Menofia University Faculty of Engineering Shebien El-kom Basic Engineering Sci. Department. First semester Examination, 2017-2018 Date of Exam : 2 / 6 / 2018	Subject: Partial Diff. Eqs. Code: BES 507 Year : Master Time Allowed : 3 hrs Total Marks: 100 Marks

Answer the following questions — ***Two Pages***

Question 1 (20 MARKS)

(A) Classify according to parabolic , elliptic or hyperbolic: of the following P.D.Es.:

(i) $2u_{xx} - 4u_{xy} - 6u_{yy} + u_x = 0$ (ii) $4u_{xx} + 12u_{xy} + 9u_{yy} - 2u_x + u = 0$

(iii) $u_{xx} - x^2 y u_{yy} = 0$ ($y>0$) (iv) $e^{2x} u_{xx} + 2e^{x+y} u_{xy} + e^{2y} u_{yy} = 0$ (5 Marks)

(B) Determine whether each of the following PDEs are linear or nonlinear , homogenous or nonhomogenous. State the order , degree and name of the dependent and independent variables :

1- $u_t = k u_{xx}$ where k is constant

2- $x u_{xx} = y\left(u_{yyy}\right)^2$

3- $u u_{xx} = xyt$ (5 Marks)

(C) (i) State the various types of boundary conditions?

(ii) In the following BCs. state the type of them:

(i) $u(0,t) = a_\circ$, (ii) $u'(L,t) = a_\circ$

(iii) $u'(0,t) = f(x)$, (iv) $u(L,t) + u'(x,10) = g(x)$

(5 Marks)

(D) Show that the set of functions $\left\{\sin\left(\frac{n\pi x}{L}\right)\right\}, n = 1,2,3,....$ *are orthogonal on the interval (0, L).* (5 Marks)

Question 2 (40 MARKS)

(A) Find the eigen values and the corresponding eigen functions of the Sturm – Louville B.V.P. $F''(x) + \lambda F(x) = 0$ *subject to the following boundary conditions:* $F(0) = 0$ and $F'(1) + F(1) = 0$ (10 Marks)

(B) Solve the following PDE $u_t = u_{xx} - 6x$, $0 < x < 1,\ t > 0$

Subject to the boundary conditions:

$$u(0,t) = 1 \quad , u_x(1,t) = 2$$

And Subject to the initial condition:

$$u(x,0) = x^3 - x$$

(15 Marks)

(C) *Solve the following PDE*

$$u_t = k u_{xx} \quad , \quad 0 < x < L, \ t > 0$$

Subject to the boundary conditions:

$$u(0,t) = 1 \quad , u(L,t) = 2$$

And Subject to initial condition:

$$u(x,0) = f(x)$$

(15 Marks)

Question 3 **(40 MARKS)**

(A) *Solve the following PDE*

$$u_{xx} + u_{yy} = (1/a) u_t , \quad 0 < x < a, \ 0 < y < b, \ t > 0$$

Subject to the boundary conditions:

$$u_x = 0 \ \ at\, x = 0, \ u_x + h_2 u = 0 \ \ at\, x = a$$

$$u = 0 \ \ at\, y = 0, \ u_y + h_4 u = 0 \ \ at\, y = b$$

$$u = f(x,y) \quad for\ t = 0$$

(20 Marks)

(B) *Solve the heat equation with steady source*

$$u_t = k u_{xx} + x \quad , \quad 0 < x < 1, \ t > 0$$

with the boundary conditions:

$$u(0,t) = 0 \quad , u(1,t) = 0$$

And initial condition: $u(x,0) = f(x)$

(20 Marks)

This exam measures the following ILOs											
Question Number	Q1-a	Q2-a			Q2-b	Q3-b			Q1-b	Q3-a	
Skills		b-i			b-i, b-iii						
	Knowledge &understanding skills				Intellectual Skills				Professional Skills		

With my best wishes

Associate Prof. Dr. Islam M. Eldesoky