Mansoura Engineering Journal (MEJ) Vol. 12, No. 2, Dec. 1987

ANALYSIS OF STRUCTURES RESTING ON ELASTIC FOUNDATION

N. S. MAHMOUD
Lecturer,
Dept. of Civil Engineering,
Mansoura University,

(Received Dec . 13, 1987, accepted Dec. 1987)

الخلاصة :

في هذا البحث تم استخدام طريقة الشرائح المحددة لتمثيل أى منشأ مرتكز على التربة التى تم تمثيلها بطريقة ونكلر .
وقد تم اخذ تأثير الاجزاء المعرضة لاجهادات ضغط فقط واهملت أى اجزاء من الشريحة معرضة لاحمال الشد .
ثم تمت مقارنة النتائج مع عدد من الطرق الاخرى ووجدت أنها دقيقة بالاضافة الى سهولة طريقة الشرائح واختصارها للوقت على الحاسب الآلى .

SUMMARY

In the present paper, the finite strip method has been used to model a structure resting on Winkler foundation over parts of the structure . Comparsion between the results based on different methods is shown. Suggestions for design charts for foundation using finite strip method conclude the paper.

1. INTRODUCTION

There are three basic types of foundations, namely an isolated footing, a strip or continuous footing, and a mat or raft foundation. Raft foundations make the settlement of the subgrade material still more even and are used if :

- The bearing capacity of the soil is low relative to the reaction from structure.
- The deformation of the foundation material exceeds the allowable value.
- The soil is weak and inhomogeneous.
- The structure is subjected to heavy and nonuniformly distributed loads.

Under load, the raft and the associated subgrade material act together, thereby forming a single system. Their interaction gives rise to what is known as the contact soil pressure. For design purpose, engineers distinguish rigid footings whose deflections under load are negligible in comparison with the deflections of the subgrade material, and flexible footings whose deflections are comparable with the deflection of the soil. There are three different methods of analysis of flexible mat foundation, the elastic plate, the finite difference and the finite element method (1,2,3,4). The fourth method developed in the present work is the finite strip method (5) .

The finite strip approach does offer a number of advantages over alternative methods of analysis (6). An important feature of the method is that it can be used for a wide range of structures and is not specific in its application.

2. MODEL OF DISCRETIZATION OF MAT FOUNDATION

In the finite strip method, the mat is idealized as an assembly of plates (strips) rigidly connected together, at their longitudinal edges (Fig. 1) . Assuming a polynomial function in the transverse direction of the strip and a series function in the longitudinal direction, the components of displacement can be represented. From this assumption, it is clear that there are two degrees of freedom, namely out-of-plan deflection (w) and a rotation about the edge (θ), at each edge of the strip as shown in Fig. 1. The applied external load can exist in the direction of each of the above deformation.

The soil is divided into finite strips identical to those of the mat and idealized as a set of isolated springs capable of resisting compression only (9). The model is based on the assumption that the settlement of the soil at a given point is independent of the settlement at the other points and is proportional to the pressure exerted at that point. This modeling incorporates a general form of the contact pressure which enables the reduction in subgrade stiffness matrix due to localized separation between the mat and the soil to be taken into account.

3. COMPARISON WITH PREVISOUELY PUBLISHED RESULTS

Two examples have been studied to investigate the accuracy of the present finite strip method. For the purpose of comparison, different methods of analysis have been considered. The description of each example is as follows :

3.1. EXAMPLE 1.

The first example is a combined footing 579.0x239.0 cm in plan and 76 cm thick (1). It is designed to support two reinforced concrete squar columns (33.0x33.0 cm and 38.1x38.1 cm). The columns transfer to the footing a total vertical load of 235.87 t. The load and dimensions of the footing are shown in Fig. 2 .

The reinforced concrete used to construct the footing is of a compressive strength equal to 225 Kg/cm^2 and the modulus of elasticity, E_c, assumed to be 211.0 t/cm^2. The soil is classified as a medium sand and the modulus of subgrade reaction assumed to be 2.44 Kg/cm^3.

