INDEXING THE CAPABILITY OF MULTI-CHARACTERISTIC PROCESSES WITH MINIMUM STATISTICS

التقييم التوافقى للعمليات المركبة باستخدام الاجراءات الاحصائية البسيطة

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خلاصة :

يفيد دليل توافق العملية بتوقع كمى لقدرة العملية على التوافق مع المواصفات، حيث يحتل تصور قدرة العملية و دليلها موقع هام ضمن ادارة الجودة الحديثة. و تتبنى الشركات المتطورة هذا التصور ضمن اجراءات تحسين الجوده و التى تهدف الى قياس و تخفيض التغاير فى مخرجات العملية المعنية. و يمكن الحاق هذه الكميات بسياسات الاحلال للتفاضل بين البدائل المتاحة. يعرف هذا البحث الأدلة القياسيه للعمليات و يقدم منهجية لتقييم العمليات المركبة والتى تحتوى على أكثر من بعد. و قد اشتمل البحث على اطارين من التحليل: (1) تحليل صناعى يصنف العمليات الى أربعة أصناف، (2) تحليل احصائى يشمل أسلوب مبسط لفرز العمليات الشادة لاستبعادها من تقييم العمليات المركبة بالاضافة الى تحليل الانحدار بين أحد أهم الأدله القياسية و احتمالية القبول، هذا مع العلم أنه قد بنيت بعض العلاقات بين مخرجات العمليات على أساس معادلات سماح احصائية أشتقت وطورت من بعض المعادلات التى صممت للتعامل مع الأبعاد المتداخلة. وأيضا أجريت التجارب على حالة عشوائية لاستكشاف العلاقات الابدالية التى توقع الباحث وجودها بين بعض حالات التقييم. و قد يبدو أن التقييم الشامل لا يلتصق مباشرة ببرنامج تحسين الجوده و لكنه دائما يدل على ماهية التراكب بين العمليات، و أيضا يمكنه على الأقل أن يدعم قرارا بالابقاء عليها. و تتناسب هذه المنهجية أساسا مع المنتجات المتقطعة دون المنتجات المتصلة والتى تحتاج الى تطوير دراسات قد تختلف كثيرا، مع العلم أنها يمكن أن تمتد الى أنظمة الخدمة.

ABSTRACT

A Process capability index provides a quantitative prediction of a stable process's ability to meet specifications. The concept of process capability and process capability indices are of great value to modern quality management. Many ever-growing firms have adopted this concept as a part of their quality improvement strategies which aimed at measuring and reducing a current proeess variation. These quantities can be merged in replacement policies to differentiate available process alternatives. This research defines the standard process capability indices, and develops a methodology for evaluating a complex process which returns more than one characteristic. Two types of analysis are proposed as a foundation: (1) industrial analysis classifies the process operations into four categories and (2) statistical analysis includes: (a) regression analysis between the most popular standard index and the probability of conforming and (b) simple screening technique used for eliminating the odd operations from the overall evaluation of a complex process. Moreover, some relationships between output characteristics are built according to modified statistical tolerance formulas extracted from formulas designed for interacting dimensions. A computational experiment was conducted on a randomly generated problem—with specific variations—to capture expected trade-offs between some formulas which were proposed for overall evaluation. Note that the overall evaluation may not be fed as a direct input to a quality program, but it demonstrates the interaction between the operations, and at least it supports a decision of maintaining all operations. This methodology has been presented to implement the processes related to discrete products, but it can be extended to service systems. For the processes related to coalesced products, the problem needs to extensive studies.

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Keywords: quality; process variation/capability indices; interacting dimensions; regression analysis; statistical tolerance formulas: characteristics; overall index/evaluation

BACKGROUND

The term process capability (PC) is always used to express the ability (adequacy) of a process to meet the imposed standard specifications. Specification instances, including tolerance limits in a manufacturing process, service level in processes such as nursing and banking, limits of shortage in an inventory system and limits of output voltage in a power line. In other words, PC refers to the inherent variability of the process around some center—the process mean is often used as center—i.e., the instantaneous reproducibility under a defined homogeneous processing environment (Juran 1988.) The defect rate of a product or a service represents a drastic function of the PC (Vaughan 1998), therefore the concept of PC is a critical issue to modern quality management. Moreover, PC is a cornerstone to make decisions about a current process and expected quality improvement programs by comparing the monitored characteristic variation to quality standards (capability study.) Decisions that may be taken include continuing with the current plan, reviewing imposed standards, changing some facility assignments, replacing some facilities, changing maintenance routines, and check control stations and workstations. The standards are often reviewed when they found unrealistic. Juran and Gryna (1980) summarized statistical calculations of quality limits. The latter decisions are regarded when management are convinced that the process fails to hold standards or when they adopted improvement programs.

