Development of Computer Vision Algorithms for Measurement and Inspection of Spur Gears

تطوير أساليب رؤية بالحاسب لقياس وفحص التروس العدلة

E.S. Gadelmawla

Production Engineering and Mechanical Design Department, Faculty of Engineering, Mansoura University, Mansoura 35516, Egypt

Email: esamy@mans.edu.eg

الملخص العربي:

تعتبر التروس من أهم الأليات الميكانيكية لنقل الطاقة والحركة. ويسبب الخطأ في التروس مشكلتين أساسيتين هما زيادة الضوضاء وزيادة التآكل، وكلاهما ضار بشكل يسبب القلق. لذلك تلعب القياسات الدقيقة للتروس دورا هاما في قياس وفحص التروس. وتعتبر الطرق الحالية لقياس التروس مستهلكة للوقت أو ذات تكلفة عالية. بالإضافة إلى ذلك، لا توجد طريقة قياس واحدة متاحة قادرة على قياس كل معاملات التروس بدقة مع تقليل وقت القياس بشكل واضح. لذلك فإن الهدف من هذا البحث هو توظيف تكنولوجيا الرؤية بالحاسب لتطوير نظام قياس غير تلامسي وسريع قادر على قياس وفحص أغلب معاملات التروس العدلة بدقة مقبولة. وقد تم إنشاء هذا النظام في معمل القياسات الميكانيكية بكلية الهندسة جامعة المنصورة، حيث يستخدم لالتقاط صور للتروس المراد قياسها أو فحصها، ثم يتم تحليل هذه الصور باستخدام برنامج تم تطويره خصيصا لقياس وفحص التروس من خلال الصور الملتقطة. وقد تم تطوير هذا البرنامج كاملا أثناء البحث باستخدام لغة البرمجة (Visual C++). وقد تم معايرة النظام بحيث يمكن استخدام النظام المتري أو النقطة (Pixel) المستخدمة في نظام الصور. وبعد المعايرة تم اختبار دقة النظام المقدم بقياس ترسين مختلفين ومقارنة نتائج القياس بالقيم الحقيقية. وقد وجد أن أكبر اختلاف بين القيم المحسوبة والقيم الحقيقية هو ٠,١٠١ مم، وذلك عند قياس ترس عدل قطره الخارجي ١٥٦ مم. ويمكن زيادة دقة القياس باستخدام النظام لقياس تروس أصغر.

Abstract:

Gears are one of the most common mechanisms for transmitting power and motion. Error in gears causes two main problems, increased acoustic noise in operation and increased wear, both of which are sufficiently troublesome to cause concern. Therefore, precision measurement of gears plays a vital role in gear measurement and inspection. The current methods of gear measurement are either time consuming or expensive. In addition, no single measurement method is available and capable of accurately measuring all gear parameters while significantly reducing the measurement time. The aim of this paper is to utilize the computer vision technology to develop a non-contact and rapid measurement system capable of measuring and inspecting most of spur gear parameters with an appropriate accuracy. The vision system has been established in the metrology lab and it is used to capture images for gears to be measured or inspected. A software (named *GearVision*) has been especially developed in-house using Microsoft Visual C++ to analyze the captured images and to perform the measurement and inspection processes. The introduced vision system has been calibrated for both metric and pixel units. After calibration, the system was verified by measuring two sample gears and comparing the calculated parameters with the actual values of gear parameters. The maximum variations between the calculated parameters and the actual values were ±0.101 mm for a spur gear with 156 mm outside diameter. This variation can be decreased by measuring small gears.

Keywords: Spur gears, Computer vision, Image processing, Measurements.

Accepted March 31, 2009.

1. Introduction

Gears are one of the most common mechanisms for transmitting power and motion. For most of the modern industrial and transport applications, gears are important and are frequently used as fundamental components [1]. Error in the manufacture of gears causes two main problems, increased acoustic noise in operation and increased wear, both of which are sufficiently troublesome to cause concern [2].

For closer control over the accuracy of gears manufacture, precision measurement of gears plays a vital role. Spur gears have the majority among all types of gears in use; therefore automating the measurement process of spur gears becomes a persisting target.

The deviation of an actual tooth from the design profile, the profile error, can be measured in a number of ways. The simplest way is to measure the tooth width at a number of pitches using an adapted caliper gauge [3]. Another method is to use gauging with a moving probe, with a displacement transducer attached, which traces the design profile. Many mechanical-probe gear inspection systems are available but these systems are not suitable for inspection of smaller gears. Some attempts have been made to develop smaller probes capable of measurement of small mechanical elements [4]. Alternatives are to use a coordinate measurement machine to measure the actual profile, or rolling the gear across a stationary probe [5-7]. Optical methods have also been employed to measure gear-tooth deformation [8], pitch errors and tooth profiles [9]. The use of laser based system to measure the thickness, pitch, and tooth flank profile of spur gears was investigated by Younes et al [10, 11].

The current methods of spur gear measurement are either time consuming or expensive. In addition, no single measurement method is available and capable of accurately measuring all gear parameters while significantly reducing the measurement time. Therefore, the measurement and inspection of spur gears has been emphasized by many researchers.

