Menofia University
Faculty of Engineering Shebien El-kom
Basic Engineering Sci. Department.
First semester Examination, 2017-2018
Date of Exam : 23 / 5 / 2018

Subject : Complex Analysis .
Code: BES 605
Year : Master Grade 600
Time Allowed : 3 hrs
Total Marks: 100 Marks

Answer the following questions

Question 1 (30 MARKS)

A) (i) Prove that the function $u = e^{-x}(x \sin y - y \cos y)$ is harmonic.

(ii) Find v such that $f(z) = u + iv$ is analytic.

(iii) Find f(z). (10 Marks)

(B) Prove that $\frac{d}{dz}(z^2\,\bar{z})$ doesn't exist any where . (i.e the function is $f(z) = z^2\,\bar{z}$ non analytic) (5 Marks)

(C) Find the orthogonal trajectories of the following families of the curves:

a) $x^3y - xy^3 = \alpha$, b) $e^{-x}\cos y + xy = \alpha$ (5 Marks)

(D) Evaluate

$$\int_{(0,3)}^{(2,4)} (2y + x^2)\,dx + (3x - y)\,dy \quad along:$$

a) The parabola $x = 2t$, $y = t^2 + 3$.

b) The straight line from (0, 3) to (2, 3).

c) Then from (2, 3) to (2, 4) astraight line from (0, 3) to (2, 4). (10 Marks)

Question 2 (40 MARKS)

If C is the curve $y = x3 - 3x2 + 4x - 1$ joining points (1, 1) and (2, 3) . Find the value of :

$$\int_c (12z^2 - 4iz)dz$$

(5 Marks)

(B) If f(z) is analytic inside and on the boundary C of a simply – connected region R , Prove Cauchy's integral formula. $F(a) = \frac{1}{2\pi i}\oint \frac{f(z)}{z-a}dz$ (5 Marks)

(C) Verify Green's theorem in the plane for $\oint_c (2xy - x^2)dx + (x + y^2)dy$

Where C is the closed curve of the region bounded by $y = x^2$ and $y^2 = x$

(10 Marks)

(D) . Evaluate:

$(a)\oint_c \frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)(z-2)}dz,\quad (b)\frac{e^{2z}}{(z+1)^4}dz$ where C is the circle $|z| - 3.$ (10 Marks)

(E) Find the residues of

$$f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$$

at all its poles in the finite plane. (10 Marks)

Question 3 **(30 MARKS)**

(A) Prove that *(i)* $\displaystyle\int_0^{\infty} \frac{\ln(x^2+1)}{x^2+1}\, dx = \pi \ln 2$

(ii) $\displaystyle\int_0^{\infty} \frac{\sin x}{x}\, dx = \frac{\pi}{2}.$

(iii) $\displaystyle\int_0^{\infty} \frac{dx}{x^4+1} = \frac{\pi}{2\sqrt{2}}$ (15 Marks)

(B) (i) Evaluate

$$\oint_c \frac{e^{3z}}{z - \pi i}\, dz$$

if C is

a) the circle $|z - 1| = 4$

b) the ellipse $|z - 2| + |z + 2| = 6$

(ii) Evaluate

$$I = \frac{1}{2\pi i} \oint_c \frac{e^{z}}{z - 2}\, dz$$

if C is

a) the circle $|z| - 3$

b) the circle $|z| = 1$

(iii) Find a transformation that maps the real axis in the w plane onto the ellipse $(x^2/a^2) + (y^2/b^2) = 1$ *in the z plane.* (15 Marks)

This exam measures the following ILOs											
Question Number	Q1-a	Q2-a	Q3-b	Q2-e	Q2-b	Q3-b	Q2-d		Q1-b	Q3-a	Q1-d
Skills			b-ii			b-i					
	Knowledge &understanding skills				Intellectual Skills				Professional Skills		

With my best wishes

Associate Prof. Dr. Islam M. Eldesoky