PC is assessed indirectly by measuring the monitored output characteristic (diameter, length, temperature, service performance, etc.) for a sufficient run. A graphical plot for the individual consecutive measurements versus specification limits could be used to judge the process attributively; see Juran and Gryna (1980), p. 273. In practice, a capability study encounters long- and short-term variables, and more than single characteristic for the same product. Thus making it is necessary to resort, in addition, to statistical techniques to provide quantitative measures for the PC (Juran and Gryna 1980, Juran 1988, Foguth and Motwani 1999.) The latter authors identified a statistical method for capability studies including material variation.

Statistically, the concept of "six sigma" or $6\sigma_X$ ($\pm 3\sigma_X$ centered at μ_X) process capability is commonly used, where X is a random variable represents the population of the monitored characteristic having μ_X mean and σ_X standard deviation (see Juran and Gryna 1980, 1993). Effective use of this concept requires that the process be statistically controlled, sample measurements are independently and identically distributed, and X is approximately normally distributed; under these conditions "six sigma" refers to the natural tolerance, which yields 99.73% confidence level (Juran and Gryna 1980). Vaughan (1998) discussed this concept and analyzed the normality condition using Chi-Square, Kolmogorov-Smirnov and normal probability plot techniques. The value of μ_X and σ_X can be estimated by mean and standard deviation of sufficient sample of measurements. But the full potential assurance of these values occurs when sources of variation are included as possible. Control charts analysis of process output exhibit most significant sources of variations such as within sample, sample to sample and time to time—short- and long-run variations (Juran and Gryna 1980.)

Six sigma PC concept was originally developed at Motorola (Gill 1990, Smith 1993, Vaughan 1998.) Other firms, including General Electric, Raytheon, Boeing, Caterpillar, IBM, and Xerox have adopted the concept as standard in their quality improvement programs (Rayner and Haavind 1990, Vaughan 1998, Conlin 1998.)

The main objective of this paper is to develop a methodology for providing an overall quantitative prediction about the adequacy of complex processes without resorting to complex statistical analysis. The rest of this paper is organized into four main sections and four Appendices, as follows: §2 aided with Appendices A&B presents a brief review for existing standard PC indices and their versions, in addition to an examination of the relationship between the most common index and the probability of conforming using regression analysis. §3 includes the main skeleton of methodology which comprises an industrial classification of operations and related statistical analysis of the problem itself—complemented by Appendix C. §4 computationally experiments the validity of an expected mutuality between two evaluation formulas by using Monte-Carlo simulation. §5 gives some concluding remarks and mentions future work.

PROCESS CAPABILITY INDICES

Intuitively, a process performs independently of the quality standards, i.e. PC is a card for the process itself. Therefore, to make actions, PC must be related to the imposed quality standard of the characteristic being monitored. A simple dimensionless ratio known as the capability index—the ratio of specification range to the confidence interval (Prasad and Bramorski 1998)—is defined and commonly used as a base to demonstrate the process or to compare between two or more processes (Rodriguez 1992.) The standard PC indices are defined in Appendix A; see Kane (1986) and Juran and Gryna (1993). Other indices were developed to accommodate several irregularities that may occur within the process, such as tool wear (Long and De Coste 1988) and material variation (Foguth and Motwani 1999.) Other problems have been manipulated in Prasad and Bramorski (1998) and some papers therein. All those indices could be integrated into the method proposed in this paper because it is only interested in how to combine several indices.

However, the yield of defectives is not a function of PC alone but also setting of the process center, target and specification limits; this issue is discussed in the most of quality control texts. Here in that concern, a regression analysis is performed for a normally distributed process as shown in Appendix B1 and summarized in Fig. 1. This process is defined by 6σ PC, C_p index, bilateral specifications, and center at the characteristic nominal value. The specification range is taken as a multiplier of σ. Where k is the multiplier, $g(k)=P_a$ is the probability of product acceptance, and C_p is the capability index. The ratio $f(k)=P_a/C_p$ demonstrates more powerful relative prediction of the process response. Such process is principally described by $P_a=2\Phi(3C_p)-1$, where Φ is the cumulative density function of standard normal distribution. A similar analysis could be conducted to the process when a parameter being off cited. For instance if the actual process center is being $\pm q\sigma$ off-target, where q is a multiplier like k, functions of k and q can be extracted with high determination factor R^2 (see Appendix B2.) Such relationships are used in §3 to approximate an evaluation for a compound process.

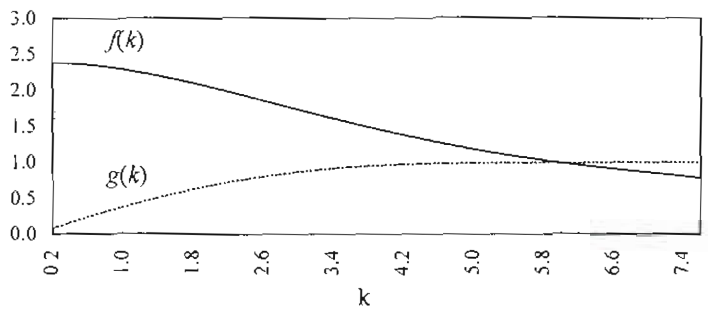

Fig. 1. Regression analysis: Effect of specifications on 6σ nominal centered process.