Recently, vision systems have been widely used in many applications [12-15]. Computer vision systems have been developed for quality control and started to be used as an objective measurement and evaluation systems [16]. Robinson et al. [17] described the design of an involute spur gear inspection system in which measurements were made using a video camera and image analysis software. They investigated the possible measurement accuracy and the possible sources of error identified. They concluded that the measurement accuracy is comparable to that of current methods for tolerance inspection of spur gears. In addition, the low cost and ease of use made image analysis measurement systems an attractive alternative. Sung et al. [18] employs wavelet transform to detect the location of tooth defects in a gear system with high accuracy. They reported that utilizing this approach might improve the ability for fault detection of a gear transmission system, especially when the faulty gear rotates in an angular speed close to those of other gears.

The aim of this paper is to utilize computer vision to develop a non-contact measurement system capable of measuring most of spur gear parameters rapidly with a reasonable accuracy. This can facilitate and speed up the measurement and inspection processes of spur gears.

2. The Proposed System

Fig. 1 shows a photograph of the proposed vision system. It consists of two main parts, hardware and a developed software. The hardware consists of three items. The first item is the backlighting table (1), which is a lighting box with diffusing surface at its front, and it is used to produce a back lighting for the gear to be

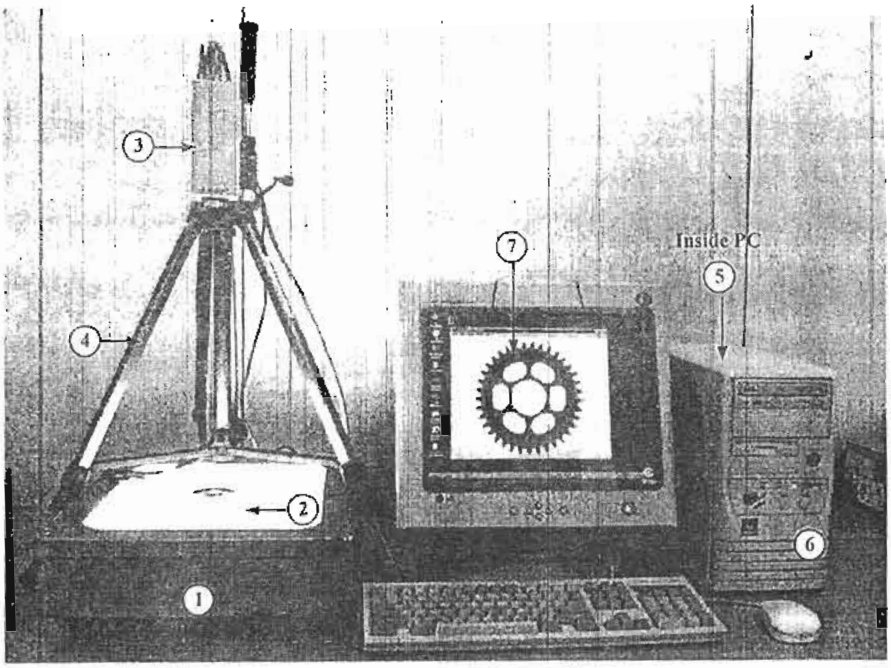


Fig 1: Photograph of the proposed vision system

measured (2). The second item is a CCD color video camera (3) and a set of lenses with different focal lengths. The camera is carried by a camera holder (4). The third item is a 24 bit per pixel (ELF VGA) frame grabber video card (5), which is installed inside the PC computer (6) and connected to the CCD camera. A capturing software (7) is provided with the frame grabber to acquire images and save it to files with various types of file formats.

The developed software (named *GearVision*) is fully written in-house using Microsoft Visual C++ as a 32-bit Windows application. It features many image processing and computer vision algorithms to measure spur gear parameters from captured images of the spur gears. Fig. 3 shows the main interface of the *GearVision* software.

To perform the measurement or inspection process, the gear to be measured is set on the backlighting table then an image is captured and saved to a BMP file using the capturing software. The captured image is then opened by the *GearVision* software and analyzed to perform the measurement and inspection processes.

3. The Developed Algorithms

To perform the measurement process, two data items are required to be entered from the user through the main interface of the *GearVision* software (Fig. 2). The first item is the value of the outer diameter (D_O) of the gear to be measured and the second data is the pressure angle (ϕ) of the gear. If the outer diameter is specified, the system will be calibrated automatically and the measurements will be performed in millimeter, otherwise, it will be performed in image pixels. If the pressure angle is specified, both base circle diameter (D_B) and base pitch (P_B) will be calculated.

Several image processing and computer vision algorithms (Gear algorithms) are applied to the captured image to perform the measurement process. Figure 3 shows a block diagram of these algorithms and the following sections explain them.

