DETERMINATION OF ELECTROMECHANICAL TRANSIENT PROCESS CHARACTERISTICS USING EQUATION OF VARIABLES

تحديد خصائص الحالات العابرة الكهروميكانيكية باستخدام معادلة المتغيرات

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ملخص البحث:

نتيجة لصعوبة حلى المعادلات اللاخطية وكذلك طول الفترة الرمعية المطلوبة لحل مثل هذه المعادلات ، بالإضافة إلى زيادة التعقيدات في الشبكات الكيربية كان لابد من المحث عن طرق رباضية أخرى عبر تقليدية.

لذلك يقدم هذا البحث حلا جديدا لهذه المشكلة مسنخدما طريقة معادلة المتغيرات، وهذه الطريقة عتربيبية تعتمد على فرص قيمة للحل غير خاضعة لأى تغيير وتتكون هذه المعادلة من عدد من المعاملات، كل معامل له معادلة نتوقف على درجة الحل ويسمى الحل الأول وبزيادة درجة الحل يكون الحل الثاني وهكدا. وهي النهاية يكون الحل التقريبي باستخدام هذه الطريقة عبارة عن محموع الحل المفروض مضافا البه المعاملات عليه يكون هداك فحرق عن الحل المضبوط يسمى بالإنجراف في قيمة المتغير وباستخدام هذا الإنجراف يمكن الوصول إلى حل نقيس سبة خطأه لانتجاوز 25٪. عن الحل المضبوط. وباستخدام هذه الحلول يمكن نحديد الخصائص الكبروميكانيكية للحالات العابرة بيسر وبسرعة كبيرة وبدقة عالية.

ولقد تم تقديم وصفا كاملا للطريقة المقترحة وكذلك التحليلات الرياضية الخاصة بها، وأيصا تم ترجمة النتائج التي توصل إليها الدحث في حالتي الخطأ الدسيط والمتسدب فيه تغير أحد عناصر الثبكة الكبربية وكذلك الخطأ الكبير نتيجة حدوث قصر أحادى الوجه، وتمت المقارنة بين الطريفة المقترحة وطريفتيس أحرتيس لتوضيح سرعة الحل، وأيضا تُمت المقارنة بين هذه الطريقة والطريقة المضبوطة والتي وضح منها مدى دقة الطريقة المقترحة.

ABSTRACT

The equation of variables (EV) is an approximation of a mathematical method. The used to linearize nonlinear equations. It is simple and accurate method and it offers quick solutions. This paper explains this equation and uses it to determine the characteristics of electromechanical transient process(EMTP) in electrical power systems.

The characteristics of EMTP are studied in this paper in two cases: first, simple disturbance / (as a result of parameter variation) and second, large disturbance (as a result of occuring single S.C.) In this paper also, a comparison between the proposed method and another three methods is presented. One of these methods has exact solution and the others have an approximate one.

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INTRODUCTION

Due to rapid growth of electrical power systems, over the past few years, stability analysis of the system became extremely difficult, even, using the present day computer facilities. Dynamic stability of electrical power systems stands as a subject of theoretical and practical interest.

Solution of non-linear equations, describing transient processes in power systems using method of optimal control theory directly, is too tidious to apply [1&2].

The output of a conventional transient stability program is analyzed using transient energy functions for individual machines [3].

The stability of power systems can be deduced from the roots of the system characteristic equation [4]. On the other hand, in a nonlinear system, such an inference is invalid [5] because its stability depends upon the initial conditions as well as, the system parameters.

This paper illustrates the characteristics of EMTP using EV and presents the comparison between EV and those methods introduced in [6].

ANALYSIS SOLUTION OF EV

The error of determination EMTP characterestics with the help of EV may be evaluated. The solution of EV is analyzed in the case of selecting undisturbed and parameter variation system[6]. Using this EV, the nonlinear differential equations will be transformed into second order linear differential equations with constant coefficients. These coefficients are illustrated and determined in [7].