PROCESS OVERALL EVALUATION

Comprehensive Description of the Problem

Quite often, characterization of process capability refers to a single characteristic resulting from an individual operation, while a product may pass more than one operation. Thus, a product quality in practice, explains the capability of one or more manufacturing/service facility. For instance, management may intend to evaluate a type of machinery, a whole production line or machine cell. Of course, the capability study will be conducted to a large number of operations through one or more sample for each operation (characteristic.) Using different statistical techniques and/or indices is not the problem—the characteristics are often found in different directions and/or different dimensions of measurement. The problem of different measuring units would be encountered in the industries such as chemical and textile industries while the problem of measuring units is inherent in other industries. It is not pessimistic to expect that a machine would demonstrate different PC's for its operations even if tooling and gauging are set carefully. Therefore, an overall evaluation is helpful to discover the source of distorting the full potential capability of a facility.

An overall evaluation requires complex statistical techniques such as multivariate analysis and design of experiment. Nevertheless, a complex analysis may lead to erroneous conclusions. Thus, in this research, a simple methodology is developed. Sufficiency of the application depends on understanding process definition and nature. Under the assumption that each individual operation results individual characteristic, the methodology is aided with the following classification for potential operations:

1-*Dimensionally Related Operations*-A set of operations performed, sequentially or not sequentially to yield a final dimension. Note that, in this class, an operation may distort the results achieved by a predecessor operation.

a-*Assembly operations*-Operations characterized by negative (subtractive) or positive (additive) accumulation of dimensions already finished. In other words, they interact two or more dimensions to create a final result. Instance of negative accumulation appears in assembling a bush and a shaft (an internal dimension) while positive accumulation appears in assembling two or more links, assembling bars, assembling metal sheets, etc. (external dimensions.)

b-*Attached operations*-Operations share their finished points, but each of them refers to a different characteristic. Also, they relate negatively such as milling and grinding a surface or positively such as coating a surface. Some of these operations may be finished in the same station, hence they become hybrid.

2-*Non Related Operations*-A set of operations performed, sequentially or not sequentially by different processing stations without sharing their finished points.

3-*Station Related Operations*-A set of operations performed using the same station (operator, machine, tank, room, etc.) The degree of relation is variable, for example two operations may be performed on the same machine but not the same spindle.

4-*Hybrid Related Operations*-A set consists of operations have both relationships—dimensionally related and station related.

Setting of tools and equipment is an important variable which may puzzle the planner and in turn leads to meaningless quantities. Other subtle variables are gauging precision and personnel practices. Also other factors may be hidden. Moreover, in the most of practical cases, those four classes of operations exist in the same process. Therefore the effective use of the cited classification needs to planner experience to group and differentiate the potential operations; thus in turn reduces the expected fallacious evaluation of the process variation.

Statistical Analysis of the Problem

Before proceeding with merging the individual indices, a screening procedure must be adopted to eliminate odd operations from the overall evaluation. Here, odd operations are defined as operations encounter special setting complexities including tools setting and gauge setting. Odd operations may result characteristics seem to be inexpressive to the overall performance. Therefore the resultant indices should be relatively verified. Appendix C proposes a preliminary statistical method which can be implemented aided with control charts to compare the potential characteristics.

Assembly operations and not related operations

For a group of assembly operations based on C_p index and bilateral specification limits, suppose that $f_i(k_i,q_i)$ be the regression function $(P_a/C_p)_i$ for part i, $f(k,q)$ be the same for the final item, and n be the number of assembled parts. Under the assumption of independence, maximum probability of accepting the final item is given by

$$P_a = \prod_{i=1}^{n}(P_a)_i \,, \tag{1}$$

where $(P_a)_i$ denotes the probability of accepting a part. Substituting by the regression functions,

$$C_p f(k,q) = \prod_{i=1}^{n}(C_p)_i f_i(k_i,q_i) \,, \tag{2}$$

then the overall index is given by

$$C_p = \prod_{i=1}^{n}(C_p)_i \frac{\prod_{i=1}^{n} f_i(k_i,q_i)}{f(k,q)} . \tag{3}$$

It is found experimentally—Appendix B1—that removing the functional fraction in eq. (3) is equivalent to taking geometric mean of the first operand, hence

$$C_p = \sqrt[n]{\prod_{i=1}^{n}(C_p)_i} \,, \tag{4}$$

which is greater than the assembly tolerance range divided by six sigma. Eq. 4 becomes closer as q approaches 0. Eq. (4) could be conducted to other disciplines of indices, but with less accuracy. Also, it is applicable for n non related operations with more accuracy, because the actual probability components in assembly is $n+1$, i.e. the assembly may be found within specifications and rejected. Hence, $(P_a)_i$ substitutes for operation i.

Station related operations

For a group of station related operations under the previous conditions, a similar procedure can be followed. The exception is that a degree of dependence exists, that is all operations belong to the same station. Effect of variances on the acceptance of station output characteristics lessens as tolerance ranges increase while the centers stay near to their processes targets. Note that the target is the center at which the process is aimed.