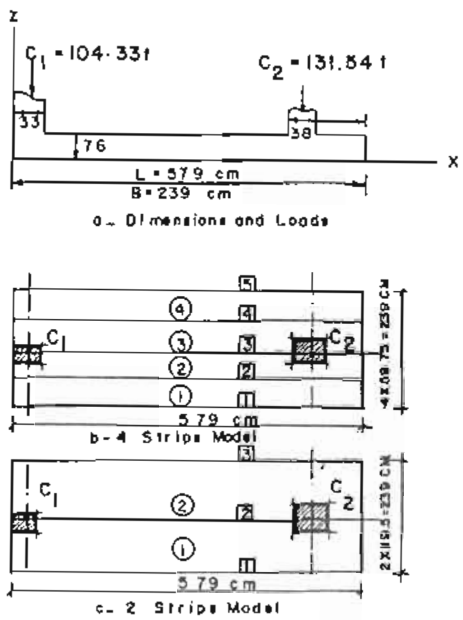


Fig. 2 Combined Footing (Example 1)

Solutions were obtained with the combined footing divided into different numbers of strips between two and four strips. The deflection along the center line of the footing and along its edge has been determined. The results compared better with finite difference results than with the results based on a beam on elastic foundation (1) as shown in Table 1. It is clear that the deflection of the footing based on finite strip method overestimates the results based on the beam on elastic foundation by up to 30 % in some point and underestimates the results by up to 40 % in other points. These ratios become 9 % and 6 % respectively when the finite strip results are compared with the finite difference results.

Because the breadth to depth ratio (b/t ≅ 3.0) is relatively small, there is no change in the deflection in the transverse direction as shown in Table 1. It is clear from this table that convergence is quite rapied with the combined footing divided into two strips.

In Table 2, the soil reaction, shear force and the bending moment have been calculated using the finite strip method. The results are compared with previsouly published results based on the finite difference, the beam on elastic

Table 1. Deflection (cm.) of the combined footing shown in Fig. 2.

Distance X / L	Finite Strip Method				Finite difference (1)	Beam on elastic foundation (1)
	2 Strip		4 Strip			
	Edge	Center line	Edge	Center line		
0.0	0.9131	0.9188	0.9119	0.9186	0.8297	1.228
0.1	0.8050	0.8098	0.8041	0.8097	0.7663	0.954
0.2	0.7070	0.7115	0.7062	0.7114	0.7090	0.716
0.3	0.6346	0.6404	0.6340	0.6404	0.6651	0.539
0.4	0.5994	0.6085	0.5990	0.6086	0.6383	0.439
0.5	0.6007	0.6143	0.6003	0.6145	0.6306	0.415
0.6	0.6255	0.6453	0.6270	0.6457	0.6419	0.466
0.7	0.6633	0.6842	0.6631	0.6848	0.6697	0.588
0.8	0.6975	0.7192	0.6975	0.7199	0.7087	0.759
0.9	0.7250	0.7456	0.7252	0.7465	0.7519	0.960
1.0	0.7480	0.7667	0.7485	0.7677	0.7937	1.158

foundation and the rigid methods. Again the finite strip results compared very well with the finite difference results while the results from the two other methods are different.

3.2 EXAMPLE 2 :

For the purpose of comparison of finite strip and finite element methods of analysis, a beam and loading as shown in Fig. 3 was chosen (5). In the finite element analysis, three cases for the subgrade beneath the beam has been considered, Winkler's subgrade and discrete springs at 2 m and 1 m spacing. The results are summarised in table 3 and in Fig. 3. Table 3 illustrates the distribution of the contact stresses while Fig. 3 illustrates the distribution and magnitude of the shear forces and bending moments. It is clear that the finite strip method results are in good agreement with the finite element results. The difference between the results from the two methods diminish as the number of springs increases in the second method.