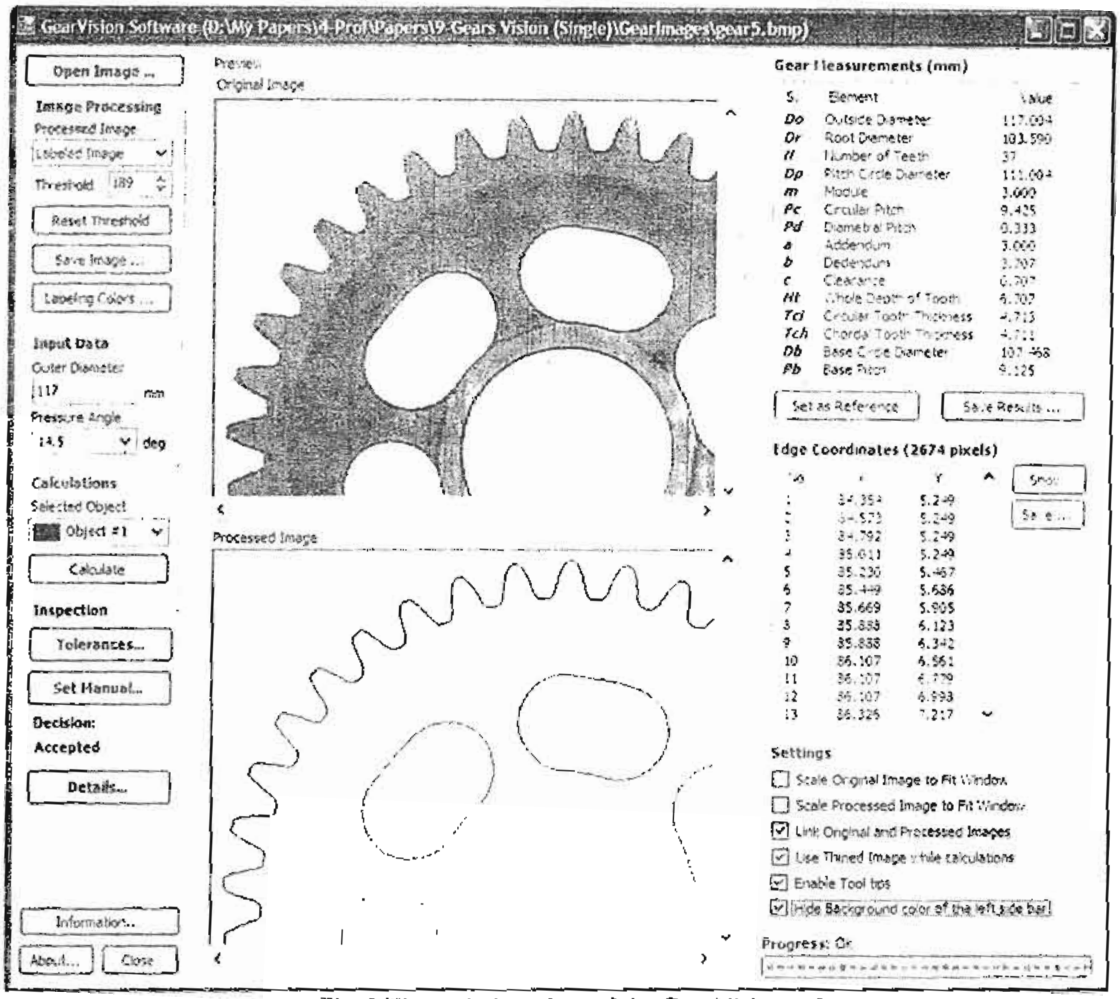


Fig. 2 The main interface of the GearVision software

3.1 Image Processing Algorithms

The image processing algorithms shown in figure 3 were discussed in details in a previous work [19]. In short, the process starts by opening the image of the gear to be measured. If the opened image is colored, then it will be converted into gray scale image. The gray image is converted into binary image based on a calculated threshold. After creating the binary image, an edge detection algorithm is applied to the binary image to extract the edge pixels, which represent the boundaries of the gear elements. A thinning algorithm is then applied to the edge pixels in order to remove extra pixels obtained by the edge detection algorithm. Fig. 4-a shows a sample gear image and Fig. 4-b shows the extracted edge pixels obtained by the edge detection algorithm after applying the thinning algorithm.

Next, a labeling algorithm is applied to the thinned image in order to mark each set of connected pixels by a unique label. As a result, the outer contour, which represents the profile of the gear teeth, can be distinguished from other contours such as the gear's hole or the internal slots. Once the outer contour is labeled, its pixels are extracted and a sorting algorithm is applied to sort the pixels sequentially according to the distances between each two successive pixels.