For a process explained by n operations, suppose that Z is an image output characteristic represents the overall response function of the process such that $Z=G(X_1, X_2, \ldots, X_n\backslash T_i)$, $0 \le Y \le 1$ is an image input characteristic represents the station status which describes the function $X_i=g_i(Y)$, and T_i is the tolerance range specified for the characteristic X_i results from operation i. As mentioned before, X's are assumed random variables approach normal distribution. The relation between those variables and operations characteristics is depicted in Fig. (2-a) as a control diagram in which the tolerance range plays the role of disturbance. The idea of disturbance comes from the fact that a tolerance is an external factor imposed by the designer; in other words if it is not being specified, the PC index cold be considered unlimited whatever the value of a process variation. Also, Fig. (2-b) is a vector imagination for the characteristics relationship with angles explain performance deviation.

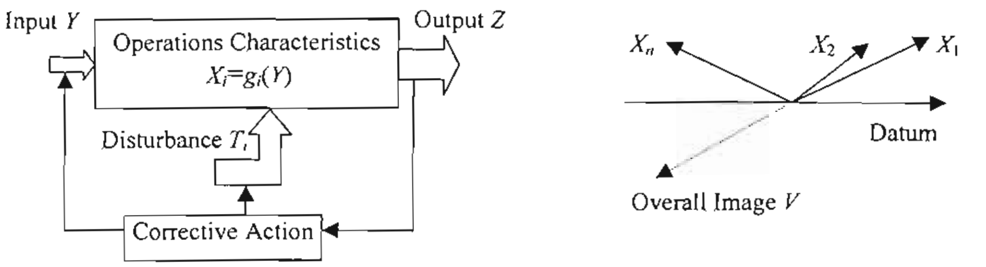

Fig. (2-a) Process control diagram. **Fig. (2-b) Characteristic vector diagram.**

Fig. 2. Relation between process paramcters.

The probability of accepting a final item—defined by Z characteristie—is given by the joint probability

$$P_a = P(Y \in W, X_1 \in T_1, X_2 \in T_2, \ldots, X_n \in T_n), \qquad (5)$$

where W be the acceptable range of status Y which is expressed as a fractional continuous random variable. Although X's are functions of Y, the setting of operations should be supposed independent, then eq. (5) can be approximated as

$$P_a \approx P(Y \in W)P(X_1 \in T_1)P(X_2 \in T_2)\ldots P(X_n \in T_n)$$

$$= P(Y \in W)\prod_{i=1}^{n}(P_a)_i, \qquad (6)$$

where $(P_a)_i = P(X_i \in T_i)$. The quantity $P(Y \in W)$ could be estimated using a pilot sample from the station output—as the fraction of accepted items—or the known conditions of the station. Eq. (6) returns its maximum value at $P(Y \in W) = 1$, then the procedure follows eqs. (1 to 4). However, If $P(Y \in W)$ is not found out, eq. (4) can be used to get the most optimistic prediction of the overall index. Therefore, weighted geometric mean becomes more realistic for the current class of operations; formally

$$C_p = \sqrt[A]{\prod_{i=1}^{n}(C_p)_i^{\alpha_i}}, \qquad (7)$$

where $A = \sum \alpha_i$ and α_i is a weight assigned to the operation i according to its effect on rejecting a station output item. Those weights should be experimented before applying the formula. Also, eq. (7) could be carefully applied to other disciplines of indices. Next, the vector image is used to extract an alternative model described by (eqs. 8 to 12.)

What complieates the problem here is that class of operations results characteristics having no subtractive or additive nature. Thus making it difficult to set an expression for the total variation. However, we can imagine the operations characteristics as vectors in an anticlockwise angular diagram as illustrated in Fig. (2-b). In turn, the tolerance should be set in the direction of its characteristic. Hence, an equivalent vector can be expressed as a function of all characteristics. Let a_i be directional parameters provide an equivalent vector V with dimensionless components such that

$$V = a_1 X_1 + a_2 X_2 + \ldots + a_n X_n. \qquad (8)$$

Whereas the mentioned angles are imaginary, it is proposed to assign $\lambda_i = 1/\mu_{X_i}$ to a_i, where $\mu_{X_i} \neq 0$ is the arithmetic mean of characteristic i, then it becomes

$$V = \lambda_1 X_1 + \lambda_2 X_2 + \ldots + \lambda_n X_n , \tag{9}$$

with mean n and standard deviation

$$\sigma_V = \sqrt{\sum_{i=1}^{n} CV_i^2} , \tag{10}$$

where CV_i is the coefficient of variation of characteristic i. Similar to setting an overall statistical tolerance range for interacting dimensions (Juran and Gryna 1980, p. 303 –310 and references therein) in addition to λ_i analyzer, equivalent tolerance range could be proposed as

$$T_\lambda = \sqrt{\lambda_1^2 T_1^2 + \lambda_2^2 T_2^2 + \ldots + \lambda_n^2 T_n^2} , \tag{11}$$