As, in the capacity of parameter variation, the admittance (y) is considered. From [6 & 8] the rotor angle (δ) is:

$$\delta(t,y) = \delta^* + L_1(t)\Delta y + \frac{1}{2!}L_2(t)\Delta y^2 + \dots + R_1,$$
(1)

where δ^* - the undisturbed value of δ

Ri - the remainder of series (1)

$$=L^{@}_{(i+1)}(t)\Delta y^{(i+1)}/(i+1)!,...@\varepsilon\Delta y = y - y^{*}$$
(2)

 $L_1(t), L_2(t)$ etc. are the first , second etc. solution of EV.

The first EV is presented in [6]. It has the following form:

$$L_1(t) = C_0^{(1)} (1 - \cos \sqrt{a_1} t), ... C_0^{(1)} = K_0^{(1)} / a_1 = -\tan(\delta') / y'$$
(3)

where

$$a_1 = \frac{2\pi f_0}{T_j} E' U^* y \cos \delta^* = \text{const.}$$

$$K_0^{(1)} = \frac{2\pi f_0}{T_j} E' U \sin \delta^* = \text{const.}$$
(4)

E'& U -the generator and infinite bus voltages (in p. u.): T_j - the generator inertia constant;

the undisturbed value of y (in p.u.)

Analysis of EMTP characteristics is carried-out for the following cases:

- i) Undisturbed stable(unstable) solution, standard solution, which is approximated with the help of variable coefficients method (VCM) [6];
- ii) Undisturbed stable(unstable) solution, standard solution unstable (stable).

The second EV is

$$L_2 + a_1 L_2 = d_2(t),$$
 $L_2(0) = L_2(0) = 0$ (5)

where
$$d_2(t) = \frac{\partial^3 f}{\partial \delta^2} L_1^2(t) + 2 \frac{\partial^2 f}{\partial \delta \partial y} L_1(t)$$
 (6)

From Eq.(6) and Eq.(3)

$$d_2 = K_0^{(2)} + K_1^{(2)} \cos \sqrt{a_1} t + K_2^{(2)} \cos 2\sqrt{a_1} t$$
 (7)

where; the upper index indicates the order of EV and the lower shows the ratio of argument.

From Eq.(6) and the initial conditions, d_2 (0) =0. From this, we get,

$$K_0^{(2)} + K_1^{(2)} + K_2^{(2)} = 0$$
 (8)

Integrating the second EV at the initial conditions and substituting in Eq.(7), we have

$$L_2(t) = C_0^{(2)} + C_1^{(2)} \cos \sqrt{a_1} t + C_2^{(2)} \cos 2\sqrt{a_1} t + R_1^{(2)} t \sin \sqrt{a_1} t, \tag{9}$$

where; L_2 (t) is determined from coefficients $k_0^{(1)}, k_1^{(2)}, k_2^{(2)}$ using the following expressions:

$$C_0^{(2)} = K_0^{(2)}/a_1$$
, $C_1^{(2)} = -(C_0^{(2)} + C_2^{(2)})$,
 $C_2^{(2)} = -K_2^{(2)}/3a_1$, $R_1^{(2)} = -K_1^{(2)}/2\sqrt{a_1}$ (10)

For the third EV

$$\ddot{L}_3 + a_1 L_3 = d_3(t)$$
 , $L_3(0) = 0$, (11)

where d₃(t) is determined from the following formula:

$$d_3(t) = \frac{\partial^3 f}{\partial \delta^3} L_1^3(t) + 3 \frac{\partial^3 f}{\partial \delta^2 \partial y} L_1^2(t) + 3 \frac{\partial^2 f}{\partial \delta^2} L_1(t) L_2(t) + 3 \frac{\partial^2 f}{\partial \delta \partial y} L_2(t)$$
 (12)