4. RESULTS AND DISCUSSION

To cover a wide range of practical cases, a typical strip (divided into two substrips) with variable flexural stiffness and variable subgrade properties were used. The strip shown in Fig. 4 has been analysed for a range

Table 2. Comparison between finite strip results and other methods

Distance	Shearing Force (ton)				Bending Moment (m.t.)				Soil Reaction (Kg.)	
X/L	Finite strip	Finite Difference	Beam on Elastic F.	Rigid design	Finite strip	Finite Difference	Beam on Elastic F.	Rigid design	Finite strip	Finite Difference
0.0	89.049	90.333	104.326	0.0	-	0.0	0.0	0.0	15.280	13.993
0.1	62.111	64.492	67.555	- 80.809	- 34.956	- 35.153	49.322	36.393	26.938	25.841
0.2	38.444	40.574	- 39.504	- 57.127	- 70.980	- 72.487	79.935	76.334	23.627	23.918
0.3	17.139	18.144	- 18.485	- 33.446	- 93.278	- 95.992	96.439	102.561	21.305	22.430
0.4	3.108	3.384	- 2.195	- 9.764	-103.217	-106.496	102.260	115.073	20.247	21.528
0.5	23.552	24.657	11.963	13.918	-101.414	-104.539	99.393	113.870	20.444	21.273
0.6	45.034	46.312	26.623	37.599	- 87.754	- 90.258	- 88.305	98.953	21.482	21.655
0.7	67.817	68.896	44.236	61.281	- 61.633	- 63.438	- 67.984	- 70.321	22.783	22.584
0.8	39.773	38.746	66.845	84.962	- 22.299	- 23.538	- 36.100	- 27.974	23.950	23.900
0.9	14.938	13.386	- 35.733	- 21.973	7.248	7.749	10.672	5.904	24.835	25.360
1.0	2.168	0.000	0.000	0.000	-	0.000	0.000	0.000	12.770	13.386

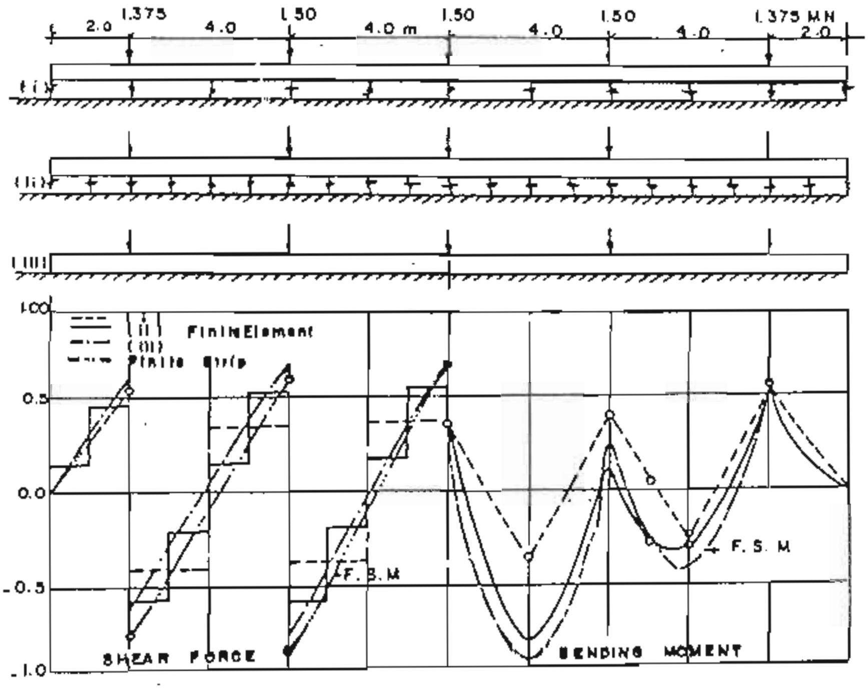


Fig. 3 Comparsion Between Finite Strip and Finite Element Results

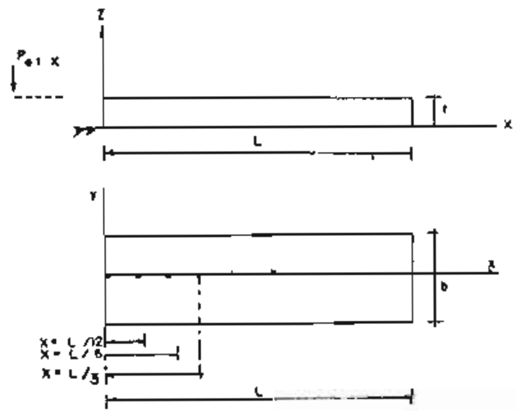


Fig. 4 Typical Strip Under Concentrated Load at Different Position.

of value of the parameter λ L between 2.0 and 5.0.

$$\lambda = \sqrt[4]{\frac{bk}{4EL}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where

L = Length of the strip

b = Strip width

K = Winkler's coefficient of subgrade reaction

E = Young's modulus

I = Moment of inertia of the strip.