This formula modifies the statistical approach to accommodate such class of relationships. Referring to the conditions described by eqs. (8 to 11), an overall index can be approximated as

$$C_p = \frac{T_\lambda}{6\sigma_V} . \tag{12}$$

Unfortunately, this alternative is restricted to the index described by eq. (12)

Attached operations and hybrid related operations

The final interrelationship between a number of n attached operations can be defined by as

$$\hat{X} = |X_1 - X_2| + |X_2 - X_3| + \ldots + |X_{n-1} - X_n| , \tag{13}$$

where the order of processing characteristic X_i is considered as declared by the formula. The formula always has minus signs even for additive relationships. If the attached operations are not hybrid, the characteristics X_i may be considered independent, then

$$\sigma_{\hat{X}} = \sqrt{(\sigma_{X_1}^2 + \sigma_{X_n}^2) + 2\sum_{i=2}^{n-1} \sigma_{X_i}^2} . \tag{14}$$

which can be compared with a tolerance formula adapted as

$$T = \sqrt{(T_1^2 + T_n^2) + 2\sum_{i=2}^{n-1} T_i^2} . \tag{15}$$

which presents another modification to the known statistical approach. Referring to the conditions defined by eqs. (13 to 15), an overall index will be at least

$$C_p = Min\{\frac{T}{6\sigma_{\hat{X}}}, \frac{T_n}{6\sigma_{X_n}}\} . \tag{16}$$

Also, the described conditions are restricted to the C_p index. If two or more operations are founded hybrid, eq. (14) should be corrected using the covariance statistic or another correction factor to substitute for dependency. However, eq. (16) applies without significant loss of accuracy because it returns a lower bound.

Finally, an important question appears, what should be done if a manufactured product: passes all classes of operations; passes more than one station; passes all classes of operations and more than one station? The answers are not so easy or unique because the built relationships between the resultant characteristics depend on a lot of factors. However the author introduces helpful guidelines for that concern:

(1) For each station, manipulate each group of attached operations separately. Let the final quantities reduce each group to a single characteristic. Note that if the last operation is attached with an operation on another station, whole group is considered in attach with this operation or its group;
(2) For each station, manipulate the group which comprises only station related operations, which in turn reduces to a single characteristic. Note that if an operation is attached with an operation on another station, whole group is considered in attach with this operation or its group;
(3) Consider the first station as a control panel and replace the operation/group results with the next attached operation/group results up to the last station;
(4) If some operations/groups stay only station related on each station, group them as in step (2) and hence manipulate the results as not related operations/groups over all stations;
(5) Manipulate the groups result in step (3) and the group results in step (4) as station related operations;
(6) Do steps (1 to 5) for each component of the product and evaluate the final assembly.

The consistent formula should be selected carefully at each grouping point. In case of enormous number of operations and relationships, the results may be meaningless if the planner is confused about the classification of operations. A planner may prefer to index each station separately by proceeding steps (1)&(2) and combine the resultant indices to a geometric mean. In this case, the accuracy of using the geometric mean depends on the number of operations attached between each two successive stations. Note that the inherent problem is always how to find an equivalent variation for a group of characteristics.

SIMULATION

The objective of this simulation study is to examine the geometric mean index (eq. 7), to demonstrate whether it can be used as an alternative for attached operations (eq. 16). Monte-Carlo simulation is used to simulate the tolerance range of ten operations respectively. A sample of five observations is made for each characteristic in each run. The distribution used to generate random tolerances was uniform distribution; two parameters must be specified for characteristics: minima and maxima of the tolerance range. Fig. 3 shows a sample of 22 runs of the results which demonstrate a repetitive behavior. Also, the experiment was conducted on a problem generated randomly with tolerance and standard deviation, whereas the results were not significantly different. Appendix D summarizes the calculations of a selected run.

The comparison shown in Fig. 3 proves that both equations approximately have the same trend, although they are slightly different in original values with sum of squares about 0.02. Moreover, this analysis showed that the last characteristic index is slightly different from the overall index if the operations are being normally interacted. However, under these conditions, geometric mean—unity weighted—can't be ignored as an alternative for these operations. This approximation becomes better as the number of characteristics increases, while the planner may be obliged to use such alternative to deal with large problems. Similar computational experiments can be conducted to compare other formulas to discover whether some approximations are valid.

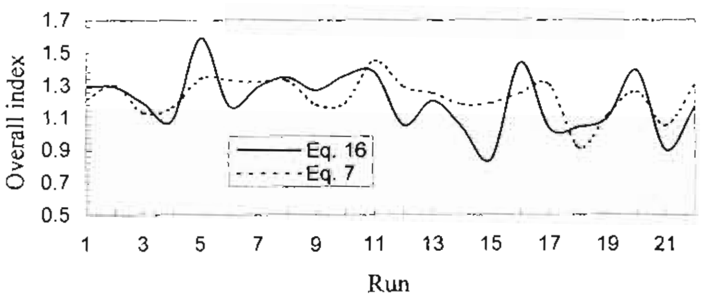

Fig. 3. Simulation results of overall index for ten operations using two formulas.