At this point, the coordinates of the pixels constituting the outer profile of the

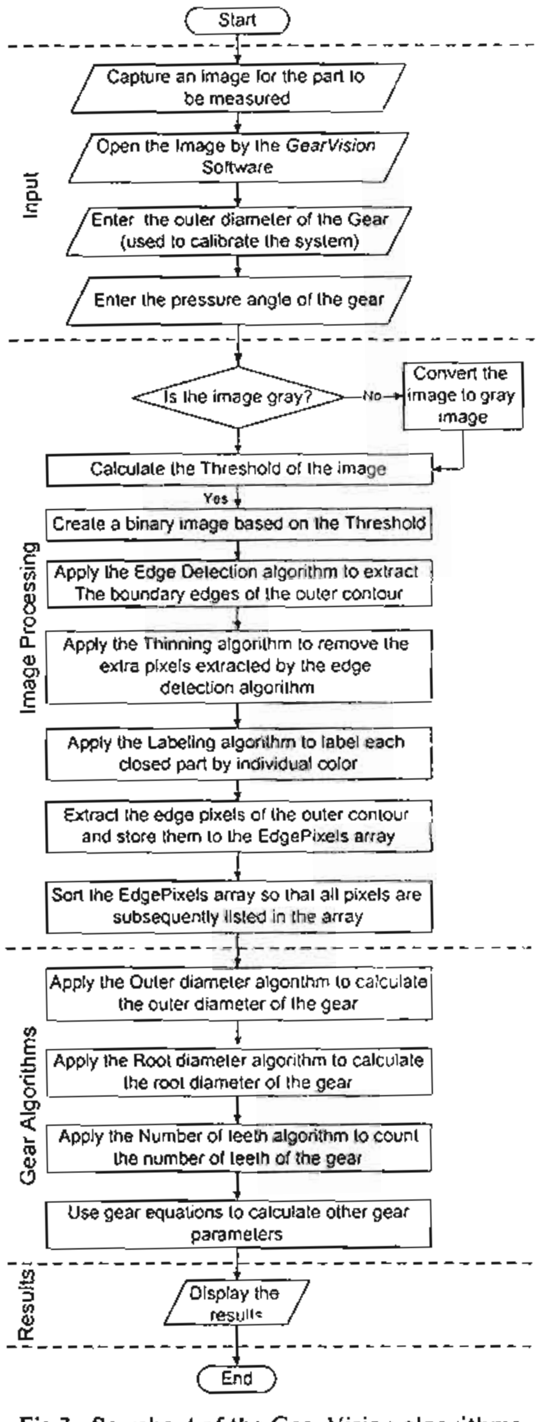


Fig 3. flowchart of the GearVision algorithms

gear (teeth profile) are known and it is required to develop some computer vision algorithms capable of utilizing these data to measure the spur gear parameters.

It is well known that many of the gear parameters are correlated to each other by known equations [20]. For example, the pitch circle diameter (D) is correlated to the number of teeth and the dimetral pitch while the module (m) is correlated to the pitch circle diameter and the number of teeth. In this situation, three computer vision algorithms were developed to calculate the outer diameter, the root diameter and the number of teeth of the gear to be measured, then other gear parameters are calculated using their equations based on these parameters.

3.2 The Outer Diameter Algorithm

The Outer diameter algorithm was developed to calculate the outer diameter (D_o) of the gear from the extracted edge pixels of the outer contour (teeth profile). This algorithm works as follows:

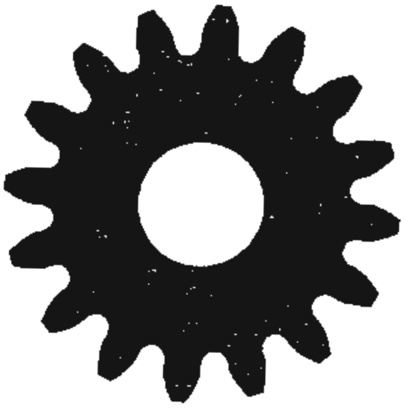
1- Search all edge pixels of the outer contour to find the following pixels:
   - The two pixels having the minimum and the maximum x coordinates. (Px_{min}, Px_{max}).
   - The two pixels having the minimum and the maximum y coordinates (Py_{min}, Py_{max}).

2- Apply the least square circle method (discussed in details in [19]) to define a primary outer circle (POC) that fit the pixels (Px_{min}, Px_{max}, Py_{min}, Py_{max}) and to calculate its center (POC_{cx}, POC_{cy}) and radius (POC_R).

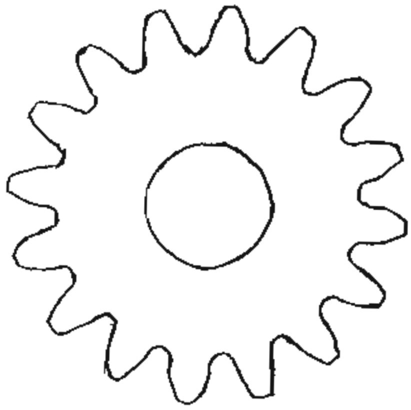
3- Search edge pixels again to extract all pixels lie (or nearly lie) on the POC (Fig. 4-c) and store their coordinates to an array called *OuterPixels*. This can be done as follows:
   - Calculate the distance (d_i) between the center of the POC (POC_{cx}, POC_{cy}) and each pixel (x_i, y_i) using equation (1).