From Eqns.(12), (3) & (9), we find

$$d_3(t) = K_0^{(3)} + K_1^{(3)} \cos \sqrt{a_1} t + K_2^{(3)} \cos 2\sqrt{a_1} t + K_3^{(3)} \cos 3\sqrt{a_1} t + K_3^{(3)} + t \sin \sqrt{a_1} t + M_2^{(3)} + t \sin 2\sqrt{a_1} t$$
(13)

From Eq.(13) and initial conditions, d3 (0)=0. Then

$$K_0^{(3)} + K_1^{(3)} + K_2^{(3)} + K_3^{(3)} = 0$$
 (14)

Integrating Eq.(11) at initial conditions and substituting in Eq.(13), we have

$$L_3(t) = C_0^{(3)} + C_1^{(3)} \cos \sqrt{a_1} t + C_2^{(3)} \cos 2\sqrt{a_1} t + C_3^{(3)} \cos 3\sqrt{a_1} t + C_3^{(3)} t \sin \sqrt{a_1} t + R_2^{(3)} t \sin 2\sqrt{a_1} t + S_1^{(3)} t^2 \cos \sqrt{a_1} t,$$
(15)

where ; $L_3(t)$ is determined from coefficients $k_0^{(3)}, k_1^{(3)}, k_2^{(3)}, k_3^{(3)}, M_1^{(3)}, M_2^{(3)}$ using the following expressions:

$$C_0^{(3)} = \frac{K_0^{(3)}}{a_1} , S_1^{(3)} = \frac{M_1^{(3)}}{4\sqrt{a_1}} , R_1^{(3)} = \frac{K_1^{(3)}}{2\sqrt{a_1}} + \frac{M_1^{(3)}}{4a_1} , C_3^{(3)} = \frac{K_3^{(3)}}{8a_1} ,$$

$$R_1^{(3)} = \frac{M_2^{(3)}}{a_1} , C_3^{(3)} = \frac{4M_2^{(3)}}{a_1} , C_3^{(3)} = \frac{K_3^{(3)}}{a_2} , C_3^{(3)} = \frac{K_3^{(3)}}{a_2} ,$$

$$R_{2}^{(3)} = \frac{M_{2}^{(3)}}{3a_{1}}, C_{2}^{(3)} = \frac{4M_{2}^{(3)}}{9a_{1}\sqrt{a_{1}}} - \frac{k^{(3)}}{3a_{1}}, C_{1}^{(3)} = -(C_{0}^{(3)} + C_{2}^{(3)} + C_{3}^{(3)})$$

From the previous analysis,the solution of i^{th} EV (general form) is possible to write in the form

$$L_{i}(t) = \sum_{n=0}^{i} C_{n}^{(i)} \cos n \sqrt{a_{1}} t + t \sum_{n=0}^{i-1} R_{n}^{(i)} \sin n \sqrt{a_{1}} t + \dots + t^{i-2} \sum_{n=0}^{i-(i-2)} Y_{n}^{(i)} \cos n \sqrt{a_{1}} t + t^{i-1} \sum_{n=0}^{i-(i-1)} Z_{n}^{(i)} \sin n \sqrt{a_{1}} t$$
(16)

As follows from the obtained equations, the solutions of EV $L_2(t), \ldots, L_i(t)$ increase without bound on time increase. These solutions pass unbounded period of time through its initial value.

The performance of the proposed approach is investigated by computer simulation of power system consisting of a single machine connected to an infinite bus through long transmission line. The data and figure of the system are given in [1].