SUMMARY AND CONCLUSION

The management of manufacturing firms and service plants encounter every day a question about their process's performance and adequacy. They can use many industrial and statistical techniques to control the processes especially control charts. The output of such analysis can be used to predict the process capability attributively and quantitatively, but it dos not provide a complete picture to make a decision of maintaining the current process or adopting a quality improvement program. Successful continuation with a current process or implementation of an improvement program necessitates understanding of the process, the desired product characteristics (Symons and Jacobs 1997), and the relationship between both. Moreover, any analysis should be concluded with the capability study—a comparative study between speeification limits and process variation—to demonstrate whether a process is adequate through a ratio known by process capability index.

The process capability indices which implement single-charaeteristie processes have received a considerable attention in the literature, but combinations of several indices or several capabilities seem to have been ignored. Although those indices are fed directly to make a decision about a simple process and used as a foundation for any quality improvement program, overall indices provide an essential prediction of processes interaction. This paper demonstrates that overall indices can be extracted by using the statistical relationships between characteristics (each characteristic results from a simple process.) It is evidence from the analysis that the nature of the relationships is not always additive or subtractive, therefore the vector diagram is proposed as a trial to create an alternative relationship similar to addition and subtraction.

The developed methodology does not involve complex statistical analysis, therefore it can be easily implemented by the practitioners. Referring to the statistical formulas especially that of tolerance, the methodology seems to be robust to some types of processes but not to others especially those related to coalesced products. Ambiguity can be eliminated by categorizing the operations included and defining the statistical relationship between the resultant characteristics. It is recommended to use the geometric mean for a rough evaluation for all classes of operations before proceeding with the consistent formula. For that concern, the experimental computations proved that the geometric mean with unity weight is an attractive alternative for the quite often performed operations—attached operations. The error appeared in the approximation is expected to be less if the number of operations is considerably large.

Some of the overall evaluation formula is restricted to the most popular process capability index C_p whereas other are common. Such limitation may exist due to the nature of specifications and/or the nature of statistical relationship between characteristics which in turn explains the manufacturing relationship. In spite of limitations, this paper incepts a new topic in process capability studies. It can be used as a foundation for developing data bases and expert systems. The author himself has a plan to build another methodology actuated with genetic algorithm to overcome some obstacles which faced him during this study.

APPENDIX A

Standard Process Capability Indices

The objective of process capability indices is to provide a relative quantitative prediction of process adequacy. Commonly used standard indices are defined as follows:

For bilateral specification limits,

$$C_p = \frac{USL - LSL}{6\sigma_X}, \qquad \text{A-1}$$

and

$$C_{pk} = C_p - \frac{|m - \mu_X|}{3\sigma_X}, \qquad \text{A-2}$$

For unilateral specification limits,

$$C_{PU} = \frac{USL - \mu_X}{3\sigma_X}, \qquad \text{A-3}$$

and

$$C_{PL} = \frac{|LSL - \mu_X|}{3\sigma_X}. \qquad \text{A-4}$$

where

X random variable denotes the measured characteristic of the process;
USL upper specification limit;
LSL lower specification limit;
m target value of the process center;
μ_X actual center line of the process;
σ_X process standard deviation (population— individual measurements— variation.)

Comments: the process must be in a state of statistical control; A-2 index account for the process having center being off target; normally, the process is considered capable to meet the specifications if it indexes at least 1; quite often, the characteristic being monitored follows known probability distribution but the normally distributed characteristic is effectively explained by the cited indices.

APPENDIX B

B1. A sample of regression and correlation analysis (Excel 97)

For approximation: Cp k/6= 1.00 1.02
$f(k)$=-0.242k+2.4773 is used Multiplier k = 6.00 6.10

k	Cp	Pa	Exp(Pa)	Pa/Cp	Exp(Pa/Cp)	n	Cp all	Cp all
0.00	0.0000	0.0000	1.0000	...	...	...	...	...
0.20	0.0333	0.0796	1.0829	2.3880	10.8917	1	1.00	1.02
0.40	0.0667	0.1586	1.1719	2.3790	10.7941	2	1.01	1.01
0.60	0.1000	0.2340	1.2636	2.3400	10.3812	3	1.02	1.01
0.80	0.1333	0.3108	1.3645	2.3310	10.2882	4	1.02	1.01
1.00	0.1667	0.3830	1.4667	2.2980	9.9543	5	1.02	1.00