$$d_i = \sqrt{(POC_{cx} - x_i)^2 + (POC_{cy} - y_i)^2} \quad (1)$$

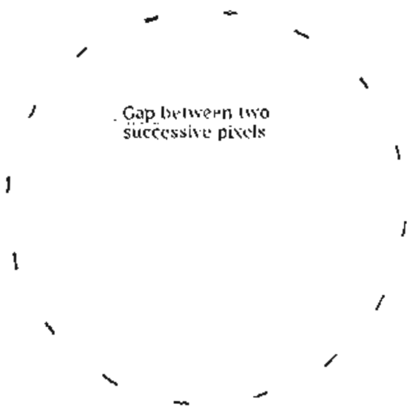
   - Extract all pixels having d_i's equal to the radius of the POC within a tolerance of ±1 pixel as follows:



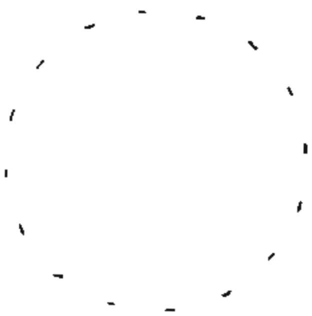
a) Original image



b) Extracted edge pixels



c) Extracted pixels of the outer circle



d) Extracted pixels of the root circle

Fig. 4 Extracting the pixels lie on the outer and root circles

If $(d_i > POC_R - 1$ AND $d_i < POC_R + 1)$
Store the pixel $P(x_i, y_i)$ to the *OuterPixels* array.

4- Apply the least square method on the *OuterPixels* array to define the actual outer circle (AOC) and to calculate its center (AOC_{cx}, AOC_{cy}) and radius (AOC_R).

5- Finally, the outer diameter (D_o) of the gear is calculated using equation (2).

$$D_o = 2 * AOC_R \tag{2}$$

3.3 The Root Diameter Algorithm

The root diameter algorithm is used to calculate the root diameter (D_R) from the extracted edge pixels. This algorithm uses the center (AOC_{cx}, AOC_{cy}) of the AOC to calculate the root diameter as follows:

1- For all edge pixels of the outer contour (teeth profile), calculate the distance (d_i) between the center of the AOC and each pixel (x_i, y_i), then find the pixel that have the minimum distance (d_{min}).

2- Define a primary root circle (PRC) which its center is the center as the AOC and its radius (PRC_R) equal to the minimum distance (d_{min}).

3- Search edge pixels again to extract all pixels lie, or nearly lie, on the PRC (Fig. 4-d) and store them to an array called *RootPixels*. This can be done by calculating the distance (d_i) between the center of the AOC (AOC_{cx}, AOC_{cy}) and each pixel (x_i, y_i), then extracting all pixels that satisfy the following criterion:

If $(d_i > PRC_R - 1$ AND $d_i < PRC_R + 1)$
  Store the pixel $P(x_i,y_i)$ to the *RootPixels* array

4- Apply the least square method on the *RootPixels* array to define the actual root circle (ARC) and to calculate its radius (ARC_R).

5- Finally, the root diameter (D_R) of the gear is calculated using equation (3).

$$D_R = 2 * ARC_R \quad (3)$$

3.4 The Number of Teeth Algorithm

The number of teeth algorithm was developed to count the number of teeth (N) from pixels extracted by the Outer Diameter algorithm, which stored to the *OuterPixels* array. If *OPXi* contains the x coordinates of the *OuterPixels* array and *OPYi* contains the y coordinates, then the following C code can be used to count the number of teeth.

```
int N = 0; double dist= 0;
for (int i=0; i< NPoints-1; i++)
{
  dist=Sqrt(Pow(OPXi[i]-OPXi[i+1],2)+
       Pow(OPYi[i]-OPYi[i+1],2));
  if (dist >= 5)
     N++;
}
```

Where: N is the number of teeth, *NPoints* is the number of pixels in the *OuterPixels* array, and *dist* is a variable stores the distance between each two successive pixels in the *OuterPixels* array.

The initial value of N is zero. When the distance between any two successive pixels in the *OuterPixels* array is greater than five the number of teeth is increased by one. The value of five was taken as an indication to the gap between the last pixel on a tooth and the first pixel on the next tooth. Usually, the distance between each two successive pixels on the same tooth does not exceed than 1.414 pixels ($\sqrt{2}$). Therefore, the value of five is safe enough to ensure that the gap between two teeth has been occurred.

3.4 Other Gear Parameters

The above three algorithms calculate the outer diameter, the root diameter and the number of teeth. Using these parameters, all other gear parameters can be calculated using their equations as follows:

3.4.1 The Diametral pitch

Having the outer diameter and the number of teeth, the diametral pitch (P) can be calculated as follows:

$$P = (N+2)/D_o \quad (4)$$

3.4.2 The Pitch Circle Diameter

Having the diametral pitch and the number of teeth, the pitch circle diameter (D) can be calculated as follows:

$$D = N/P \quad (5)$$

3.4.3 The Module

Having the pitch circle diameter (D) and the number of teeth, the module (m) can be calculated as follows:

$$m = D/N \quad (6)$$

3.4.4 The Circular Pitch

The circular pitch (p) can be calculated as follows:

$$p = \pi D/N \quad (7)$$

3.4.5 The Addendum

Having the outer diameter (D_o) and the pitch circle diameter (D), the addendum (a) can be calculated as follows:

