Cos } \varphi_\ell)^2$

$$b = -2.N.I_D.(R_S.\sin(\beta+\delta) + X_S.\cos(\beta+\delta))$$

$$.(V_o - X_S.I_L.\sin\varphi_\ell + R_S.I_L.\cos\varphi_\ell)$$

$$c = (N.I_D)^2.(R_S.\sin(B+\delta) + X_S.\cos(B+\delta))^2$$

$$-(-R_S.I_L.\sin\varphi_\ell - X_S.I_L.\cos\varphi_\ell)^2$$

Equation 14 gives two values for Sin δ. Accordingly, two values in turn for the angle will be obtained . The acceptable value of δ is to have the smallest value.

2.3.3. Third Approach

Here, the control of V_D is applied as well as the excitation control to hold the advance angle of the inverter and the terminal voltage, respectively be constant at their initial values at t = 0 (i.e. β_o and V_o) .Therefore, the load angle δ_{n-1} can be directly determined where $\beta_{n-1} = B_o$

$$\delta_{n-1} = (B+\delta)_{n-1} - B_{n-1}$$

And eqns. variables $(V_o, I_L, \ , B, \ )_{n-1}$ can be stated as knowns

Applying eqns. 7 and 8 ,a relation of the inverter current can be obtained

$$(I_N)_{n-1} = C_1.V_o / (C_2 + C_3 . \tan\delta) \quad (15)$$

Where

$$C_1 = A/Z_L + (1+\beta/Z_L).\tan\delta$$

$$C_2 = R_S.\sin\beta + X_S.\cos\beta$$

$$C_3 = R_S.\cos\beta - X_S.\sin\beta$$

$$A = R_S.\sin\varphi_\ell + X_S.\cos\varphi_\ell$$

$$B = R_S.\cos\varphi_\ell - X_S.\sin\varphi_\ell$$

Having I_N, the current in the DC-link can be given by:

$$(I_D)_{n-1} = (I_N)_{n-1} / N$$

And the controlled voltage can be deduced by :

$$(V_D)_{n-1} = 3.N.V_o\cos\beta + R_D I_D + [(I_D)_{n-1} - (I_D)_{n-2}].L_D/T_D \quad (16)$$

So, accordingly to the approach applied the rest of the initial conditions required per solution progress can be decided.

3. PROGRAM ORGANIZATION

The proposed mathematical model had been written in the form of FORTRAN program for the mini-computer installed in the Electrical Engineering Department of EL-Mansoura University. The criteria being applied to the results of each time interval will be discussed in the following section. Most of these criteria are suggested to stop the calculations when the system shows tendency to one of the following kinds of instability:

(a) Load Angle $\delta \geqslant \pi/2$
This criterion indicates that the synchronous compensator is going out of stability.

(b) Negative-current in the DC-link
This condition occurs when $V > V_D$ in the first approach .Negative current in the DC-link can't actually be happened because of the nature of the inverter as long as $\beta < \pi/2$. In this case I_D will stop to flow and the calculations will be continued by $I_D = 0$

(c) Advanced Angle $\beta \geqslant \pi/2$
It will be indicated ,here, the advance angle β is going above its upper limit. This means that the inverter has to reverse the power flow. Naturally, this can't be happened, where the DC-link is the only source for active power.

(d) Advance Angle $\beta \leqslant \gamma$
To achive this criterion , the commutation angle γ must be firstly determined, (6) :

$$\gamma = \text{Arc cos}\ (\ 1-\ 2.\ X'_S.\ I_D\ /\ (\ \sqrt{6}\ V\ ))\qquad (17)$$

The physical concepts of the inverter show that the advance angle β must be greater than γ . If $\beta \leqslant \gamma$, an inverter failure will give alarm.

The model had been tested using the following data :

(a) Load
KVA = 4, V = 220 Volt , f = 50 Hz, P.F. = 0.8 lag.

(b) Synchronous Compensator
KVA =5.23 P.F = 0.15 leading P_{f+w} = 600 Watt, R_S= 0.33 , X_P= 0.66
X_S =5.3 , X'_S = 0.55 Ohm, R_K = 0.15 ,and X_E = 0.0 Ohm and J = 0.308 $Kg.m^2$.

(c) Inverter and DC -link
$\beta_o = 36^{\circ}$, R_D = 0.25 Ohm and L_D = 200 mh.