EXAMPLE 1:

n=0, 1, 2,....,i

In this part, the obtained solutions of EV will be applied on a mathematical example to show the character of approximated standard characterestics. Data of this example are P_0 = 0.6 p.u.; $\dot{\text{EU}}$ = 1.2 p.u. ; T_j =7 s.; δ =26 deg.; y^* = 1.14 p.u. Coefficient values of unbounded terms and obtained solutions of first three EV are listed in table 1. Solutions of first three EV depending on time are shown in Fig.1. With the help of these solutions and the following formula

$$\delta(t,y) = \delta' + L_1(t)\Delta y + \frac{1}{2!}L_2(t)\Delta^2 y + \frac{1}{3!}L_3(t)\Delta^3 y$$
,

we shall consider three approximated standard characterestics. EMTP: single phase short circuit (S.C.) y = 0.34 p.u., $\Delta y = -0.8$ p.u. (Fig.2); perturbation y = 0.82 p.u.; $\Delta y = -0.32$ p.u. (Fig.3); disturbance near to critical value for admittance $y_c = \frac{P_0}{EU}$; y = 0.6 p.u.; $\Delta y = -0.54$ p.u. (Fig.4). Curves in these figures are obtained with the help of first, second and third EV. Also steady state undisturbed value of angle δ^* and standard characteristic $\delta_0(t)$.

On the basis of analyzed solutions EV, it is possible to describe the characteristics of EMTP. Thus, in the case of S.C., calculate first three EV exact values in large range of angle variations with time (divergence larger than 5 deg. step at 0.58 s. and angle value 160 deg.).

After recovery of S.C. (i.e. parameter variation is decayed), EV and its solutions are varied. Besides initial conditions of these new EV, will be greater than zero($t=t_{dis}$) where t_{dis} is the removed fault time.

After decaying of parameter variations, the first EV have the following expressions:

$$\ddot{L}_{1}^{(0)} + a_{1}L_{1}^{(0)} = 0, \quad L_{1}^{(0)}(\tau = 0) = L_{1}(t = t_{dis})$$

$$\dot{L}_{1}^{(0)}(\tau = 0) = \dot{L}_{1}(t = t_{dis})$$
(17)

where $\tau = t - t_{dis}$; upper index (0) is meaning that the solutions of EV are after recovery of S.C.

Solution of (17) has the following form

$$L_1^{(0)}(\tau) = A_0^{(1)} \cos(\sqrt{a_1} \tau - \phi_1^{(1)})$$
 (18)

where , the upper index indicates identity value for order data of EV; and lower index shows the ratio of solution argument. Solution of amplitude values $A_0^{(1)}$ and its angle $\phi_1^{(1)}$ depends on solution values of

$$\ddot{L}_1 + a_1 L_1 = d_1 = k_0^{(1)}; \quad L_1(0) = \dot{L}_1(0) = 0$$

and its first derivative at ending variation moment (tdis). These values are determined as:

$$\phi_1^{(1)} = \arctan \frac{\dot{L}_1(t_{dis})}{\dot{L}_1(t_{dis})} = \arctan \frac{\sin \sqrt{a_1} t_{dis}}{1 - \cos \sqrt{a_1} t_{dis}}, A_0^{(1)} = \frac{\dot{L}_1(t_{dos})}{\cos \phi_1^{(1)}}$$
(19)

After a little manipulation in expression for solving first EV(18) it is possible to eliminate time τ as follows

$$L_1^{(0)}(t) = A_0^{(1)} \cos(\sqrt{a_1}t - \theta), \quad \theta(t_{dis}) = \sqrt{a_1}t_{dis} + \phi_1^{(1)}$$
(20)

Fig.5 shows the relations between coefficient L and time in the case of disconnected S.C. (t_{dis} =0.25 s.). The solution of first EV is also shown in Fig.5 " curve no.1". This curve consists of two parts: first part represents Eq. (3) (from origin point to $t = t_{dis}$) and the second represents Eq.(18) (from $t > t_{dis}$). In Fig. 5, curve no.2 represents the same solution of EV without disconnecting S.C.. Bias quantity of extremal values of these two curves (1 & 2) is equal to

$$\Delta = (\pi - \phi_1^{(1)}) / \sqrt{a_1}$$

Curve no.3 represents the solution in the case of $t=t_{mdis}=\sqrt{a_1}=0.4229$ s., at which extremal value of $L_m^{(1)}=2C_0$, is reached.