Continued: **Appendix B1**

1.20	0.2000	0.4514	1.5705	2.2570	9.5544	6	1.02	1.00
1.40	0.2333	0.5160	1.6753	2.2114	9.1287	7	1.02	1.00
1.60	0.2667	0.5762	1.7793	2.1608	8.6776	8	1.02	1.00
1.80	0.3000	0.6318	1.8810	2.1060	8.2153	9	1.02	1.00
2.00	0.3333	0.6826	1.9790	2.0478	7.7508	10	1.02	1.00
2.20	0.3667	0.7286	2.0722	1.9871	7.2943	11	1.02	1.00
2.40	0.4000	0.7698	2.1593	1.9245	6.8517	12	1.02	1.00
2.60	0.4333	0.8064	2.2398	1.8609	6.4297	13	1.02	1.00
2.80	0.4667	0.8384	2.3127	1.7966	6.0289	14	1.02	1.00
3.00	0.5000	0.8664	2.3783	1.7328	5.6565	15	1.02	1.00
3.20	0.5333	0.8904	2.4361	1.6695	5.3095	16	1.02	1.00
3.40	0.5667	0.9108	2.4863	1.6073	4.9893	17	1.02	1.00
3.60	0.6000	0.9282	2.5300	1.5470	4.6974	18	1.02	1.00
3.80	0.6333	0.9426	2.5666	1.4883	4.4296	19	1.02	1.00
4.00	0.6667	0.9544	2.5971	1.4316	4.1854	20	1.02	1.00
4.20	0.7000	0.9642	2.6227	1.3774	3.9647	21	1.02	1.00
4.40	0.7333	0.9722	2.6438	1.3257	3.7649	22	1.02	1.00
4.60	0.7667	0.9786	2.6607	1.2764	3.5838	23	1.02	1.00
4.80	0.8000	0.9836	2.6741	1.2295	3.4195	24	1.02	1.00
5.00	0.8333	0.9876	2.6848	1.1851	3.2711	25	1.02	1.00
5.20	0.8667	0.9906	2.6928	1.1430	3.1362	26	1.02	1.00
5.40	0.9000	0.9930	2.6993	1.1033	3.0142	27	1.02	1.00
5.60	0.9333	0.9948	2.7042	1.0659	2.9033	28	1.02	1.00
5.80	0.9667	0.9962	2.7080	1.0306	2.8026	29	1.02	1.00
6.00	1.0000	0.9974	2.7112	0.9974	2.7112	30	1.02	1.00
6.20	1.0333	0.9980	2.7129	0.9658	2.6269	31	1.02	1.00
6.40	1.0667	0.9986	2.7145	0.9362	2.5502	32	1.02	1.00
6.60	1.1000	0.9990	2.7156	0.9082	2.4798	33	1.02	1.00
6.80	1.1333	0.9994	2.7167	0.8818	2.4153	34	1.02	1.00
7.00	1.1667	0.9996	2.7172	0.8568	2.3556	35	1.02	1.00
7.20	1.2000	0.9996	2.7172	0.8330	2.3002	36	1.02	1.00
7.40	1.2333	0.9998	2.7177	0.8106	2.2494	37	1.02	1.00
7.60	1.2667	0.9998	2.7177	0.7893	2.2019	38	1.02	1.00

Primal correlation analysis:				*Average Index*	1.02	1.00
Correlation:	k&Pa	=	*0.8664*	Error = ±1%		
Correlation:	Cp&Exp(Pa)	=	*0.9024*		2.48	Absc.
Correlation:	k&Pa/Cp	=	*-0.9926*	*Linear Regression*	-0.24	Slope
Correlation:	k&Exp(Pa/Cp)	=	*-0.9609*			

B2. Bilateral shift of the process center from the target

Suppose that the process target (aimed center) is m, hence the actual process center, lower specification limit, upper specification limit, lower standard normal variate, and upper standard normal variate are respectively

$$\mu_X = m \pm q\sigma_X, \quad \text{B2-1}$$

$$LSL = m - \frac{k\sigma_X}{2}, \quad \text{B2-2}$$

$$USL = m + \frac{k\sigma_X}{2}, \quad \text{B2-3}$$

$$Z_L = \frac{LSL - \mu_X}{\sigma_X} = -\frac{k}{2} \pm q, \quad \text{B2-4}$$

and

$$Z_U = \frac{USL - \mu_X}{\sigma_X} = \frac{k}{2} \pm q. \quad \text{B2-5}$$

Note that in eqs. B2-4 and B2-5, the sign is set opposite the shift sign and the probability P_a is calculated as the area between Z_L and Z_U.

APPENDIX C

Preliminary Statistical Analysis

Table 1 shows a part of analyzing the relationship between process characteristics resulting from a single station. The analysis is based on coefficients of variation and individual indices which are used to calculate some effective quantities. The purpose is to exhibit a comparative study in order to screen the operations which return unrealistic indices from the view of the overall performance of the station.

Table 1. Relationship analysis of station characteristics.