4. RESULTS

Fig. 3 indicates the behaviour of the DC current I_D resulting on applying the three approaches. The behaviour of a group of other variables such as β, γ , ($\varphi_S + \delta$),V, I_S, (w_r/w_o),(w_v/w_o) and (w_s/w_o), of the first approach only are given in Fig. 4. The first approach, which does not apply any type of control, show unstable operation. The DC-link current increases in an oscillatory manner with diverging envelope to give an inverter failure at advance angle β less than the commutation angle. The angle ($\varphi_s + \delta$), Fig. 4 indicates the mode of operation of the synchronous machine while it is oscillating around w_o. At moments, at which ($\varphi_s + \delta$) = π/2, the machine will act as ideal compensator and doesn't exert the synchronous torque. For ($\varphi_s + \delta$) less or greater than π/2, the machine will act as a motor or generator, respectively. In this case, TS will be positive or negative, consequently. It is evident that V and I_S are also oscillating with ascending exponential envelop. The relative speeds (w_r/w_o), (w_v/w_o) and (w_s/w_o) show the variation in speed of the corresponding variables : E_f, V and I_S, respectively. Either of them is pulsating with a different speed and manner with respect to the rigid reference w_o. In accordance with the results of the second and third approache Fig 4 shows the behaviour of I_D which is also oscillating with converging envelope to reach its steady value in few milli-seconds. It is a also evident that I_D resulting from the third approach reaches its steady value sooner than that resulting from the second approach. Changes in other variables aren't remarkable.

5. CONCLUSION

Quasi-steady behaviour of a system consisting of a synchronous compensator in conjunction with a rigidly impulsed inverter supplying an isolated load had been formulated through an adequate mathematical model The model gives three approaches for solving such problem. The first approach doesn't consider any control concepts ,the second approach considers the excitation control, while the third one considers the excitation and DC voltage behind the DC link controls . Application of the mathematical model to a system having a conventional compensator shows the necessity of continous excitation control , else the system will go out of stability or has an inverter failure. The application of continuous control of both excitation voltage and the voltage behind the DC-. link results in quick stable operation. The mathematical model gives the opportunity to have an optimum dimensioning of each element in the system.

6. REFERENCES

1-Concordia,Ch. " Synchronous Machines, theory and performance ", John Wiley& Sons, Inc, NewYork, 1951.

2 Miller,T.J.E. " Reactive power Control in Electric Systems",John Wiley & Sons, Inc, NewYork, 1982.

3-Hindmarsh,J.," Electrical Machines and their Applications" Pergramon Press, Oxford, Third addition, 1977.

4-AEG. Telefunken " Shaft driven alternator " reference list 1980

5-EL-Shamoty,M.M., Abou-Tabl .H.A." Synchronousation of a synchronous machine to a static Inverter with rigid frequency of steering impulses" Interntional Conf On Elec. Machines 1984,Proceding Part 2, pp 731-734.

6-Scheafer. J. " Rectifer Circuits Theory and Design " John Wiley and Sons, 1965.

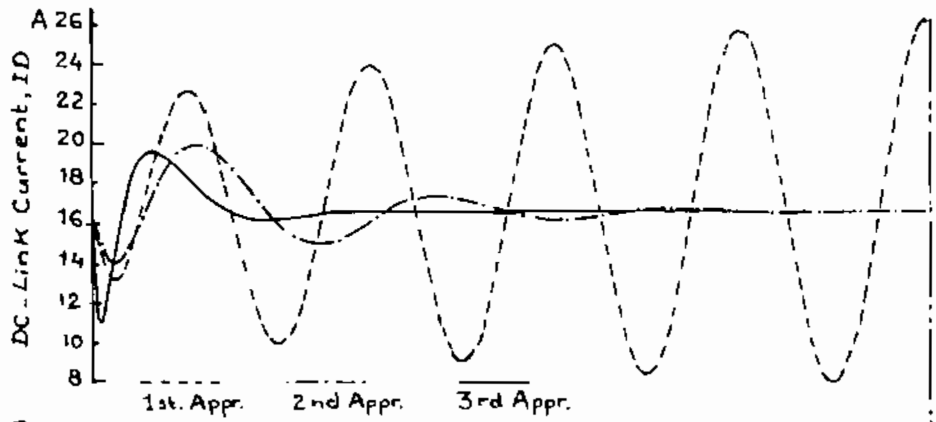
A
DC - Link Current, ID
26
24
22
20
18
16
14
12
10
8
1st. Appr.
2nd Appr.
3rd Appr.