Table 3. Comparsion between finite element method and finite strip method results

Contact Stress (KN/m²) \ Distance X/L		0.0	0.1	0.2	0.3	0.4	0.5
Finite Element Method (5)	(i)	133	177	184	195	187	194
	(ii)	136	176	183	194	187	194
	(iii)	137	175	182	194	187	194
Finite Strip	129	129	153	177	198	211	216

The solution of the finite strip method for strip with constant EI and acted by a concentrated load at L/2,L/4 or at left end are shown in Fig. 5.

When the load applies at L/2, the max deflection at that point increases as λ L increases,while when the load applies at any of the two other points (0,L/4) this deflection decreases asλL increases. The deflection curves for $\lambda L<2$, and for any case of loading, are approximately linear. Thus, for $\lambda L\leq2$, the analysis based on a rigid beam can be used. For $\lambda L>2$ the flexibility of the foundation must be considered in the analysis. For load at L/2 the varition of the central deflection, for all values of λ L, is within 20% where as, for load at L/4,this value increases to 50% and 100% at L/2 and L/4 respectively. Thus, effect of this flexibility depends on the eccentricity of the applied load, and as the eccentricity decreases this effect decreases.

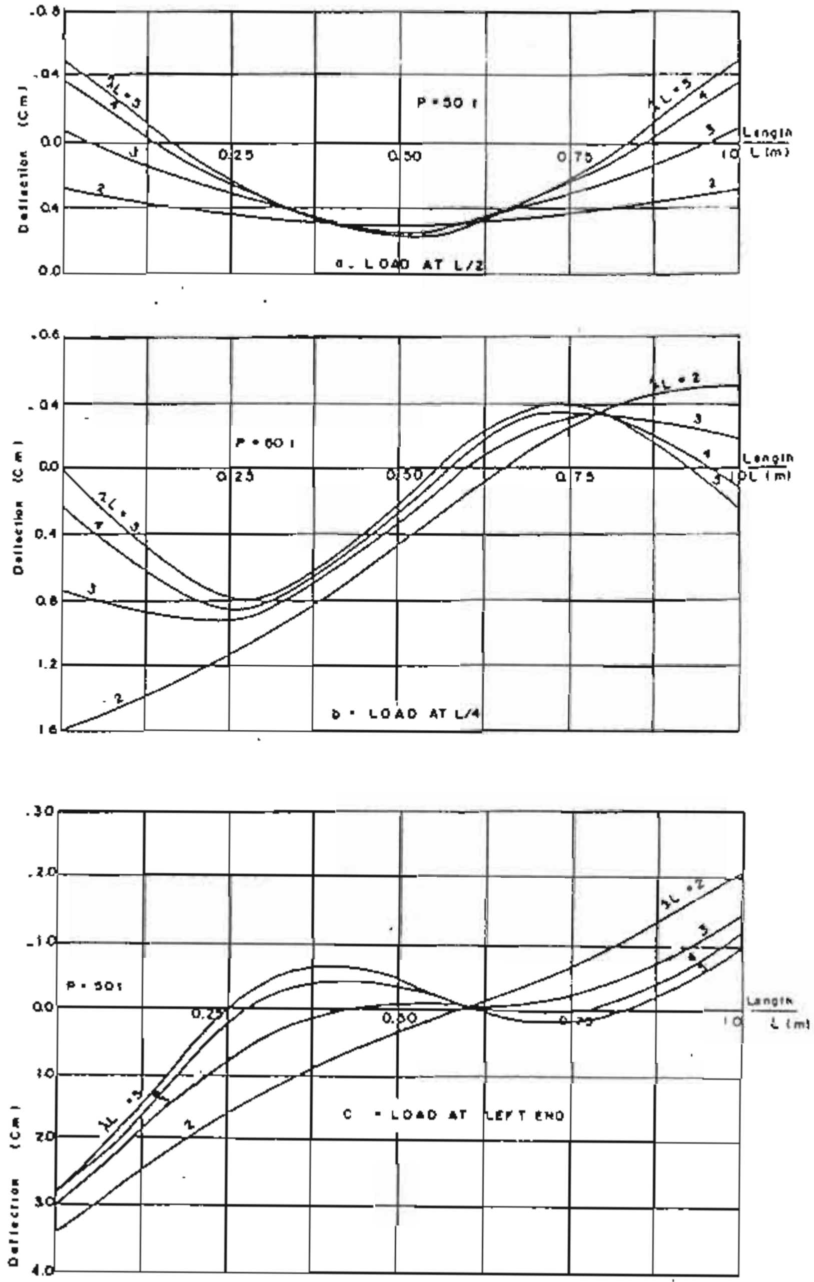


Fig 5 Relation Between Deflection and λL For Different Cases of Loading

Each curve shown in Fig. 6 corresponds to a strip acted by a load at the left end (x=0), x=L/12, x=L/6, ... from the left end. The deflection is shown in terms of δ_0, where δ_0 is the average deflection of the strip. This average deflection decreases as the parameter λL increases and/ or as the eccentricty of the load decreases.

These curves are useful in practical problems, and can be used as design charts for isolated footing, combined footing, strip footing and mat foundation. First divide the footing into a number of strips bxt, where b is the width of the strip and t its thickness. Knowing the coefficient of subgrade reaction k, the value of λ is computed from Eq. (1). Choose the chart corresponding to λL, where L is the length of the strip. For every applied concentrated load P_i choose the applicable curve and from the equilibrium equation.