For the second EV

$$L_{2}^{(0)} + a_{1}L_{2}^{(0)} = d_{2}(\tau), \quad L_{2}^{(0)}(\tau = 0) = L_{2}(1 = t_{dis})$$

$$L_{2}^{(0)}(\tau = 0) = L_{2}(t = t_{dis})$$
(21)

where

$$d_{2}(\tau) = \frac{\partial^{2} f}{\partial \delta^{2}} L_{1}^{(0)^{2}}(\tau) = l_{0}^{(2)} + l_{2c}^{(2)} \cos 2\sqrt{a_{1}} \tau + l_{2a}^{(2)} \sin 2\sqrt{a_{1}} \tau$$
 (22)

The right hand side of Eq.(22) is accomplished with the substitution of solution in Eq.(18). Integrating the second EV at given initial conditions and using Eq.(22), we obtain

$$L_{2}^{(0)}(\tau) = A_{0}^{(2)} + A_{1}^{(2)} \cos(\sqrt{a_{1}}\tau - \phi_{1}^{(2)}) + A_{2}^{(2)} \cos(2\sqrt{a_{1}}\tau - \phi_{2}^{(2)})$$
(23)

The solution of amplitudes and phases of Eq. (23) are determined through coefficients $l_0^{(2)}, l_{2c}^{(2)}, l_{2s}^{(2)}$ as follows:

$$A_{0}^{(2)} = \frac{l_{0}^{(2)}}{a_{1}}, \ \phi_{1}^{(2)} = \arctan \frac{L_{2}(l_{dis}) + 2l_{2s}^{(2)}/3\sqrt{a_{1}}}{L_{2}(l_{dis})\sqrt{a_{1} - (l_{0}^{(2)} - l_{2s}^{(2)}/3)/\sqrt{a_{1}}}}$$

$$\phi_{2}^{(2)} = \arctan \frac{l_{2s}^{(2)}}{l_{2s}^{(2)}}, \ A_{1}^{(2)} = [L_{2}(l_{dis}) - (l_{0}^{(2)} - l_{2s}^{(2)}/3)]/\cos\phi_{1}^{(2)};$$

$$A_{2}^{(2)} = l_{2s}^{(2)}/(3a_{1}\cos\phi_{2}^{(2)})$$
(24)

The solution of ith EV after disconnecting S.C. may be written as follows:

$$L_{i}^{(0)}(\tau) = A_{v}^{(i)} + \sum_{n=1}^{i} A_{n}^{(i)} \cos(n\sqrt{a_{1}}\tau - \phi_{n}^{(i)}) + \tau \sum_{n=1}^{i} Q_{n-2}^{(i)} \cos[(n-2)\sqrt{a_{1}}\tau - \phi_{n-2}Q^{(i)}] + \tau^{2} \sum_{n=1}^{i} V_{n-3}^{(i)} \sin[(n-3)\sqrt{a_{1}}\tau - \phi_{n-3}V^{(i)}] + \dots$$
(25)

where n=1,2,....i.

The final admittance variations for i the solution of ith EV are

$$L_{i} = \begin{cases} L_{i}(t), 0 < t < t_{dat}, & a \sin Eq.(16) \\ L_{i}(t_{dis}) = L_{i}^{(0)}(\tau = 0), t = t_{du} \\ L_{i}^{(0)}(\tau), t_{dis} < t < t_{s}, \tau = t - t_{dis} & a \sin Eq.(25) \end{cases}$$
(26)

Notice that the obtained expressions are possible to apply by considering a series number of iterations. Corresponding initial conditions are very necessary to be computed continuously.

EXAMPLE 2:

In this section, consider a numerical example to indicate the behaviour of approximate characteristics after decaying of parameter variations. We used the same data in the previous example. Fig.6 shows the solutions $L_1, L_2 \& L_3$, which are determined from Eq.(26), at $t_{\rm dis}$ =0.25 s.. From this figure, we notice that the curves from origin point to t= $t_{\rm dis}$ are as in Fig. 1. Amplitude and phase values of corresponding solutions, which are determined from Eqns.(19),(24),..., are given in table 2.