$$a = (D_o - D)/2 \quad (8)$$

3.4.6 The Dedendum

Having the root diameter (D_R) and the pitch circle diameter (D), the dedendum (b) can be calculated as follows:

$$b = (D - D_R)/2 \quad (9)$$

3.4.7 The Clearance

Having the addendum (a) and the dedendum (b), the clearance (c) can be calculated as follows:

$$c = (b - a) \tag{10}$$

3.4.8 The Whole Depth

The whole depth (h_t) can be calculated as follows:

$$h_t = (a + b) \tag{11}$$

3.4.9 The Circular Tooth Thickness

The circular tooth thickness (T_{Circ}) is calculated as follows:

$$T_{Circ} = \pi m/2 \tag{12}$$

3.4.10 The Chordal Tooth Thickness

The chordal tooth thickness (T_{Chor}) is calculated as follows:

$$T_{Chor} = mN\sin(90/N) \tag{13}$$

3.4.11 The Chordal Dedendum

The chordal dedendum (b_{chor}) is calculated as follows:

$$b_{Chor} = \frac{mN}{2}\left[1 + \frac{2}{N} - \cos\left(\frac{90}{N}\right)\right] \tag{14}$$

3.4.12 The Base Circular Diameter

The base circular diameter (D_B) is calculated as follows:

$$D_B = D\cos\phi \tag{15}$$

3.4.13 The Base Pitch

The base pitch (P_B) is calculated as follows:

$$P_B = \pi m \cos\phi \tag{16}$$

4. System Calibration

The system calibration is performed by calculating the pixel size in both x and y directions according to the actual size of the object to be measured as follow:

1- The user enters the outer diameter (D_o) of the measured gear (in millimeter) to the GearVision software.

2- The software searches the edge pixels of the outer contour to find the two pixels having the minimum and the maximum x coordinates (X_{max}, X_{min}), then it calculates the maximum diameter of the gear in x direction ($Dmax_x$) as follows:

$$Dmax_x = Abs(X_{max} - X_{min})$$

3- A calibration factor in x direction (CF_x) is calculated as follows:

$$CF_x = D_o / Dmax_x$$

4- Similarly, a calibration factor in y direction (CF_y) is calculated by applying step 2 to find the two pixels having the minimum and the maximum y coordinates, then CF_y is calculated as follows:

$$CF_y = D_o / Dmax_y$$

5- The final calibration factor (CF) is calculated as follows:

$$CF = \sqrt{CF_x \times CF_y}$$

6- The calculated outer diameter and root diameter are multiplied by the calibration factor (CF). All other gear parameters are automatically calibrated because they are directly or indirectly calculated from these two parameters.

5. Inspection of Spur Gears

The inspection process is performed by comparing the measurements of the gear to be inspected and the measurements of a standard gear within specified tolerances. The Inspection section in Fig. 2 deals with this process. The inspection process is performed as follows:

1. *Entering the standard gear parameters:*

   The standard gear parameters can be entered to the GearVision software by two methods. In the first, the standard gear is measured by the vision system

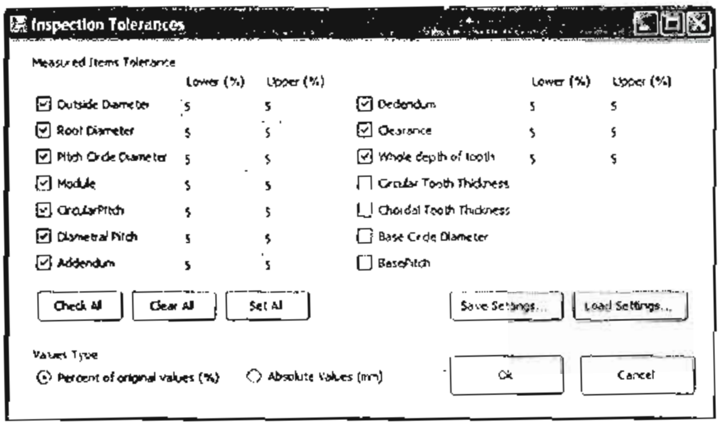


Fig. 5 Inspection tolerance dialog box

and the calculated parameters are stored as reference values by clicking the *Set as Reference* button in the gear measurement section. One reference gear could be used to check many gears. In the second method, the reference values are set manually by clicking the *Set Manual* button.

2. *Selecting the gear parameters to be inspected:*

   This can be done through the Inspection Tolerances dialog box shown in Fig. 5, which can be displayed by clicking the *Tolerances* button. The parameters to be inspected should be checked and their allowable tolerances are assigned. Each parameter can be assigned unique tolerance values (Lower and Upper) or the same tolerance values can be set for all parameters by clicking the *Set All* button in the main interface.

3. *Measuring the gear to be inspected:*

   The gear to be inspected is measured by the vision system, then the calculated parameters are compared with the standard gear parameters according to the tolerances given through the inspection tolerance dialog box.