$$P_i = bk \int_0^L \delta \, d L \qquad ....(2)$$

$$\int_0^L \delta d L = C \delta_0 L \qquad ....(3)$$

The coefficient C depends on the area under the curve and is shown in Fig. 6. Obtaining δ_0 the actual deflection curve due to the applied load P_i can be calculated. Superimposing the actual deflection curves for all the other applied concentrated loads, the overall deflection is known and thus the distribution of the stresses on soil can be obtained from

$$\sigma = K \delta . \qquad ....(4)$$

hence the actual bending moment and shearing force on each strip can be calculated.

For the purpose of illustration assume a column footing 400x200 cm in plan and 40 cm thick. It is designed to support a reinforced concrete column (120x30cms) that transfers to the footing a vertical load of 200 t. The load and dimension of the footing is shown in Fig. 7. The Winkler's coefficient of subgrade reaction = 0.0025 t/cm³ and the Young's modulus = 210 t/cm². From Eq. (1).

$$\lambda = 4.86 \times 10^{-3} \text{ cm}^{-1} \qquad ....(5)$$

$$\lambda L = 1.94 \qquad ....(6)$$

Using Fig. 6.a. where $\lambda L = 2.0$ and choose the curve for $x = \frac{L}{2}$, the value of C is found to be 0.980. The average deflection δ_o can be calculated from Eq. (2) and Eq. (3).

$$\delta_o = 1.020 \text{ cm} \qquad \text{...........(7)}$$

The deflection and the stress on soil are given in Table 4. From this table the bending moment and the shearing force can be calculated at different sections. The design bending moment according to ACI Code = 45.56 t.m where the stress on soil is assumed to be uniform and equal 2.5 kg/cm² as shown in Fig. 7. The corresponding bending moment based on finite strip method = 40.132 t.m.