Fig.7 & Fig.8 show approximate curves(time interval 0.05 s) δ_0 with the help of $L_1, L_2 \& L_3$ (curves 1,2&3). Fig.7 shows the disconnecting single phase S.C. process (y=0.34p.u , Δ y=-0.8 p.u.) within 0.25 s., but Fig.8 shows finishing parameter variations(y=0.82 p.u., Δ y=-0.32 p.u.) within 0.25 s. Also, these figures show the undisturbed angle value δ^* .

A comparison between the proposed and exact solutions of the problem under study in the case of system parameter variation is shown in Fig. 9 This figure shows that the error of the proposed solution may be neglected.

FINAL RESULTS ANALYSIS:

As we conclude from figures, the obtained analytical solutions Eq.(26) permit, steady-state solutions give good approximate characterestics EMTP at disconnecting S.C. (finished admittance variations). In addition, in the case of small variations, curves L_2, L_3 practically coinside with standard curve in Fig.8. But in the case of S.C. Sequence of using EV may achieve an accuracy near to standard characteristics $\delta_0(t)$.

An approach to obtain the family of EMTP characteristics with the help of corresponding EV analytical solutions, at steady-state undisturbed values, have series of advantages in comparison with the other methods.

As shown from table 3, the proposed method solves the example under study in 1.5 sec.. Also, the method is very fast compared with the other methods.

CONCLUSIONS:

From the previous analysis, figures and results, we conclude the following:-

- 1-With the help of EV solutions, characteristics family may be rapidly obtained. Rapidity of this method is greater than interval sequence method (nearly 10 times) and from 20 to 25 times greater than conventional method in [6].
- 2-in the case of steady-state, undisturbed values can describe undisturbed transient process.
- 3- This method is simple ,responding and accurate.
- 4- This method is applied on a simple electrical system (generator with infinite bus.),
- 5- Comparing the results of this method with the exact solution, we find that the error of this method may be neglected.

Table 1

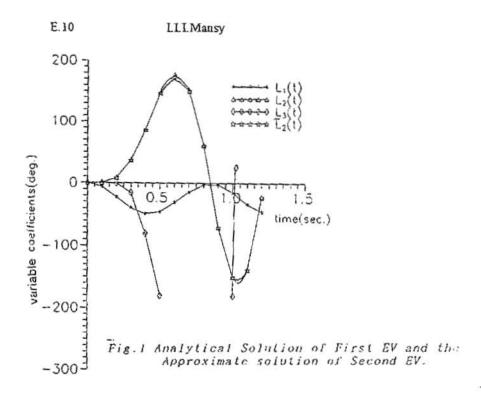
i	k (i)	k'ii	Х.,	K _s	M.	M ₂	
1 2 3	-1352.7 2796.5 -9484.8	-2937.7 10178.7	141.13 -707.9	47.9	~35632.4	-3424	4.7
i	c',,,	c', ,	C, 1,	C,''	R: '	2	8,,,
1 2 3	-24.51 50.68 -171.9	-49.83 164.11	-0.85 7.91	-0.11	-198.19 846.4	20.71	-1199.1

Table 2

i	A (1)	A(i)	λ, ()	y,,,	Δ,,,	$\phi_{i}^{(1)}$	$\phi_2^{(1)}$	φ'ιι'	φ'i)
2 3	-39.25 6.56 -16.73	41.31 38.51					-73.55 70.25		53.19

. Table 3 Execution time comparison for the proposed method and other methods

Methods	1.5 14.35 33.52		
proposed method interval seq. method approximation method			
in [5]	1		