Fig.3

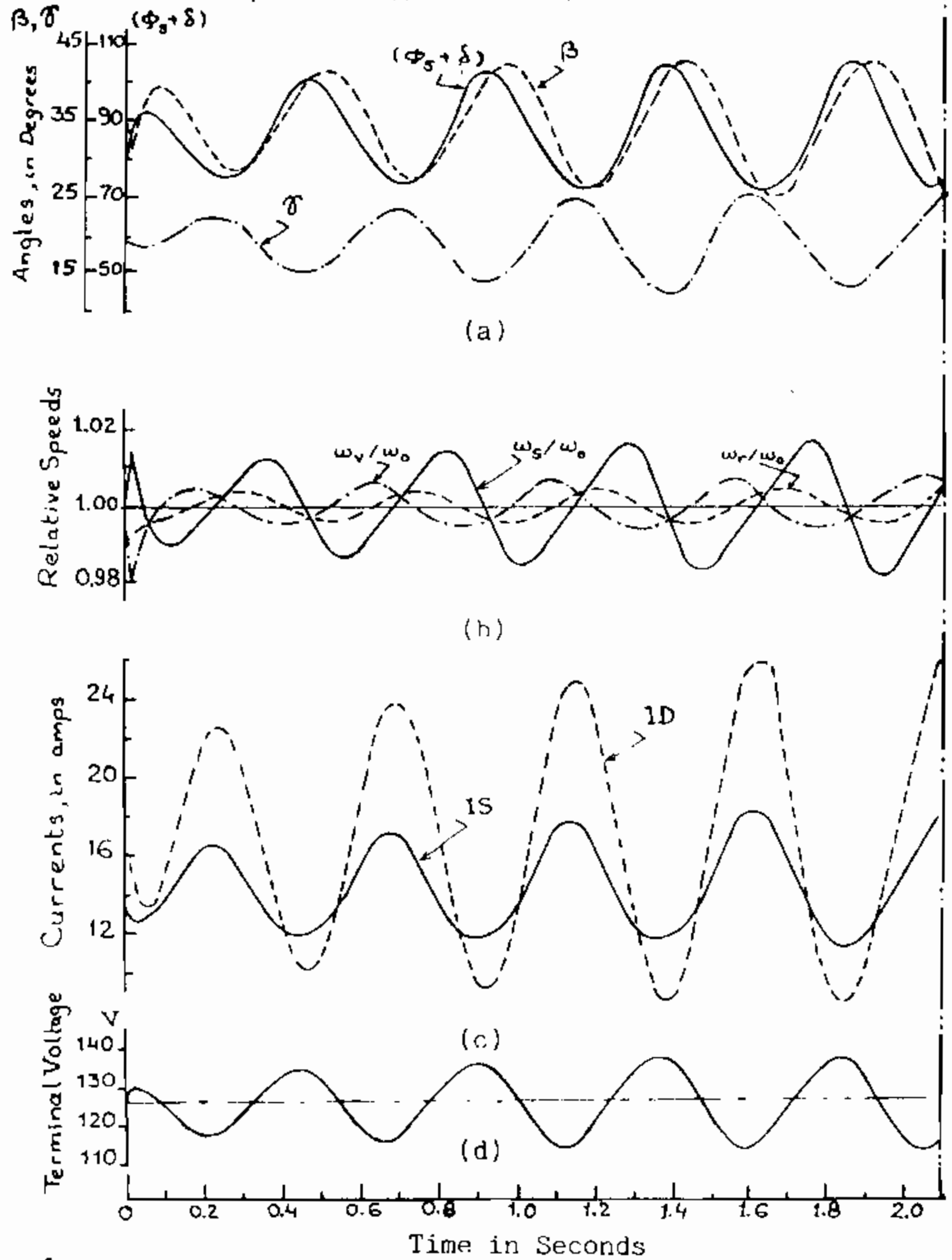
β, γ
(Φs + δ)
Angles, in Degrees
45
35
25
15
110
90
70
50
(Φs + δ)
β
γ
(a)
Relative Speeds
1.02
1.00
0.98
ωv/ωo
ωs/ωo
ωr/ωo
(b)
Currents, in amps
24
20
16
12
ID
IS
(c)
Terminal Voltage
V
140
130
120
110
(d)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
Time in Seconds

Fig. 4