4. *Taking an inspection decision:*

   The inspection decision is displayed automatically in the Inspection section. The decision will be "Accepted" if all inspected parameters for the measured gear meet the corresponding parameters of the standard gear within the specified tolerances. Otherwise, the decision will be "Rejected". The details of the comparison process can be displayed through the Inspection Details dialog box (Fig. 6), which can be displayed by clicking the Details button.

6. Verification and discussion

To verify the introduced system, two images were captured for two different gears, then the proposed algorithms were applied to each image to calculate the gear parameters. A comparison between the actual and the calculated values of each gear parameters as well as the variation between the two values are listed in table 1. Positive variations mean that the calculated values are greater than the actual values and

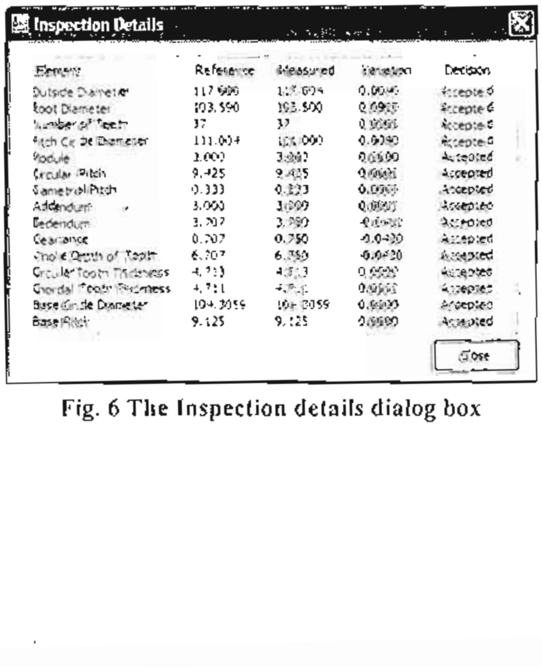


Fig. 6 The Inspection details dialog box

Table 1: Comparison between actual and calculated values of the different parameters of two sample spur gears

No.	parameter	Symbol	Test Gear 1 ($\phi = 20$)			Test Gear 2 ($\phi = 14.5$)		
			Actual	Calculated	Variation	Actual	Calculated	Variation
1	Outside Diameter	D_o	108.000	108.003	0.003	156.000	156.004	0.004
2	Root diameter	D_R	81.000	81.074	0.074	129.000	129.101	0.101
3	Number of teeth	N	16.000	16.000	0.000	24.000	24.000	0.000
4	Dimetral Pitch	P	0.167	0.167	0.000	0.167	0.167	0.000
5	Pitch Circle Diameter	D	96.000	96.000	0.000	144.000	144.000	0.000
6	Module	m	6.000	6.000	0.000	6.000	6.000	0.000
8	Circular pitch	p	18.850	18.850	0.000	18.850	18.850	0.000
9	Addendum	a	6.000	6.002	0.002	6.000	6.002	0.002
10	Dedendum	b	7.500	7.463	-0.037	7.500	7.450	-0.050
11	Clearance	C	1.500	1.462	-0.038	1.500	1.448	-0.052
12	Whole depth of tooth	h_t	13.500	13.465	-0.035	13.500	13.452	-0.049
13	Circular Tooth thickness	T_{Circ}	9.425	9.425	0.000	9.425	9.425	0.000
14	Chordal Tooth thickness	T_{Chor}	9.425	9.425	0.000	9.425	9.425	0.000
15	Base diameter	D_b	90.211	90.210	0.000	139.413	139.413	0.000
16	Base circular pitch	P_B	17.713	17.713	0.000	18.249	18.249	0.000

vice versa. The actual values of gear parameters were obtained from the MITCalc software (Mechanical, Industrial and Technical Calculations), which is a software package for gear design and calculations [21].

As discussed earlier, D_o, D_R, and N are first calculated by the developed software, then other gear parameters are calculated based on these parameters. Therefore, only the variations of gear parameters based on these three parameters are affected, while the variations of other gear parameters are zero.

It can be seen that the variations between the actual and the calculated values for the second gear (Test gear 2) are greater than those of the first gear (Test gear 1). This refers to the fact that increasing the dimensions of the measured gear increases the pixel size for the captured images, which decreases the accuracy of the vision system. This makes the vision system more suitable for measuring small spur gears.

7. Conclusions

A vision system has been introduced as a new non-contact measurement system for measurement and inspection of spur gears from their captured images. A software has been developed in-house to analyze the captured images and perform the measurement and inspection processes using developed many image processing and computer vision algorithms.

The introduced vision system has been calibrated and verified by measuring two sample gears and comparing the calculated parameters with the actual values of gear parameters. The maximum variations between the calculated parameters and the actual values were ±0.101 mm for a spur gear with 156 mm outside diameter. The accuracy of the system is affected by the size of the gear to be measured. Hence, the variation between the calculated parameters and the actual values can be decreased by measuring small gears.

References

1. Amarnath M., Sujatha C., Swarnamani S., "Experimental studies on the effects of reduction in gear tooth stiffness and lubricant film thickness in a spur geared system", Tribology International, Vol. 42, pp. 340–352, 2009.