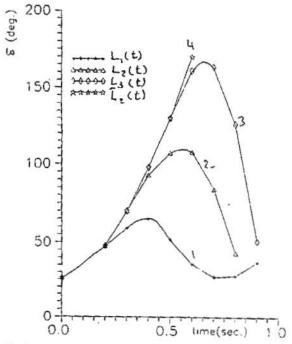


Fig. 2 Approximate Characteristics of EMTP With the Help of Analytical solutions for a System Parameter Variation (x=-3.8)

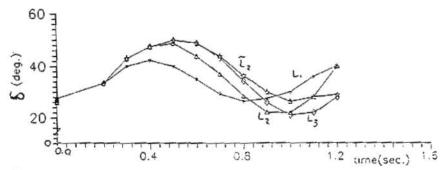


Fig.3 Approximate Characteristics of EMTP With the NeIp of Analytical solutions for a System Parameter Variation ($\triangle y=-0.32$)

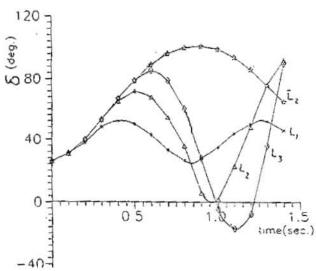
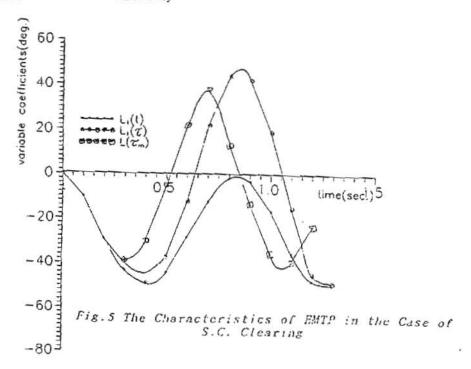


Fig.4 Approximate Characteristics of EMTP With the Help of Analytical solutions for a System Parameter Variation(4y=-0.54)



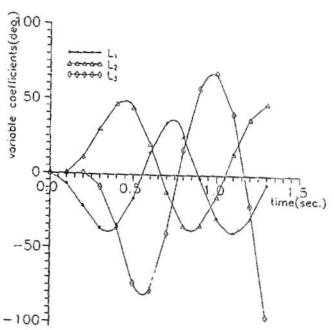
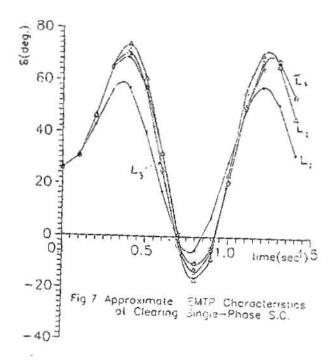


Fig. 6 Analytical Solution of First EV at Disconnecting S.C.



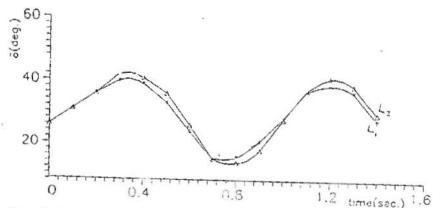


Fig. 8 Approximate EMTP Characteristics following removal of Parameter Variations

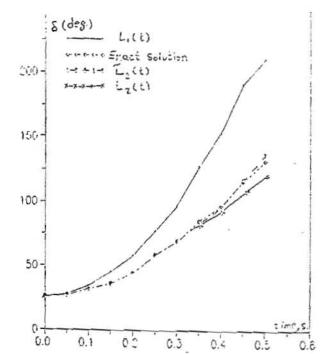


Fig. 9 Approximate and Exact characteristics of EMTP

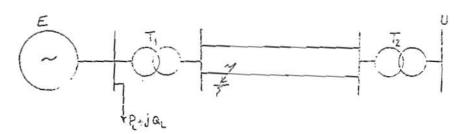


Fig. 10 System under study (Generator and Infinite-Bus)

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