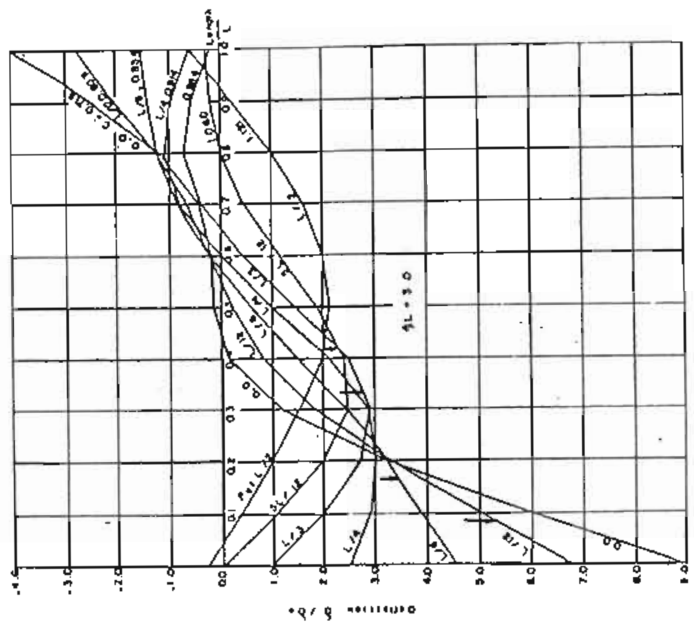
It is clear that the ACI Code overestimates the present results by about 13% . As the factor λL decreases the footing will act as a rigid plate and the finite strip results will become closer to ACI Code results.

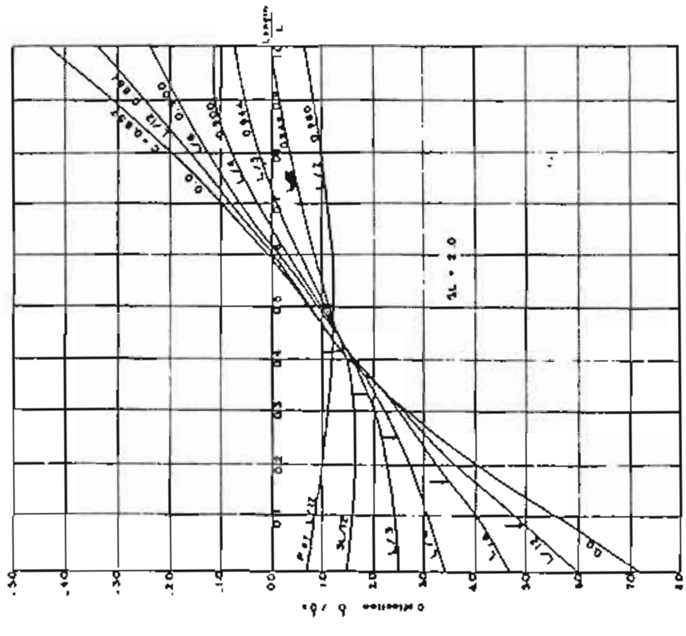
Table 4. Deflection and stress under isolated footing.

Position	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Defle. (δ), cm	0.737	0.888	1.027	1.150	1.233	1.265	1.260	1.150	1.013	0.849	0.665
Stress (σ), kg/cm²	1.750	2.170	2.550	2.875	3.120	3.163	3.120	2.875	2.550	2.170	1.750

5. CONCLUSION

The finite strip method, used for a wide range of structures, has been developed by the author to analyse structures resting on elastic foundation. The analysis involving the concept of the modulus of subgrade reaction is applicable to localized compressed soil only. A computer program based on the present method has been developed and the results have been tested for different structures. For practical purpose, design charts for a wide range of λL(2,3,4 and 5) are presented with applicable illustration example. From these results, it is clear that :