Oper.	T_i	I_i	CV_i	1	2	3	...	n	$RMS_i(I_i - I_j)_{i \neq j}$	$D_i = \lvert I_i - I'_i \rvert$
1	T_1	I_1	CV_1		$I_1 - I_2$	$I_1 - I_3$	...	$I_1 - I_n$	RMS_1	D_1
2	T_2	I_2	CV_2	$I_2 - I_1$		$I_2 - I_3$	...	$I_2 - I_n$	RMS_2	D_2
3	T_3	I_3	CV_3	$I_3 - I_1$	$I_3 - I_2$		...	$I_3 - I_n$	RMS_3	D_3
...	...	...	...	...	...	...	...	...	...	...
n	T_n	I_n	CV_n	$I_n - I_1$	$I_n - I_2$	$I_n - I_3$	...		RMS_n	D_n
Aver.		I_{aver}	CV_{aver}	$aver_1$	$aver_2$	$aver_3$	...	$aver_n$	RMS_{aver}	D_{aver}

n number of operations (characteristics) included in the process;
I_i general process capability index for operation i;
CV_i coefficient of variation of operation i characteristic;
RMS root mean square;
D_i absolute shift of operation i index from the other operations mean index;
I'_i mean index excluding i as $\sum_{j \neq i} I_i$.

The main assumptions underlying such analysis can be stated as: (i) the dependency between characteristics should be reduced as possible (a production point results from operation settings), (ii) the process is in a state of statistical control for each characteristic, (iii) the actual process center for each characteristic is near to the nominal dimension, and (iv) the specification limits of each characteristic are checked statistically (Juran and Gryna 1993). The procedure of screening is summarized below:

(1) Carry out an individual analysis for each operation to make a decision about the adequacy, tolerance range validity, and process shift. Principally, the operations which register indices below the required minimum values should be verified. Such verification discovers the causes of deviation which may be consolidated to inadequacy of the process itself and/or the measuring sets;

(2) Define a quantity to represent an index for the mutual difference between the capability indices. Here a parameter, $\ddot{I}$, is proposed to be

$$\ddot{I} = \begin{vmatrix} 0 & I_{12} & I_{13} & \cdots & I_{1n} \\ I_{21} & 0 & I_{23} & \cdots & I_{2n} \\ I_{31} & I_{32} & 0 & \cdots & I_{3n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ I_{n1} & I_{n2} & I_{n3} & \cdots & 0 \end{vmatrix} \geq 0, \qquad \text{C-1}$$

where $I_{ij} = |I_i - I_j|$. The index $\ddot{I}$ equals zero in one of two cases—equality of all indices or equality of at most two rows of the difference matrix. However, the control is defined as a smaller $\ddot{I}$ identifies a better relationship while the technique may be terminated at this step. This parameter can help a planner to identify the shift between the station operations in process adequacy. Up to date, the author does not recommend values for such index, but he leaves that to the planner sense. However, If $\ddot{I}$ registers a significant value proceed to the next step;

(3) Arrange the operations in an ascending order according to their coefficient of variations. Coefficients of variation give an expectation of the operations that may be comparatively odd;

(4) Arrange the operations in an ascending order according to RMS_i;

(5) Arrange the operations in an ascending order according to D_i;

(6) Rank the operations in an inverse order according to (steps 3 to 5.) Find the total rank of each operation. The odd operations are expected to be at the tail of the current list. Each extreme explains different conditions; quite often, top extreme explains representative operations while bottom extreme explains inadequacy of measuring sets and/or incompatibility to other operations. Note that the ranking increment—in steps 3 to 6—should be assigned carefully by the planner. After this step, the control is transferred to the overall evaluation.

APPENDIX D

A Sample of Simulating an Overall Index (C_p): Comparison between Geometric Mean & Equivalent Variance Formulas (Eq. 16&Eq.7) (Excel 97)

			Random Observations for Tolerance					*Mean*		
Oper.	σ_i	σ_i^2	1	2	3	4	5	T_i	T_i^2	C_p
1	0.30	0.09	0.83	0.08	3.46	0.12	3.50	1.60	2.55	0.89
2	0.40	0.16	5.16	3.25	2.28	4.90	1.09	3.34	11.14	1.39
3	0.20	0.04	2.57	0.58	2.74	0.42	1.22	1.51	2.27	1.25
4	0.10	0.01	0.81	0.17	0.86	0.02	1.45	0.66	0.44	1.11
5	0.10	0.01	0.90	0.50	0.45	0.89	0.41	0.63	0.40	1.05
6	0.20	0.04	0.17	2.68	0.72	1.74	2.82	1.63	2.64	1.35

Continued: **Appendix D**

7	0.10	0.01	0.02	0.91	0.65	0.84	0.29	0.54	0.29	0.90
8	0.20	0.04	0.20	1.96	2.75	1.70	0.25	1.37	1.88	1.14
9	0.01	0.00	0.10	0.12	0.09	0.13	0.06	0.10	0.01	1.66
10	0.50	0.25	0.26	4.53	6.42	2.40	1.58	3.04	9.24	1.01
SUM		0.96							49.92	

$\sigma_{\dot{x}}$ = 0.98 *Overall Standard Deviation and Tolerance* T = 7.07

Calculations:

$\frac{T}{6\sigma_{\dot{x}}}$ =1.20 and $\frac{T_{10}}{6\sigma_{10}}$ =1.01

C_p from eq.16 =1.01

$Min(\frac{T}{6\sigma_{\dot{x}}}, \frac{T_{10}}{6\sigma_{10}})$

C_p from eq.7 =1.15

Geometric Mean

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