2. Merritt H.E., "***Gears***", 3rd edition, London: Pitman, 1954.
3. Jain R.K., "Engineering metrology", 17th edition, Khanna publishers, 2-B, Nath Market, Nai Sarak, Delhi 110 006, 1999.
4. Enami K., Kuo C., Nogami T., Hiraki M., Takamasu K. and Ozono S., "Development of nano-Probe System Using Optical Scanning" Proc. IMEKO-XV, pp 189 – 192, 1999.
5. Gao C.H., Cheng K., Webb D., "Investigation on sampling size optimization in gear tooth surface measurement using a CMM", International Journal of Advanced Manufacturing Technology, Vol. 24, pp 599–606, 2004.
6. Lotze W., Haertig F., "3D gear measurement by CMM", Proceedings of Laser Metrology and Machine Performance, Birmingham, UK, pp333-344, 2001.
7. Cheng K., Gao C.H., Harrison D.K., "3D surface profile deviation evaluation based on CMM measured data", Proceedings of the 16th International Conference on Computer Aided Production Engineering, Edinburgh, UK, 7-9 Aug, pp503-510, 2000.
8. Jianfeng L., Wenxin L., Chuanmin Z. and Zhiren T., "Measurement of gear-tooth instantaneous deformation by optical method", Proc. SPIE ,Vol. 2899, pp 117 – 122, 1996.
9. Eiki Okuyama, Satoshi Kiyono, and Hitoshi Moritoko, "Investigation of an optical noncontact gear geometry measurement system: measurement of pitch errors and tooth profiles", Precision Engineering, Vol. 16, No. 2, pp 117-123, 1994.
10. Younes M.A., Khalil A.M., Damir M.N., "Automatic measurement of spur-gear dimensions using laser light - Part 1 measurement of tooth thickness and pitch", Optical engineering, Vol. 44, No 8, pp. 087201.1-087201.13, 2005.
11. Younes M.A., Khalil A.M., Damir M.N., "Automatic measurement of spur gear dimensions using laser light, part 2: measurement of flank profile", Optical Engineering, Vol. 44, No 10, pp. 103603.1-103603.7, 2005.
12. Gadelmawla E.S., "A vision system for surface roughness characterization using the gray level co-occurrence Matrix", NDT&E International, Vol. 37, pp. 577–588, 2004.
13. Kerr D., Pengilley J., Garwood R., "Assessment and visualization of machine tool wear using computer vision", International Journal of Advanced Manufacturing Technology, Vol. 28, pp 781-791, 2006.
14. Franco Pedreschi, Jorge León, Domingo Mery, Pedro Moyano, "Development of a computer vision system to measure the color of potato chips", Food Research International, Vol. 39, pp. 1092-1098, 2006.
15. Armstrong P.J., Antonis J., "The development of an active computer vision system for reverse engineering", Journal of engineering manufacture, Part B, Vol. 214, pp 615-618, 2000.
16. Brosnan T., Sun, D.W., "Improving quality inspection of food products by computer vision", Journal of Food Engineering, Vol. 61, pp 3–16, 2004.
17. Robinson M. J., Oakley J. P., and Cunningham M. J., "The accuracy of image analysis methods in spur gear metrology", Measurement Science and Technology, vol. 6, pp. 860-871, 1995.
18. Sung C. K., Tai H. M., Chen H. M., "Locating defects of gear system by the technique of wavelet transform", Mechanism and Machine Theory, Vol. 35, No. 8, pp.1169–1182, 2000.
19. Gadelmawla E.S., Khalifa W.M., Elewa, I.M. "Measurement and Inspection of roundness using computer vision", Mansoura Engineering Journal (MEJ), faculty of engineering,

Mansoura university, Egypt, Vol. 33, No. 3, pp. M20-M32, 2008.
20. Erik Oberg, Franklin D. Jones, Holbrook L. Horton, and Henry H. Ryffel, "Machinery's Handbook", Industrial Press Inc., New York, 200 Madison Avenue New York, New York 10016-4078, 26th Edition, pp. 1998-2016, 2000.
21. Mechanical, Industrial and Technical Calculations (MITCalc) software, www.mitcalc.com

Nomenclature

a	Addendum
b	Dedendum
C	Clearance
D	Pitch circle diameter
D_B	Base circle diameter
D_O	Outside diameter
D_R	Root diameter
h_t	Tooth depth
m	Module
N	Number of teeth
p	Circular pitch
P	Diametral pitch
P_B	Base pitch
T_{Chor}	Chordal tooth thickness
T_{Circ}	Circular tooth thickness

Greeks

ϕ	Pressure angle

Abbreviations

POC	Primary outer circle
POC_{cx}	x coordinate of the primary outer circle
POC_{cy}	y coordinate of the primary outer circle
POC_R	Radius of the primary outer circle
AOC	Actual outer circle
AOC_{cx}	x coordinate of the actual outer circle
AOC_{cy}	y coordinate of the actual outer circle
AOC_R	Radius of the actual outer circle
PRC	Primary root circle
POC_R	Radius of the primary root circle
ARC	Actual root circle
ARC_R	Radius of the actual root circle