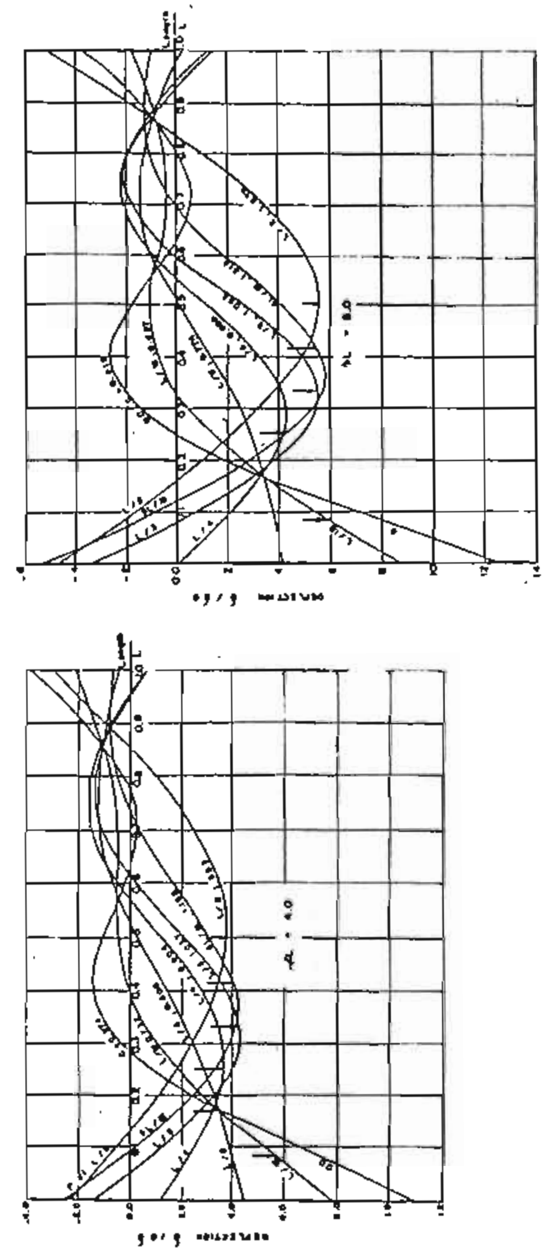


Fig. 8 Average Deflection for Different Load Position.

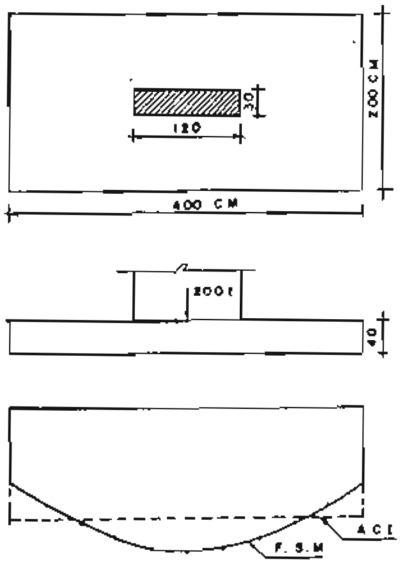


Fig. 7 Distribution of Stress Under Footing

1. The finite strip method results are in good agreement with previous published results.
2. The results based on Winkler's analysis tend towards the rigid-beam solution for λ L less than 2.0.
3. The ACI Code overestimates the present results by a factor of safety that depends on the rigidity and the length of the footing and the type of subgrade soil. (i.e. on the factor λL).
4. For $\lambda L>2$ (within the considered range) the conventional methods of foundation design overestimate the actual straining actions and underestimate the maximum stress on soil.

6. REFERENCES

1. Winterkorn, H.F. and Fang, H.Y. "Foundation Engineering Handbook" Van Nostrand Reinhold Company, New York, 1975.
2. Punmia, A.C. "Soil Mechanics and Foundations" Standard Book House, Delhi, 1982.
3. Just, D.J., Starzewski, K. and Ronan, P.B. "Finite Element Method of Analysis of Structures Resting On Elastic Foundation." An Int. Symposium on the Interaction of Str. and Foundation,", Univ. of Birmingham, U.K., July, 1973.
4. Haddadin, M. "Mats and Combined Footings-Analysis By Finite Element Method" ACI Journal, Dec., 1971.
5. Mahmoud, N.S. "A Finite Strip Method For Analysis of Mat Foundation.", First Egyptian Str. Eng. Conference, Cairo University, April, 1985.
6. Cheung. U.K. "The Finite Strip Method in Structural Analysis". Pergamon, 1976.
7. Mahmoud, N.S. "Inelastic Stability of Plate Str. Using the Finite Strip Method." Ph. D. Thesis, Univ. of Sheffield, 1981.